Pattern models:

A Dynamic Epistemic Logic For Distributed System Analysis

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Agenda

1. Context
2. Pattern models
3. Parametrized pattern models

Context
Preliminaries

- Some authors in distributed computing still refer to knowledge only informally
  - It is important to develop tools to formalize such a concept

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  - Pfleger & Schmid (2018)
  - Goubault, Ledent & Rajsbaum (since 2018)
    - Kripke frames cat. and simplicial complex cat. are equivalent

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Epistemic logic

Syntax

$$\mathcal{L}_K \ni \varphi ::= p \mid \neg\varphi \mid \ \varphi \wedge \varphi \mid K_a\varphi$$

where $a \in A$, and $p \in P$

No update modality

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Semantics

$$\begin{array}{ll}
M, w \models p & \text{iff } \ p \in L(w) \\
M, w \models \neg\varphi & \text{iff } M, w \not\models \varphi \\
M, w \models \varphi \wedge \psi & \text{iff } M, w \models \varphi \text{ and } M, w \models \psi \\
M, w \models K_a\varphi & \text{iff } M, w' \models \varphi \text{ for all } w' \text{ such that } \ w \ \sim_a \ w'
\end{array}$$

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Distributed computing models

Agents (processes)

- State machines
- Private input (local state)
- Execute a protocol of communication
  - All gathered information is sent (full-information)

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Dynamic-network models

- Synchronous (round closed) communication
- An adversary decides who communicates with whom
  - Picks a communication graph in every round

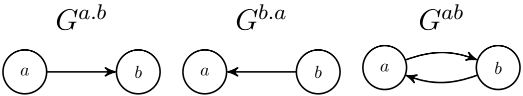
    $a \rightarrow b$ *reflexive relation
  - Oblivious
    - any communication graph in a given set of communication graphs may occur in any round

Context

Iterated immediate snapshot model (IIS)

IIS model

- processes write to a shared memory and then take a snapshot
  - Concurrent read

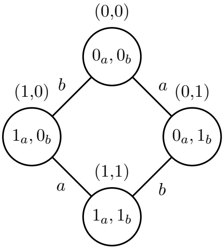


Context

Iterated immediate snapshot model (IIS)

Epistemic model (Kripke models with equivalence relations)

- A triple $(W, \sim, L)$
  - Worlds
  - Indistinguishability relations
  - True valued propositions



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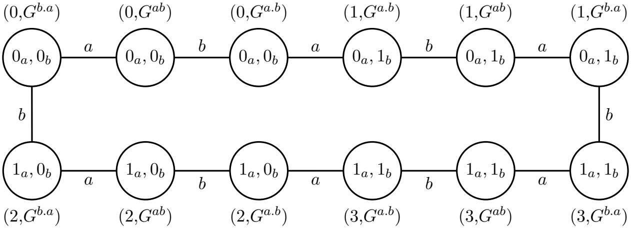

Action models

- Update mechanism

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  - Action model (Structure) $M = (E, R, Pre)$
    - Events
    - Indistinguishability relations
    - Precondition formulas

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  - Action model (Structure) $\mathsf{M} = (\mathsf{E}, \mathsf{R}, \mathsf{Pre})$
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  - Restricted modal product
    - $M \otimes \mathsf{M} = M'$

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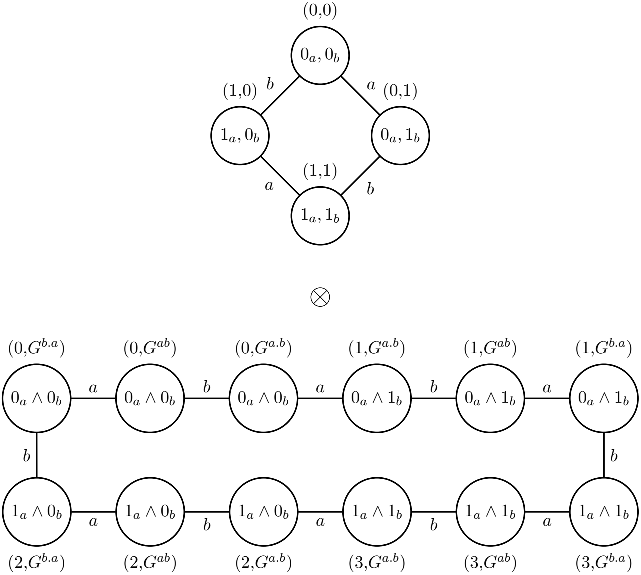

