

# Quantifying voter-controlled privacy

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Trustworth Voting project, University of Surrey

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- Express voting systems formally (syntax, semantics)
- Parametrise over voters' choice  $\gamma$
- Determine trace set
- Compare trace set for  $\gamma_1$  with trace set for  $\gamma_2$

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- secret initial knowledge
- encryption,  $\{m\}_k, \{m\}_{pk(A)}$
- *homomorphic encryption*,  $\{\!\{m\}\!}_k$
- *blind signatures*,  $\llbracket m \rrbracket_k$

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- secret initial knowledge
- encyption,  $\{m\}_k, \{m\}_{pk(A)}$
- *homomorphic encyption*,  $\{\{m\}\}_k$
- *blind signatures*,  $\llbracket m \rrbracket_k$
  
- privacy-enhancing communication
  - a. public channel
  - b. anonymous channel
  - c. untappable channel
    - authority → voter
    - voter → authority
    - voter ↔ authority

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- 1 voter, 1 vote.
- every vote has equal weight.
- election process is phased.
- how voters vote is given ( $\gamma$ ).

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- voters  $v \in \mathcal{V}$ , candidates  $c \in \mathcal{C}$
- choice function  $\gamma: \mathcal{V} \rightarrow \mathcal{C}$
- variables  $\text{var} \in \text{Vars}$ , keys  $k \in \text{Keys}$ , nonces  $n \in \text{Nonces}$
- pairing, encryption

Terms:  $\varphi ::= \text{var} \mid c \mid n \mid k \mid (\varphi_1, \varphi_2) \mid \{\varphi\}_k.$

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Terms:  $\varphi ::= \text{var} \mid c \mid n \mid k \mid (\varphi_1, \varphi_2) \mid \{\varphi\}_k.$

- syntactical equivalence:  $\varphi_1 = \varphi_2$
  - $\text{vc} \in \text{Vars}$  parametrises choice
  - variable substitution:  $\sigma = \text{var} \mapsto \varphi_1$   
application to  $\varphi_2$ :  $\sigma(\varphi_2)$
- No bound variables!

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First idea:

$$\text{match}(\varphi_{cl}, \varphi_o) \equiv$$

$$\varphi_o = \varphi_{cl} \vee \varphi_o \in \text{Vars} \vee$$

$$\langle \exists \varphi'_{cl}, \varphi'_o, k: (\varphi_{cl} = \{\varphi'_{cl}\}_k \wedge \varphi_o = \{\varphi'_o\}_k) \wedge \text{match}(\varphi'_{cl}, \varphi'_o) \rangle$$

$$\vee \langle \exists \varphi'_{cl}, \varphi''_{cl}, \varphi'_o, \varphi''_o: \varphi_{cl} = (\varphi'_{cl}, \varphi''_{cl}) \wedge \varphi_o = (\varphi'_o, \varphi''_o) \wedge$$

$$\text{match}(\varphi'_{cl}, \varphi'_o) \wedge \text{match}(\varphi''_{cl}, \varphi''_o) \rangle$$

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$$\text{match}(\varphi'_{cl}, \varphi'_o) \wedge \text{match}(\varphi''_{cl}, \varphi''_o) \rangle$$

**However:**

$$\text{match}( (\{\varphi_1, k1\}_k, k), (\{\varphi_1, k1\}_{\text{var}}, \text{var}) )?$$

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$$\text{match}(\varphi'_{cl}, \varphi'_o) \wedge \text{match}(\varphi''_{cl}, \varphi''_o) \rangle$$

However:

$$\text{match}( (\{\varphi_1, k1\}_k, k), (\{\varphi_1, k1\}_{\text{var}}, \text{var}) )?$$

Solution:

$$\text{match}(\varphi_{cl}, \varphi_o, \sigma) \equiv \sigma(\varphi_o) = \varphi_{cl} \wedge \text{dom}(\sigma) = \text{fv}(\varphi_o).$$

