Expressivity of Some Versions of APAL

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PAL and APAL

- Public announcement logic (PAL): A dynamic operator represents the consequences of information change.
  - $[\psi]\varphi$: after truthful public announcement of ψ, φ is true.
  - PAL is as expressive as epistemic logic (EL).
- Arbitrary public announcement logic (APAL): A quantifier over PAL formulas.
  - $[!]\varphi$: after any truthful public announcement, φ is true.
  - APAL is more expressive than PAL.
  - APAL is undecidable and has an infinitary axiomatization.

SAPAL, FSAPAL, SCAPAL

- *Subset* version of APAL (SAPAL): quantify over public announcements only containing a **subset** of all atoms. ($[Q]\varphi$)
- *Finite* subset version of APAL (FSAPAL): quantify over public announcements only containing a **finite subset** of all atoms.
- *Scope* version of APAL (SCAPAL): quantify over announcements only containing atoms occurring in formulas within the **scope** of the quantifier. ($[\subseteq]\varphi$).

IPAL, QIPAL

- *Imply* version of APAL (IPAL):
  - quantify over announcements implying a given formula. ($[\psi^{\downarrow}]\varphi$)
  - quantify over announcements implied by a given formula. ($[\psi^{\uparrow}]\varphi$)
- QIPAL: ψ may contain quantifier.
- IPAL: ψ is quantifier-free.

Language $\mathcal{L}_{PAL}$ and $\mathcal{L}_{APAL}$

Given a countable set $\mathbf{P}$ of atoms and a finite set $\mathbf{A}$ of agents, $p \in \mathbf{P}$, $a \in \mathbf{A}$, and $Q \subseteq \mathbf{P}$

Definition ($\mathcal{L}_{PAL}$)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi$$

Definition ($\mathcal{L}_{APAL}$)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [!]\varphi$$

Language $\mathcal{L}_{SAPAL}$, $\mathcal{L}_{FSAPAL}$ and $\mathcal{L}_{SCAPAL}$

Definition ($\mathcal{L}_{SAPAL}, \mathcal{L}_{FSAPAL}$)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [Q]\varphi$$

If the Q in $[Q]\varphi$ is always finite, we get $\mathcal{L}_{FSAPAL}$.

Definition ($\mathcal{L}_{SCAPAL}$)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [\subseteq]\varphi$$

Language $\mathcal{L}_{QIPAL}$ and $\mathcal{L}_{IPAL}$

Definition ($\mathcal{L}_{QIPAL}$ and $\mathcal{L}_{IPAL}$)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [\varphi^{\downarrow}]\varphi \mid [\varphi^{\uparrow}]\varphi$$

If the ψ in $[\psi^{\downarrow}]\varphi$ and $[\psi^{\downarrow}]$ is restricted to $\mathcal{L}_{PAL}$, we get $\mathcal{L}_{IPAL}$.

Semantics

Definition (Semantics)

Given model $M = (S, \sim, V)$, $s \in S$, we indctively define $M, s \models \varphi$ as:

$$
\begin{array}{lll}
\ldots & & \\
M,s \models [\psi]\varphi & \text{iff} & M,s \models \psi \text{ implies } M|\psi, s \models \varphi \\
M,s \models [!]\varphi & \text{iff} & \text{for any } \psi \in \mathcal{L}_{PAL} : M,s \models [\psi]\varphi \\
M,s \models [Q]\varphi & \text{iff} & \text{for any } \psi \in \mathcal{L}_{PAL}|Q \ : M,s \models [\psi]\varphi \\
M,s \models [\subseteq]\varphi & \text{iff} & \text{for any } \psi \in \mathcal{L}_{PAL}|P(\varphi) : M,s \models [\psi]\varphi \\
M,s \models [\chi^{\downarrow}]\varphi & \text{iff} & \text{for any } \psi \in \mathcal{L}_{PAL} \text{ implying } \chi : M,s \models [\psi]\varphi \\
M,s \models [\chi^{\uparrow}]\varphi & \text{iff} & \text{for any } \psi \in \mathcal{L}_{PAL} \text{ implied by } \chi : M,s \models [\psi]\varphi
\end{array}
$$

where $M|\varphi = (S', \sim', V')$ is such that
$S' = [\![\varphi]\!]_M = \{s \in S \mid M, s \models \varphi\}$, $\sim'_a = \sim_a \cap ([\![\varphi]\!]_M \times [\![\varphi]\!]_M)$, and
$V'(p) = V(p) \cap [\![\varphi]\!]_M$.
ψ implies χ means $\models \psi \to \chi$, ψ is implied by χ means $\models \chi \to \psi$.

