Branching Bisimulation Games

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Open University of the Netherlands & Radboud Universiteit Nijmegen

Joint work with Tim Willemse (TU/e) and David de Frutos Escrig (UCM Madrid)

25 October 2016

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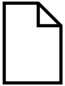
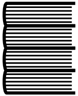
Context



Specification



Implementation

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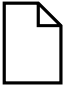

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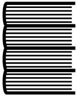

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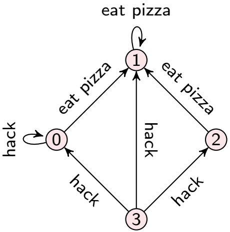

$L = \langle S, Act, \rightarrow \rangle$ Labelled Transition System

Strong Bisimulation

A strong bisimulation is a relation $R \subseteq S \times S$ on the states of an LTS $\langle S, Act, \rightarrow \rangle$ such that when $s\,R\,t$:

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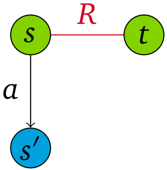

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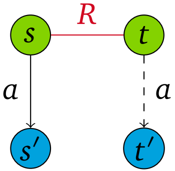

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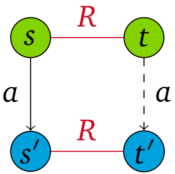

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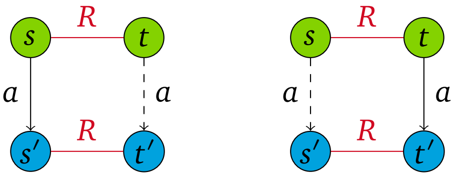

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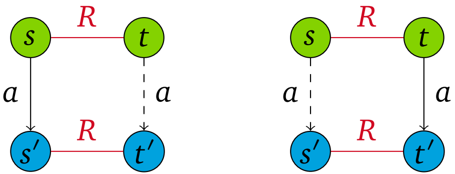

States s, t are bisimilar ($s \leftrightarrow t$) iff $s\ R\ t$ for some bisimulation R

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Strong Bisimulation Games

Stirling's bisimulation game

- Ehrenfeucht-Fraïssé game
- player Spoiler (S) tries to disprove bisimilarity of s and t
- player Duplicator (D) tries to prove bisimilarity of s and t
- S wins all plays in which D 'gets stuck'
- D wins all other plays, i.e. both infinite plays and all plays in which S 'gets stuck'
- game is played in rounds (*ad infinitum* if possible)

Strong Bisimulation Games

Round starting in $[(s,t)]$:

1. S moves from configuration $[(s,t)]$ by:
   - selecting $s \xrightarrow{a} s'$ and moving to $\langle (s,t),(a,s') \rangle$, *or*
   - selecting $t \xrightarrow{a} t'$ and moving to $\langle (t,s),(a,t') \rangle$
2. D responds from configuration $\langle (u,v),(a,u') \rangle$ by:
   - moving $v \xrightarrow{a} v'$ and continue in configuration $[(u',v')]$.

$s \Leftrightarrow t$ iff Duplicator has a strategy to win all plays starting in (s,t)

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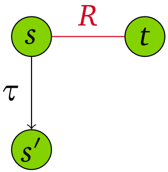

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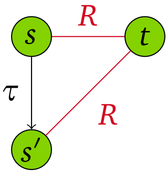

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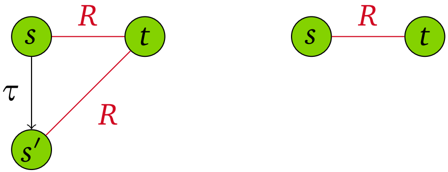

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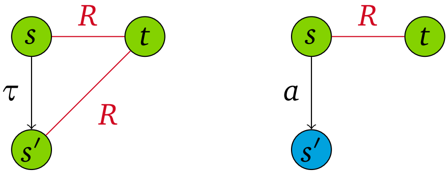