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$$K \cup \{\varphi\} \vdash \varphi$$

$$K \vdash \varphi_1, K \vdash \varphi_2 \implies K \vdash (\varphi_1, \varphi_2)$$

$$K \vdash (\varphi_1, \varphi_2) \implies K \vdash \varphi_1, K \vdash \varphi_2$$

$$K \vdash \varphi_1, K \vdash k \implies K \vdash \{\varphi_1\}_k$$

$$K \vdash \{\varphi_1\}_k, K \vdash k^{-1} \implies K \vdash \varphi_1$$

**closure:**  $\overline{K} = \{\varphi \mid K \vdash \varphi\}$

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## Phases, communication of terms:

$$Ev = \{ \quad s(a, a', \varphi), r(a, a', \varphi), \\ as(a, a', \varphi), ar(a', \varphi), \\ us(a, a', \varphi), ur(a, a', \varphi), \\ ph(i) \\ | a, a' \in Agents, \varphi \in Terms, i \in \mathbb{N} \}.$$

## Event order:

$$P ::= \delta \mid ev.P \mid P_1 + P_2 \mid P_1 \lhd \varphi_1 = \varphi_2 \triangleright P_2 \mid ev.X(\varphi_1, \dots, \varphi_n).$$

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State of agent is knowledge + process:

$$Agstate = \mathcal{P}(Terms) \times Processes.$$

**Definition 1 (voting system)** A voting system  $\mathcal{VS} \in VotSys$  specifies the state of each agent:

$$VotSys = Agents \rightarrow Agstate.$$

Instantiation of a voting system  $\mathcal{VS}$  with choice function  $\gamma$  is denoted as  $\mathcal{VS}^\gamma$ .

$$\mathcal{VS}^\gamma(a) = \begin{cases} \mathcal{VS}(a) & \text{if } a \notin \mathcal{V} \\ (\pi_1(\mathcal{VS}(a)), \sigma_a(\pi_2(\mathcal{VS}(a)))) & \text{if } a \in \mathcal{V} \end{cases}$$

where  $\sigma_a = vc \mapsto \gamma(a)$ .

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- system semantics in terms of an LTS
- paths in LTS  $\implies$  traces
- set of traces specifies the behaviour of the system

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## Deconstruction of terms:

$$\varphi \sqsubseteq \varphi$$

$$\varphi_1 \sqsubseteq (\varphi_1, \varphi_2)$$

$$\varphi \sqsubseteq \{\varphi\}_k$$

$$\varphi_2 \sqsubseteq (\varphi_1, \varphi_2)$$

$$k^{-1} \sqsubseteq \{\varphi\}_k$$

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## Deconstruction of terms:

$$\begin{array}{lll} \varphi \sqsubseteq \varphi & & \\ \varphi_1 \sqsubseteq (\varphi_1, \varphi_2) & \varphi_2 \sqsubseteq (\varphi_1, \varphi_2) & \\ \varphi \sqsubseteq \{\varphi\}_k & k^{-1} \sqsubseteq \{\varphi\}_k & \end{array}$$

## Readability of terms:

$$\text{Rd}(knw_a, \varphi_{cl}, \varphi_o, \sigma) \equiv \text{match}(\varphi, \varphi_o, \sigma) \wedge \\ \forall_{\varphi' \sqsubseteq \varphi_o} : knw_a \cup \{\varphi_{cl}\} \vdash \sigma(\varphi').$$

## ■ send:

$$\frac{k\text{nw}_a \vdash \varphi \quad \text{fv}(\varphi) = \emptyset}{(K_I, k\text{nw}_a, s(a, x, \varphi).P) \xrightarrow{s(a, x, \varphi)} (K_I \cup \{\varphi\}, k\text{nw}_a, P)}$$

