Interative Hybrid Probabilistic Model Counting

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Relational Probabilistic Problems

- Many probabilistic problems
  - require hybrid reasoning
  - have logical structure
  - deal with rare observed events, e.g. diagnostic problems
- Representation of such problems: probabilistic logics
  - capture and allow exploiting structure
  - no direct support for hybrid reasoning
  - can be extended with continuous distributions

Probabilistic Logic Programming

- Knowledge base: Probabilistic Facts & Deterministic Rules (Sato's Distribution Semantics) [Sato, 1995]
  - Probabilistic Facts

$$0.2 : low\_price(apple)$$

  - Deterministic Rules (Closed-World Assumptions, à la *Prolog*)

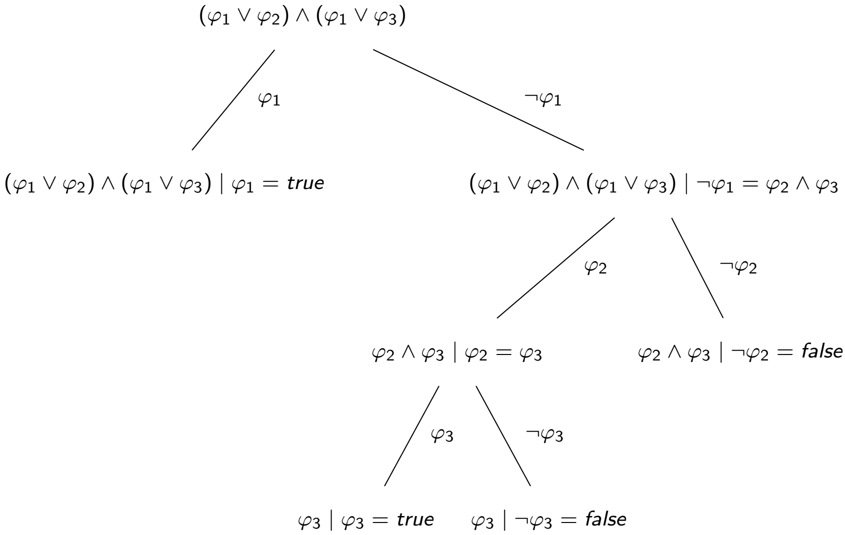
$$buy(Fruit) \leftarrow low\_price(Fruit)$$

  means

$$buy(Fruit) \iff low\_price(Fruit)$$

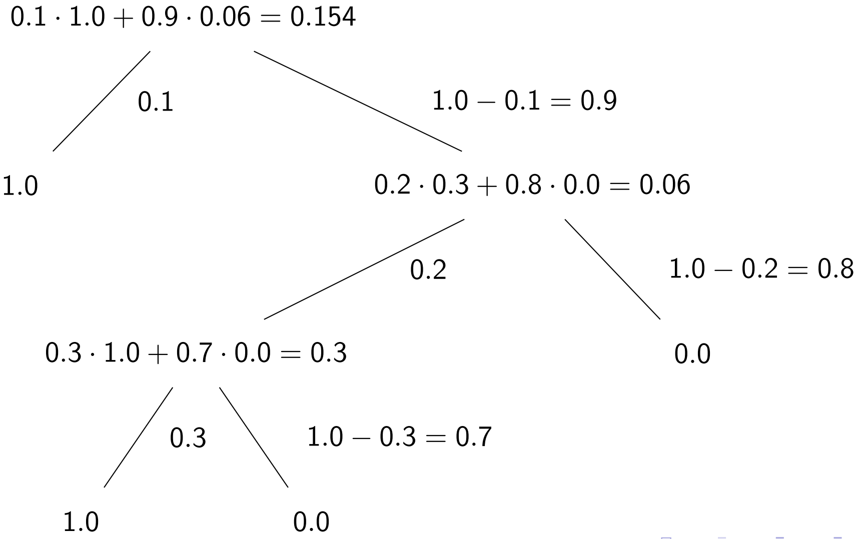
  - $P(buy(apple)) = P(low\_price(Fruit)) = 0.2$
  - Expressive enough for *Bayesian Networks*
  - Exact inference feasible for many real worlds problems by transforming the problem into a weighted model counting (WMC) problem