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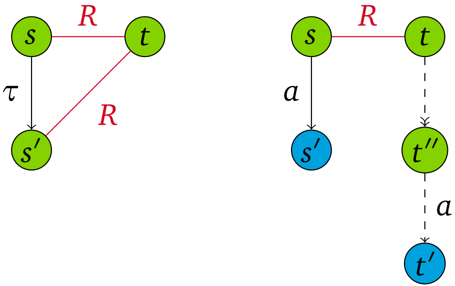

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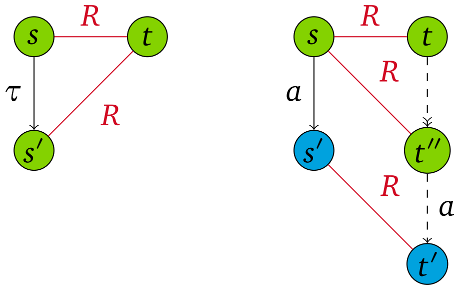

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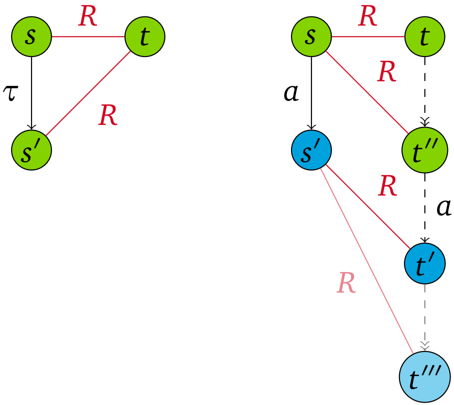

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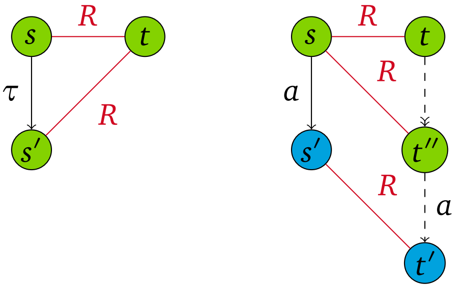

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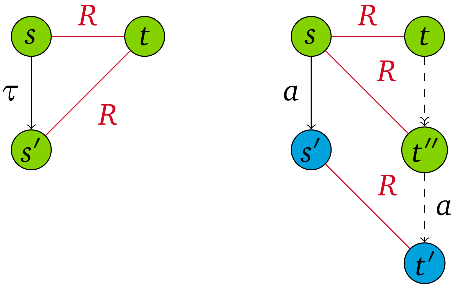

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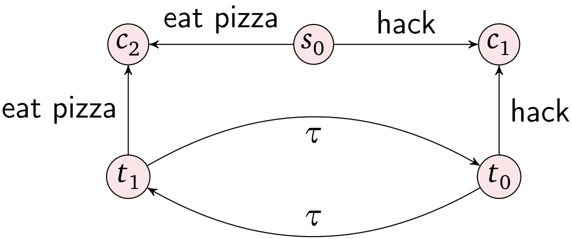

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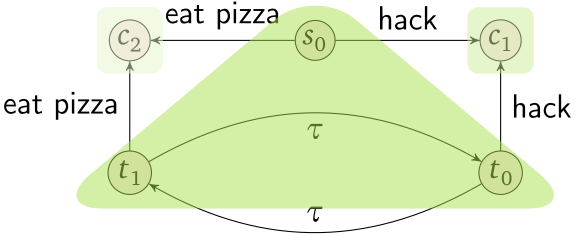

Branching Bisimulation Games

An attempt by Bulychev et al.