## ■ send:

$$\frac{k\text{nw}_a \vdash \varphi \quad \text{fv}(\varphi) = \emptyset}{(K_I, k\text{nw}_a, s(a, x, \varphi).P) \xrightarrow{s(a, x, \varphi)} (K_I \cup \{\varphi\}, k\text{nw}_a, P)}$$

## ■ receive:

$$\frac{K_I \vdash \varphi' \quad \text{fv}(\varphi') = \emptyset \quad \text{Rd}(k\text{nw}_a, \varphi', \varphi, \sigma)}{(K_I, k\text{nw}_a, r(x, a, \varphi).P) \xrightarrow{r(x, a, \varphi')} (K_I, k\text{nw}_a \cup \{\varphi'\}, \sigma(P))}$$

■ send:

$$\frac{k\text{nw}_a \vdash \varphi \quad \text{fv}(\varphi) = \emptyset}{(K_I, k\text{nw}_a, s(a, x, \varphi).P) \xrightarrow{s(a, x, \varphi)} (K_I \cup \{\varphi\}, k\text{nw}_a, P)}$$

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■ anonymous receive:

$$\frac{K_I \vdash \varphi' \quad \text{fv}(\varphi') = \emptyset \quad \text{Rd}(k\text{nw}_a, \varphi', \varphi, \sigma)}{(K_I, k\text{nw}_a, ar(a, \varphi).P) \xrightarrow{ar(a, \varphi')} (K_I, k\text{nw}_a \cup \{\varphi'\}, \sigma(P))}$$

## ■ send:

$$\frac{k\text{nw}_a \vdash \varphi \quad \text{fv}(\varphi) = \emptyset}{(K_I, k\text{nw}_a, s(a, x, \varphi).P) \xrightarrow{s(a, x, \varphi)} (K_I \cup \{\varphi\}, k\text{nw}_a, P)}$$

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## ■ anonymous receive:

$$\frac{K_I \vdash \varphi' \quad \text{fv}(\varphi') = \emptyset \quad \text{Rd}(k\text{nw}_a, \varphi', \varphi, \sigma)}{(K_I, k\text{nw}_a, ar(a, \varphi).P) \xrightarrow{ar(a, \varphi')} (K_I, k\text{nw}_a \cup \{\varphi'\}, \sigma(P))}$$

## ■ untappable receive:

$$\frac{\text{fv}(\varphi') = \emptyset \quad \text{Rd}(k\text{nw}_a, \varphi', \varphi, \sigma)}{(K_I, k\text{nw}_a, ur(x, a, \varphi).P) \xrightarrow{ur(x, a, \varphi')} (K_I, k\text{nw}_a \cup \{\varphi'\}, \sigma(P))}$$

■ non-deterministic choice:

$$\frac{(K_I, knw_a, P_1) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}{(K_I, knw_a, P_1 + P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}$$

■ non-deterministic choice:

$$\frac{(K_I, knw_a, P_1) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}{(K_I, knw_a, P_1 + P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}$$

■ conditional choice:

$$\frac{(K_I, knw_a, P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_2) \quad \varphi_1 \neq \varphi_2 \quad \text{fv}(\varphi_1) = \text{fv}(\varphi_2) = \emptyset}{(K_I, knw_a, P_1 \triangleleft \varphi_1 = \varphi_2 \triangleright P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_2)}$$

■ non-deterministic choice:

$$\frac{(K_I, knw_a, P_1) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}{(K_I, knw_a, P_1 + P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_1)}$$

■ conditional choice:

$$\frac{(K_I, knw_a, P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_2) \quad \varphi_1 \neq \varphi_2 \quad \text{fv}(\varphi_1) = \text{fv}(\varphi_2) = \emptyset}{(K_I, knw_a, P_1 \triangleleft \varphi_1 = \varphi_2 \triangleright P_2) \xrightarrow{ev} (K'_I, knw'_a, P'_2)}$$