Expressivity

For comparing expressivity between logic L and L', we introduce the following notations:

- $L \preceq L'$: L' is at least as expressive as L , iff for $\varphi \in \mathcal{L}_L$ there is a $\varphi' \in \mathcal{L}_{L'}$ such that φ is equivalent to φ'.
- $L \prec L'$: L is strictly less expressive than L' iff $L \preceq L'$ but $L' \npreceq L$;
- $L \asymp L'$: L and L' are incomparable in expressivity iff $L \npreceq L'$ and $L' \npreceq L$.

Strategy

Proof strategy for $L \not\preceq L'$:

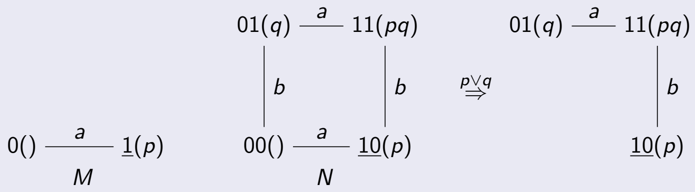
- φ is a L-formula, and therefore there are two classes of pointed-models such that φ is true on every model in one class, but is false on every model in the other class.
- Suppose there is a corresponding L'-formula ψ, and its modal depth is n, using finite atoms within Q. Show that there is a pair of models from each class such that these models are modal equivalent with respect to L'-formula up to modal depth n or restricted to the subset Q.
- As ψ cannot be true on one model and false on the other, there is a contradiction.

FSAPAL,SCAPAL vs. APAL

Proposition

APAL $\not\preceq$ *FSAPAL (SCAPAL)*

Proof.



$N, 10 \models \langle ! \rangle (K_a p \wedge \neg K_b K_a p)$
$M, 1 \not\models \langle ! \rangle (K_a p \wedge \neg K_b K_a p)$
$M, 1 \models \psi$ iff $N, 10 \models \psi$ for $\psi \in \mathcal{L}_{FSAPAL}$ (Let q not occur in ψ)

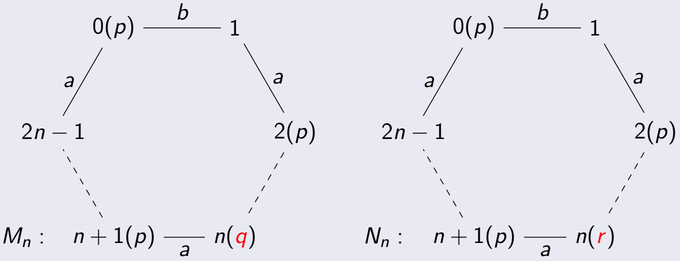
□

FSAPAL, SCAPAL vs. APAL

Proposition

FSAPAL (SCAPAL) $\not\preceq$ *APAL*

Proof.



$M_n, 0 \models \langle\{q\}\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$
$N_n, 0 \not\models \langle\{q\}\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$
$M_n, 0 \models \psi$ iff $N_n, 0 \models \psi$ with $d(\psi) < n$ (n is odd) □

SCAPAL vs. FSAPAL

Proposition

$SCAPAL \preceq FSAPAL$

Proof.

$\vDash [\subseteq]\varphi \leftrightarrow [\{var(\varphi)\}]\varphi$. □

Proposition

$FSAPAL \not\preceq SCAPAL$

Proof.

Same strategy. Details omitted. □

IPAL vs. APAL, FSAPAL, SCAPAL

Proposition

$APAL \preceq IPAL$

Proof.

$\vDash [\top^{\downarrow}]\varphi \leftrightarrow [!]\varphi$ □

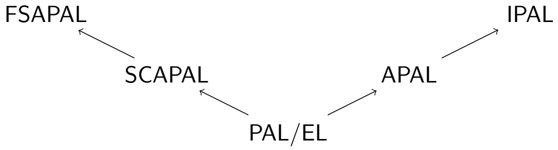
Proposition

$APAL \prec IPAL$

Proposition

$IPAL \asymp FSAPAL$, $IPAL \asymp SCAPAL$

Expressivity Hierarchy



Expressivity hierarchy of logics presented in this work. An arrow means larger expressivity. Assume transitivity. Absence of an arrow means incomparability.

Conclusion

- We investigated the expressivity of the FSAPAL, SCAPAL and IPAL.
- One of our motivations was to "tame" APAL. However, these versions of APAL also have undecidability of SAT problem and infinitary axiomatizaitons.
- As results of expressivity show, FSAPAL and SCAPAL are incomparable to APAL, and not "tameable" as we thought.

Thank you!