- S moves from configuration $[(s,t)]$ by:
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S wins all plays in which D gets stuck, D wins all other plays

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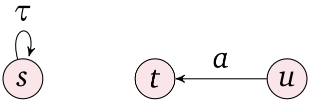
Branching Bisimulation Games

Problem with Bulychev's Definition



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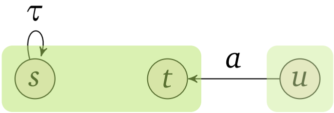

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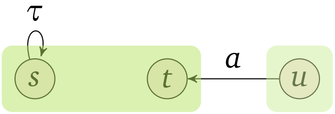

$u \overset{?}{\leftrightarrow}_b s$

- $[(u,s)]$
  - $\rightarrow \langle (u,s),(a,t)\rangle \rightarrow [(u,s)] \rightarrow \cdots$
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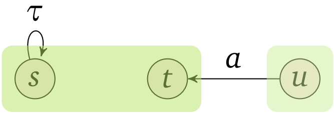

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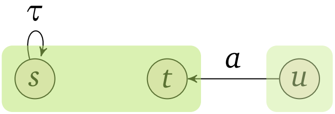

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Definition only works for LTS without divergence

Branching Bisimulation Games

- observation: after one τ-step by Duplicator, we forget what Duplicator was mimicking

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We play on configurations with

- challenges $c \in (A \times S) \cup \{\dagger\}$
- rewards $r \in \{*, \checkmark\}$

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D wins a play if S gets stuck, and all infinite plays

Branching bisimulation games

- S moves from configuration $[(s,t),c,r]$ by:
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D wins a play if S gets stuck, or she gets infinitely many $\checkmark$ rewards

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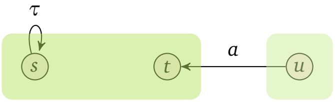
Rewards for Duplicator



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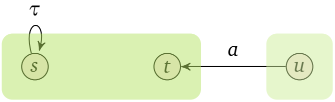

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- $[(u,s),\dagger,*] \rightarrow \langle(u,s),(a,t),*\rangle \rightarrow [(u,s),(a,t),*] \rightarrow \cdots$

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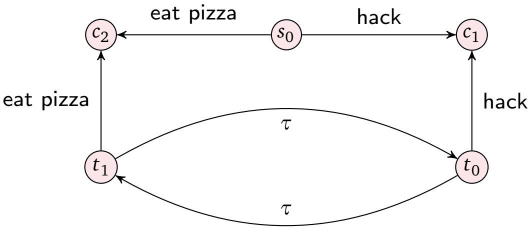
Not quite there yet...



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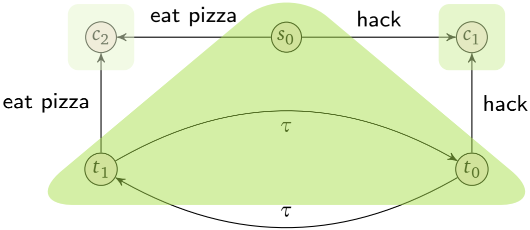

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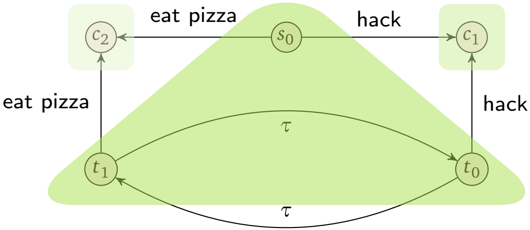

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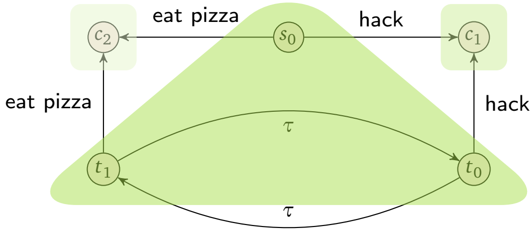

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$[(s_0, t_1), \dagger, *] \to \langle (s_0, t_1), (\text{hack}, c_1), * \rangle \to [(s_0, t_0), (\text{hack}, c_1), *] \to \langle (s_0, t_0), (\text{eat pizza}, c_1), * \rangle \to [(s_0, t_1), (\text{eat pizza}, c_1), *] \to \cdots$