■ guarded recursion:

$$\frac{(K_I, knw_a, \sigma(P)) \xrightarrow{ev} (K'_I, knw'_a, P') \quad X(\text{var}_1, \dots, \text{var}_n) = P \quad \sigma = \text{var}_1 \mapsto \varphi_1 \circ \dots \circ \text{var}_n \mapsto \varphi_n}{(K_I, knw_a, X(\varphi_1, \dots, \varphi_n)) \xrightarrow{ev} (K'_I, knw'_a, P')}$$

■ phase synchronisation:

$$i \in \mathbb{N}$$

$$\text{Phase} \subseteq \{a @ (knw_a, P_a) \in S \mid \exists P'_a: (K_I, knw_a, P_a) \xrightarrow{ph(i)} (K_I, knw_a, P'_a)\}$$

$$\text{Aut} \subseteq \{a \in \text{Agents} \mid \exists knw_a, P_a: a @ (knw_a, P_a) \in \text{Phase}\}$$

$$\begin{aligned} \text{Phase}' = \{a @ (knw_a, P'_a) \mid \exists P_a: & a @ (knw_a, P_a) \in \text{Phase} \wedge \\ & (K_I, knw_a, P_a) \xrightarrow{ph(i)} (K_I, knw_a, P'_a)\} \end{aligned}$$

$$(K_I, S) \xrightarrow{ph(i)} (K_I, \text{Phase}' \cup S \setminus \text{Phase})$$

■ non-synchronous events:

$$\frac{(K_I, knw_a, P) \xrightarrow{ev} (K'_I, knw'_a, P') \quad ev \in Ev_{nosync} \quad a @ (knw_a, P) \in S}{(K_I, S) \xrightarrow{ev} (K'_I, \{a @ (knw'_a, P')\} \cup S \setminus \{a @ (knw_a, P)\})}$$

■ non-synchronous events:

$$\frac{(K_I, knw_a, P) \xrightarrow{ev} (K'_I, knw'_a, P') \quad ev \in Ev_{nosync} \quad a @ (knw_a, P) \in S}{(K_I, S) \xrightarrow{ev} (K'_I, \{a @ (knw'_a, P')\} \cup S \setminus \{a @ (knw_a, P)\})}$$

■ untappable communication:

$$\frac{\begin{array}{c} (K_I, knw_a, P_a) \xrightarrow{us(a,b,\varphi)} (K_I, knw_a, P'_a) \\ (K_I, knw_b, P_b) \xrightarrow{ur(a,b,\varphi)} (K_I, knw'_b, P'_b) \\ s_0 = \{a @ (knw_a, P_a), b @ (knw_b, P_b)\} \quad s_0 \subseteq S \end{array}}{(K_I, S) \xrightarrow{uc(a,b,\varphi)} (K_I, \{a @ (knw_a, P'_a), b @ (knw'_b, P'_b)\} \cup S \setminus s_0)}$$

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$$Tr(\mathcal{VS}^\gamma) = \{ \alpha \in Labels^* \mid \alpha = \alpha_0, \dots, \alpha_{n-1} \wedge \\ \exists s_0, \dots, s_n \in State : s_0 = (K_I^0, \mathcal{VS}^\gamma) \wedge \\ \forall 0 \leq i < n : s_i \xrightarrow{\alpha_i} s_{i+1} \}$$

 Traces of  $\mathcal{VS}$ :

$$Tr(\mathcal{VS}) = \bigcup_{\gamma \in \mathcal{V} \rightarrow \mathcal{C}} Tr(\mathcal{VS}^\gamma)$$

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Original idea:

Can the intruder tell for  $t$ , if  $t \in Tr(\mathcal{VS}^{\gamma_1})$  or  $t \in Tr(\mathcal{VS}^{\gamma_2})$ ?

New idea:

When can the intruder distinguish  $Tr(\mathcal{VS}^{\gamma_1})$  from  $Tr(\mathcal{VS}^{\gamma_2})$ ?