WMC: based on a DPLL-like procedure



WMC on this tree

$P(\varphi_1) = 0.1 \qquad P(\varphi_2) = 0.2 \qquad P(\varphi_3) = 0.3$



Hybrid Probabilistic Reasoning

- Hybrid probabilistic logic programs

$$fails(Comp) \leftarrow \mathbf{FailCause}(Comp, Cause) = true$$
$$fails(Comp) \leftarrow \mathbf{Temp} > \mathbf{Limit}(Comp)$$
$$fails(Comp) \leftarrow subcomp(Subcomp, Comp),\ fails(Subcomp)$$

$$\mathbf{FailCause}(engine, noFuel) \sim \{0.0002\colon\ true, 0.9998\colon\ false\}$$
$$\mathbf{Temp} \sim \Gamma(20.0, 5.0)$$
$$\mathbf{Limit}(engine) \sim \mathcal{N}(65.0, 5.0)$$
$$subcomp(fuelPump, engine)$$
$$\mathbf{Limit}(fuelPump) \sim \mathcal{N}(80.0, 5.0)$$
$$\ldots$$

- Probability of query event q, given evidence e: $P(q \mid e)$

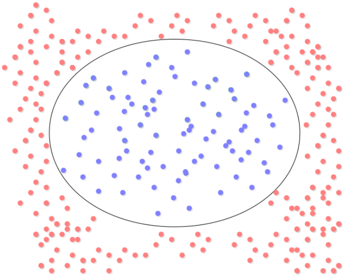
$$P\big(fails(fuelPump) \mid fails(engine)\big)$$

- How to do inference?

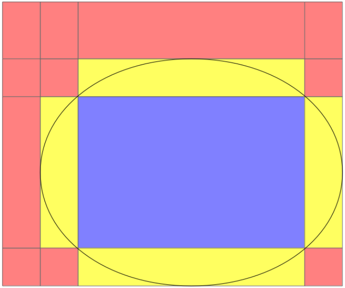
Inference Methods

Method	Exact	Rejection / Importance Sampling	MCMC	**IHPMC**
Works for	finite problems only	(virtually) all problems	(virtually) all problems	large class of hybrid problems
Quality guarantee	no error	probabilistic	none	bounded error
Structure-sensitive	yes	no	hand-tailored solution often required	yes
Sensitive to rare evidence	no	yes	no	no

IHPMC Basic Idea



$$\widetilde{P}(q) = \frac{\#{\color{blue}\bullet}}{\#{\color{blue}\bullet} + \#{\color{red}\bullet}}$$



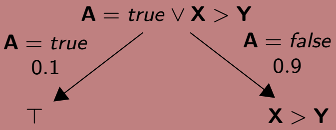
$$\underline{P}(q) = P({\color{blue}\blacksquare})$$

$$\overline{P}(q) = P({\color{blue}\blacksquare}) + P({\color{yellow}\blacksquare})$$

$$\widetilde{P}(q) = P({\color{blue}\blacksquare}) + P({\color{yellow}\blacksquare})/2 \pm P({\color{yellow}\blacksquare})/2$$

Exploiting Structure

Hybrid Probability Tree (HPT)

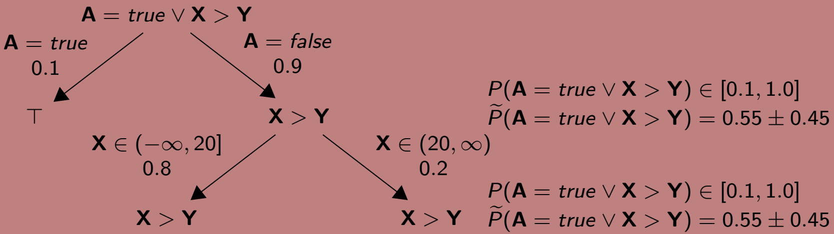


$$P(\mathbf{A} = \textit{true} \vee \mathbf{X} > \mathbf{Y}) \in [0.1, 1.0]$$
$$\widetilde{P}(\mathbf{A} = \textit{true} \vee \mathbf{X} > \mathbf{Y}) = 0.55 \pm 0.45$$