Branching bisimulation games

Punishing Spoiler

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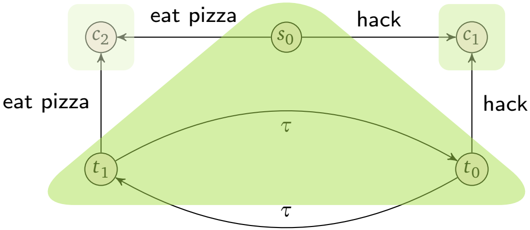
Example



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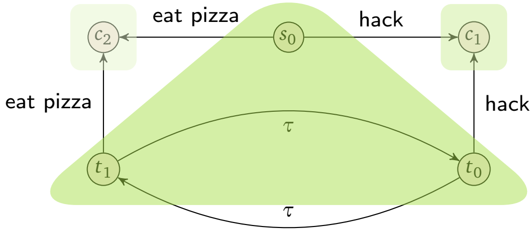

Reconsider the case where S alternates challenge between eat pizza and hack:

$[(s_0, t_1), \dagger, *] \to \langle (s_0, t_1), (\text{hack}, c_1), * \rangle \to [(s_0, t_0), (\text{hack}, c_1), *] \to$
$\langle (s_0, t_0), (\text{eat pizza}, c_1), \checkmark \rangle \to [(s_0, t_1), (\text{eat pizza}, c_1), *] \to \cdots$

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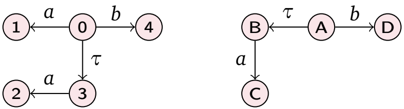
Branching Bisimulation Games

Example



Spoiler (computer) plays against Duplicator (you) showing $0 \not\leftrightarrow_b A$:

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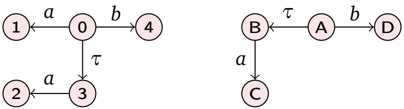
Branching Bisimulation Games

Example

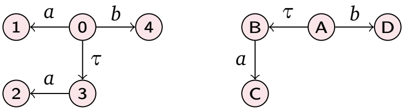


Spoiler (computer) plays against Duplicator (you) showing $0 \not\leftrightarrow_b A$:

`Spoiler challenges` $0 \xrightarrow{a} 1$

Branching Bisimulation Games

Example



Spoiler (computer) plays against Duplicator (you) showing $0 \not\Leftrightarrow_b A$:

`Spoiler challenges` $0 \xrightarrow{a} 1$

`Your response:` $A \xrightarrow{\tau} B$`; you continue playing from` $((0, B), (a, 1))$

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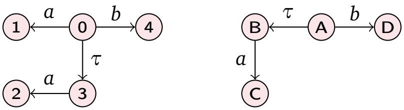
Branching Bisimulation Games

Example

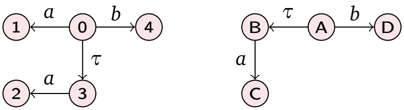


Spoiler (computer) plays against Duplicator (you) showing $0 \not\leftrightarrow_b A$:

`Spoiler challenges` $0 \xrightarrow{a} 1$
`Your response:` $A \xrightarrow{\tau} B$`; you continue playing from` $((0,B),(a,1))$
`Spoiler drops challenge` $(a,1)$ `and challenges` $0 \xrightarrow{b} 4$`. You earn` ✓

Branching Bisimulation Games

Example



Spoiler (computer) plays against Duplicator (you) showing $0 \not\leftrightarrow_b A$:

Spoiler challenges $0 \xrightarrow{a} 1$

Your response: $A \xrightarrow{\tau} B$; you continue playing from $((0,B),(a,1))$

Spoiler drops challenge $(a,1)$ and challenges $0 \xrightarrow{b} 4$. You earn ✓

You cannot respond. You lose.