When he can reinterpret  $t$  as  $t'$ .

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**Definition 2 (reinterpretation (GHP05))** Let  $\rho$  be a permutation on the set of terms Terms and let  $K_I$  be a knowledge set. The map  $\rho$  is a semi-reinterpretation under  $K_I$  if it satisfies the following.

$$\begin{aligned}\rho(p) &= p, \text{ for } p \in \mathcal{C} \cup \text{Nonces} \cup \text{Keys} \\ \rho((\varphi_1, \varphi_2)) &= (\rho(\varphi_1), \rho(\varphi_2)) \\ \rho(\{\varphi\}_k) &= \{\rho(\varphi)\}_k, \text{ if } K_I \vdash \varphi, k \vee K_I \vdash \{\varphi\}_k, k^{-1}\end{aligned}$$

Map  $\rho$  is a reinterpretation under  $K_I$  iff it is a semi-reinterpretation and its inverse  $\rho^{-1}$  is a semi-reinterpretation under  $\rho(K_I)$ .

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**Definition 3 (trace indistinguishability)** Traces  $t, t'$  are indistinguishable for the intruder, notation  $t \sim t'$  iff there exists a reinterpretation  $\rho$  such that

$$obstr(t') = \rho(obstr(t)) \wedge \overline{K_I^t} = \rho(\overline{K_I^{t'}}).$$

**Definition 4 (choice indistinguishability)** Given voting system  $\mathcal{VS}$ , choice functions  $\gamma_1, \gamma_2$  are indistinguishable to the intruder, notation  $\gamma_1 \simeq_{\mathcal{VS}} \gamma_2$  iff

$$\begin{aligned} \forall t \in Tr(\mathcal{VS}^{\gamma_1}) : \exists t' \in Tr(\mathcal{VS}^{\gamma_2}) : t \sim t' \quad \wedge \\ \forall t \in Tr(\mathcal{VS}^{\gamma_2}) : \exists t' \in Tr(\mathcal{VS}^{\gamma_1}) : t \sim t' \end{aligned}$$

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**Definition 5 (choice group)** *The choice group for a voting system  $\mathcal{VS}$  and a choice function  $\gamma$  is given by*

$$cg(\mathcal{VS}, \gamma) = \{\gamma' \mid \gamma \simeq_{\mathcal{VS}} \gamma'\}.$$

*The choice group for a particular voter  $v$ , i.e. the set of candidates indistinguishable from  $v$ 's chosen candidate, is given by*

$$cg_v(\mathcal{VS}, \gamma) = \{\gamma'(v) \mid \gamma' \in cg(\mathcal{VS}, \gamma)\}.$$