- Similar to binary WMC
- Exploits logical structure
- Search towards hyperrectangles with high probability

Exploiting Structure

Hybrid Probability Tree (HPT)



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Exploiting Structure

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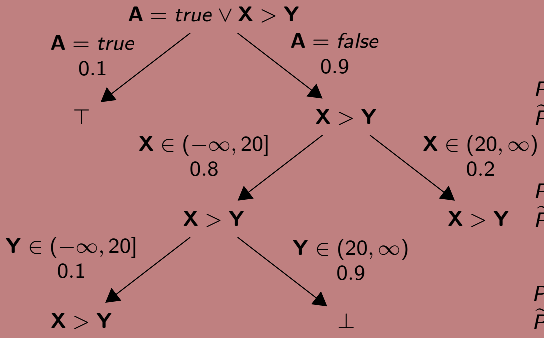

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$P(\mathbf{A} = \mathit{true} \vee \mathbf{X} > \mathbf{Y}) \in [0.1, 0.352]$
$\widetilde{P}(\mathbf{A} = \mathit{true} \vee \mathbf{X} > \mathbf{Y}) = 0.226 \pm 0.126$

- Similar to binary WMC
- Exploits logical structure
- Search towards hyperrectangles with high probability

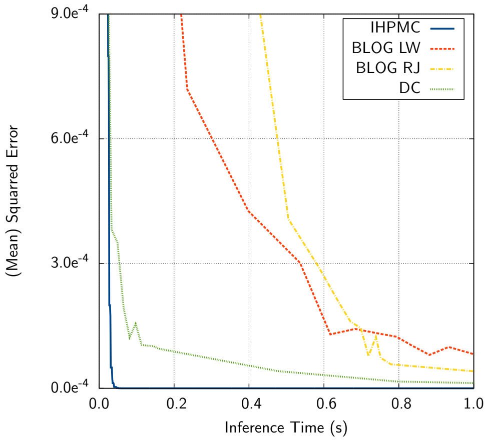
Theoretical property

Approximations with arbitrary precision can be computed

For all events q and e and every maximal error ϵ, IPHMC can in finite time find an approximation such that:

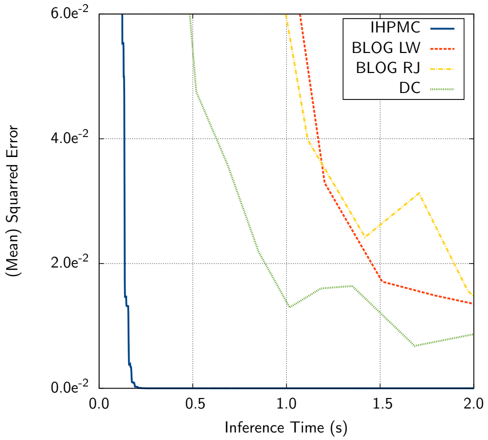
$$P(q \mid e) - \underline{P}(q \mid e) \leq \epsilon \text{ and}$$
$$\overline{P}(q \mid e) - P(q \mid e) \leq \epsilon$$

No Evidence



$$P(fails(9)),\ p = 0.01,\ \mu = 60.0$$

Rare Observed Event



$$P(\mathit{fails}(9) \mid \mathit{fails}(0)),\ p = 0.0001,\ \mu = 60.0$$

Conclusions

- IHPMC provides alternative to sampling
  - insensitive to rare observed events
  - no hand-tailoring
  - bounded error
  - may fail, **but lets the user know!**
- Try it: `http://www.steffen-michels.de/ihpmc`