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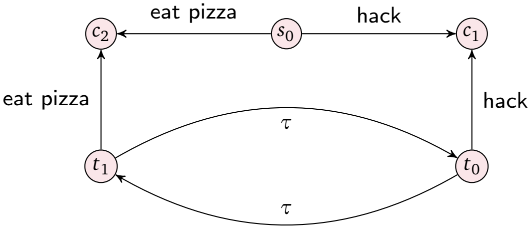
Extensions

Branching Bisimulation with Explicit Divergence



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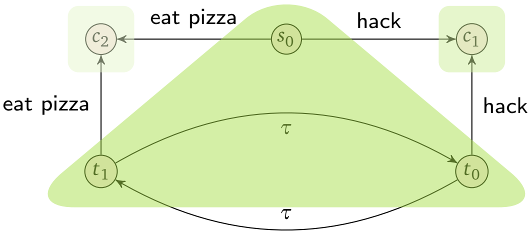
Extensions

Branching Bisimulation with Explicit Divergence



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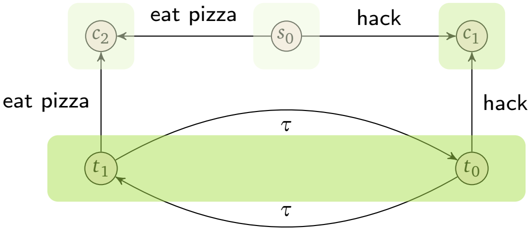
Extensions

Branching Bisimulation with Explicit Divergence



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Extensions

Branching Bisimulation with Explicit Divergence

- S moves from configuration $[(s,t),c,r]$ by:
  - selecting $s \xrightarrow{a} s'$ and moving to
    - $\langle (s,t),(a,s'),* \rangle$ if $c = (a,s')$ or $c = \dagger$, *and to*
    - $\langle (s,t),(a,s'),\checkmark \rangle$, otherwise; *or*
  - selecting $t \xrightarrow{a} t'$ and moving to $\langle (t,s),(a,t'),\checkmark \rangle$
- D responds from a configuration $\langle (u,v),(a,u'),r \rangle$ by:
  - not moving if $a = \tau$ and propose configuration $[(u',v),\dagger,\checkmark]$, or
  - moving $v \xrightarrow{a} v'$ if available and continue in $[(u',v'),\dagger,\checkmark]$, or
  - moving $v \xrightarrow{\tau} v'$ if possible and continue in $[(u,v'),(a,u'),*]$

D wins a play if S gets stuck, or she gets infinitely many $\checkmark$ rewards

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Extensions

Branching Bisimulation with Explicit Divergence

- S moves from configuration $[(s,t),c,r]$ by:
  - selecting $s \xrightarrow{a} s'$ and moving to
    - $\langle(s,t),(a,s'),*\rangle$ if $c=(a,s')$ or $c=\dagger$, *and to*
    - $\langle(s,t),(a,s'),\checkmark\rangle$, otherwise; *or*
  - selecting $t \xrightarrow{a} t'$ and moving to $\langle(t,s),(a,t'),\checkmark\rangle$
- D responds from a configuration $\langle(u,v),(a,u'),r\rangle$ by:
  - not moving if $a=\tau$ and propose configuration $[(u',v),\dagger,*]$, or
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D wins a play if S gets stuck, or she gets infinitely many $\checkmark$ rewards