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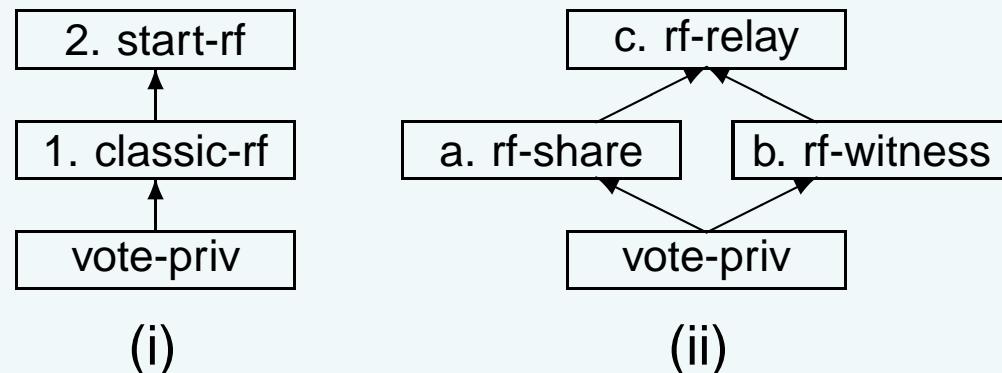
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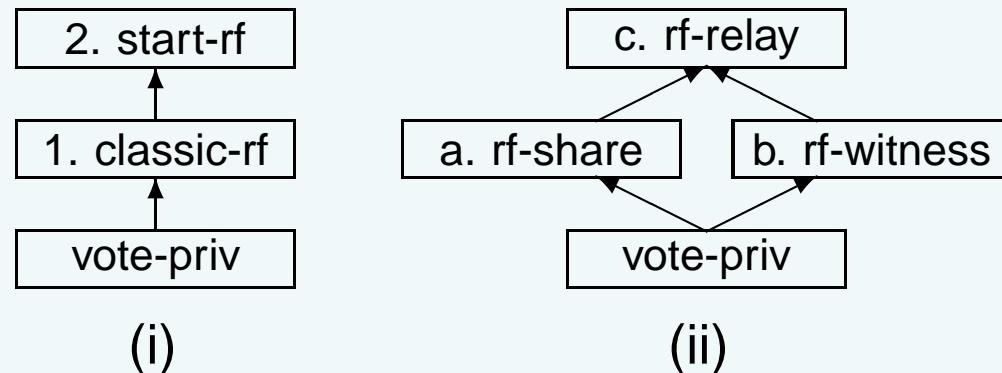
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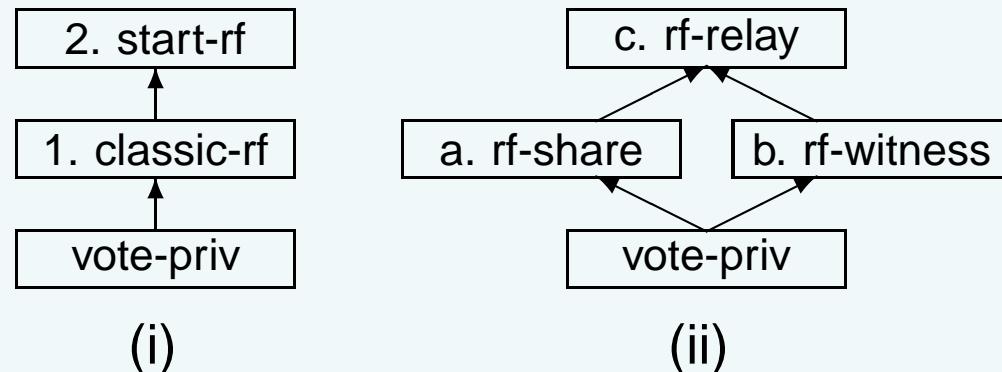
## Privacy techniques:

- secret initial knowledge
- specific communication channels
  - untappable channel

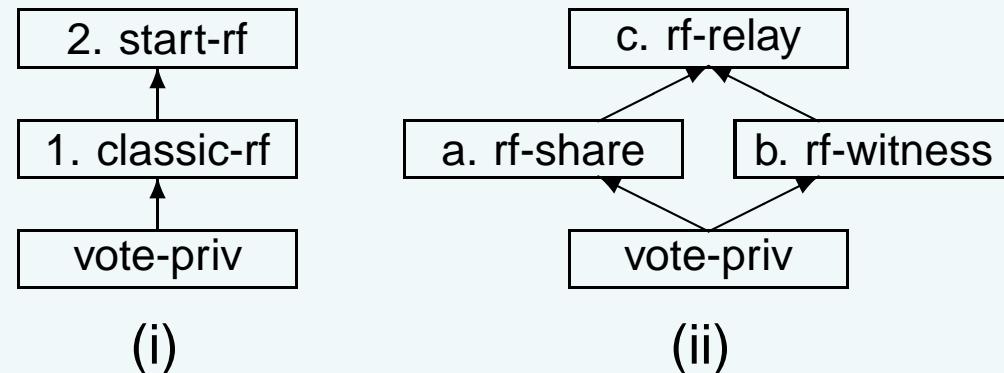




- transform processes using  $\Theta_i$ , where  $i \in \{1, 2, a, b, c\}$ .