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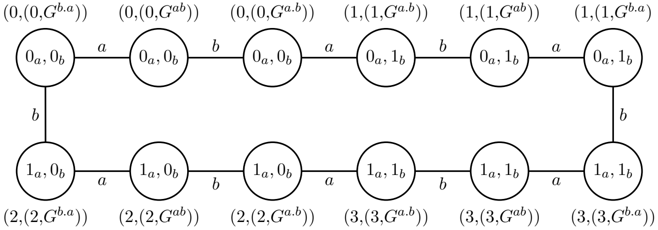

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Modeling epistemic change with action models through rounds of communication has drawbacks

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- Direct application is simple but inefficient
- Finding compact action models is not clear

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We propose *pattern models*, a dynamic epistemic logic

- Similar to reading event models (Baltag and Smets)

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  - The events are communication graphs
  - The restricted modal product allows preconditions
  - A protocol can be a parameter of the product (not only full-information)

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Let $Ga = \{b \in A \mid bGa\}$ be the in-neighbourhood of a in G

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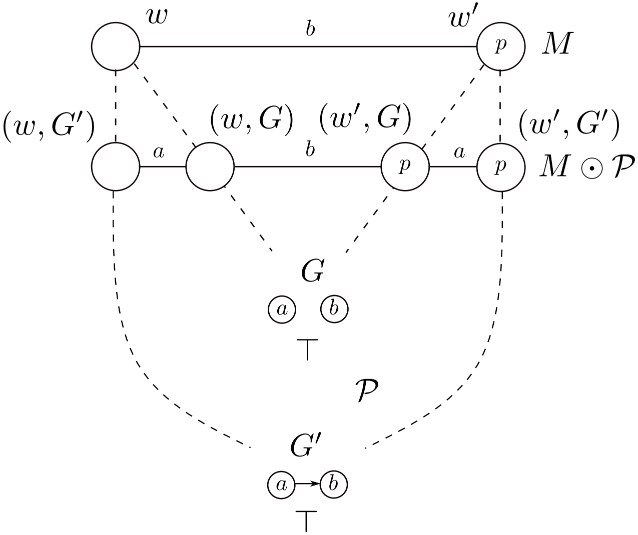

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- The full-information protocol is not practical in real systems

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Msg, set of message contents

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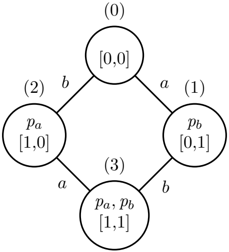

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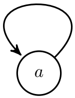
G



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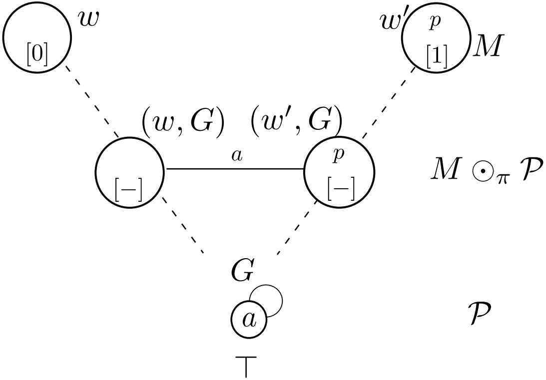
Parametrized pattern models



Pattern models

- This parametrized version is useful for designing efficient protocols
  - Automated formal verification of protocols

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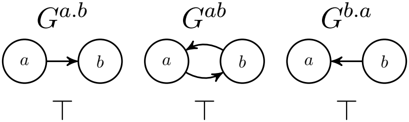
  - Proof of correctness of such pattern models
    - The correctness is still valid with the parametrized version
  - Oblivious dynamic-network models requires constant space



рахмат
danke
謝謝
ngiyabonga
spas
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Баярлалаа
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kia ora
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faafetai lava
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dank je
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matondo
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grazzi
tapadh leat
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thank you
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dhanyavadagalu
shukriya
mercè
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kam sah hamnida
rahmat
ευχαριστώ
diolch
তোমাকে ধন্যবাদ
감사합니다
xièxie
merci