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Extensions

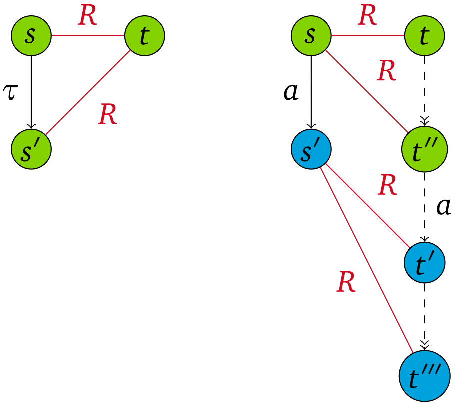
Branching Simulation

- S moves from configuration $[(s,t),c,r]$ by:
  - selecting $s \xrightarrow{a} s'$ and moving to
    - $\langle (s,t),(a,s'),* \rangle$ if $c = (a,s')$ or $c = \dagger$, *and to*
    - $\langle (s,t),(a,s'),\checkmark \rangle$, otherwise; *or*
  - ~~selecting $t \xrightarrow{a} t'$ and moving to $\langle (t,s),(a,t'),\checkmark \rangle$~~
- D responds from a configuration $\langle (u,v),(a,u'),r \rangle$ by:
  - not moving if $a = \tau$ and propose configuration $[(u',v),\dagger,\checkmark]$, or
  - moving $v \xrightarrow{a} v'$ if available and continue in $[(u',v'),\dagger,\checkmark]$, or
  - moving $v \xrightarrow{\tau} v'$ if possible and continue in $[(u,v'),(a,u'),*]$

D wins a play if S gets stuck, or she gets infinitely many $\checkmark$ rewards

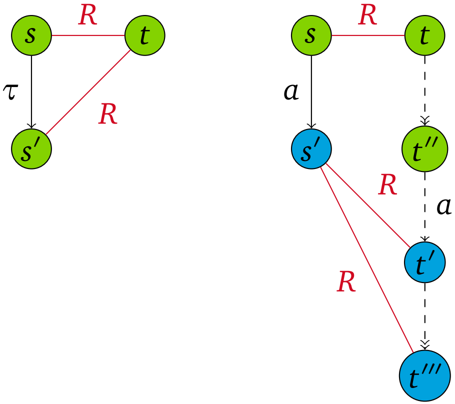
Extensions

Other weak behavioural equivalences



Extensions

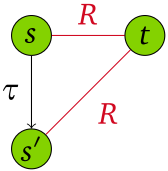
Other weak behavioural equivalences

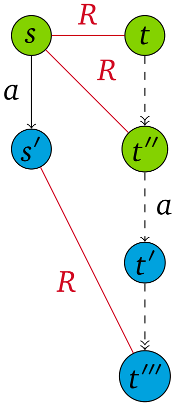


delay bisimulation

Extensions

Other weak behavioural equivalences





delay bisimulation η-bisimulation

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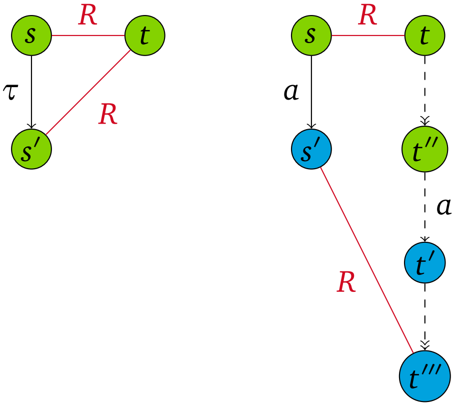
Extensions

Other weak behavioural equivalences



delay bisimulation η-bisimulation   weak bisimulation

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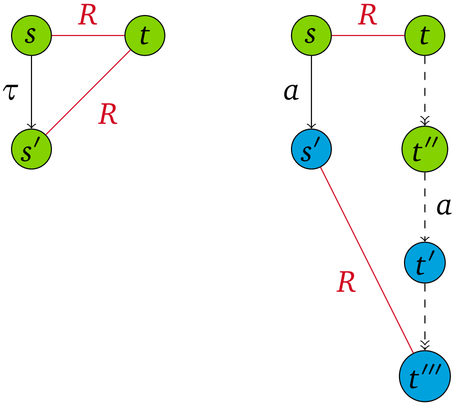
Extensions

Other weak behavioural equivalences



delay bisimulation η-bisimulation   weak bisimulation

Summary

- Presented games for:
  - Branching bisimulation
  - Divergence preserving branching bisimulation
  - Branching simulation
- Require no preprocessing of input LTS
- Spoiler's winning strategy explains why games are not (bi)similar; this enables debugger-like applications

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Future work

- Interactive implementation of our games (proof-of-concept available)

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Future work

- Interactive implementation of our games (proof-of-concept available)
- Applications in education?

Thank you