- transform processes using  $\Theta_i$ , where  $i \in \{1, 2, a, b, c\}$ .
  - transform events using  $\theta_i$



- transform processes using  $\Theta_i$ , where  $i \in \{1, 2, a, b, c\}$ .
  - transform events using  $\theta_i$
  - coercion resistance  $i$ :  $\forall v, \gamma: cg_v^i(\mathcal{VS}, \gamma) = cg_v(\mathcal{VS}, \gamma)$

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■  $\theta_a(v, ev) =$

$$\begin{cases} ur(ag, v, \varphi) . is(v, \varphi) & \text{if } ev = ur(ag, v, \varphi) \\ ev & \text{otherwise} \end{cases}$$

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- $\theta_a(v, ev) = \begin{cases} ur(ag, v, \varphi) . is(v, \varphi) & \text{if } ev = ur(ag, v, \varphi) \\ ev & \text{otherwise} \end{cases}$
- $\theta_b(v, ev) = \begin{cases} is(v, \text{vars}(v, \varphi)) . ir(v, \text{vars}(v, \varphi')) . us(v, ag, \varphi') & \text{if } ev = us(v, ag, \varphi), \text{ for } \varphi' = \text{freshvars}(v, \varphi) \\ ev & \text{otherwise} \end{cases}$

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- conspiracy
- event transformation**
- process transformation
- conspiracy-resistance

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- $\theta_a(v, ev) = \begin{cases} ur(ag, v, \varphi) . is(v, \varphi) & \text{if } ev = ur(ag, v, \varphi) \\ ev & \text{otherwise} \end{cases}$
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- $\theta_c(v, ev) = \theta_b(v, \theta_a(v, ev))$

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$$\Theta_2(v, P) = \text{is}(knw_v).P$$

$$\Theta_i(v, P) = \begin{cases} \delta & \text{if } i \neq 1 \wedge P = \delta \\ \text{is}(v, knw_v).\delta & \text{if } i = 1 \wedge P = \delta \\ \theta_i(v, ev).\Theta_i(v, P) & \text{if } P = ev.P \\ \Theta_i(v, P_1) + \Theta_i(v, P_2) & \text{if } P = P_1 + P_2 \\ \Theta_i(v, P_1) \triangleleft \varphi_1 = \varphi_2 \triangleright \Theta_i(v, P_2) & \text{if } P = P_1 \triangleleft \varphi_1 = \varphi_2 \triangleright P_2, \\ & \text{for } \varphi_1, \varphi_2 \in Terms \\ \theta_i(v, ev).Y(\varphi_1, \dots, \varphi_n), & \text{for fresh } Y(\text{var}_1, \dots, \text{var}_n) = \Theta_i(v, P') \\ & \text{if } P = X(\varphi_1, \dots, \varphi_n) \wedge X(\text{var}_1, \dots, \text{var}_n) = P' \end{cases}$$

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classical notion:

$$\forall v, \gamma: |cg_v^1(\mathcal{VS}, \gamma)| > 1.$$

Our definition:

**Definition 6 (conspiracy-resistance)** We call voting system  $\mathcal{VS}$  conspiracy-resistant for conspiring behaviour  
 $i \in \{1, 2, a, b, c\}$  iff

$$\forall v \in \mathcal{V}, \gamma \in \mathcal{V} \rightarrow \mathcal{C}: cg_v^i(\mathcal{VS}, \gamma) = cg_v(\mathcal{VS}, \gamma).$$

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## Conclusions:

- we can quantify privacy in voting
- possibility to detect new attacks
- considering the exact choice group aids in reasoning about privacy

## Future work:

- consider transformations of authorities
- defense strategies
- automated application
- extend with probabilism (election result)

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Thank you for your attention.

Questions?