Formal Abstractions for Packet Scheduling

Mohan, Liu, Foster, Kappé, Kozen

SDN made networks programmable.

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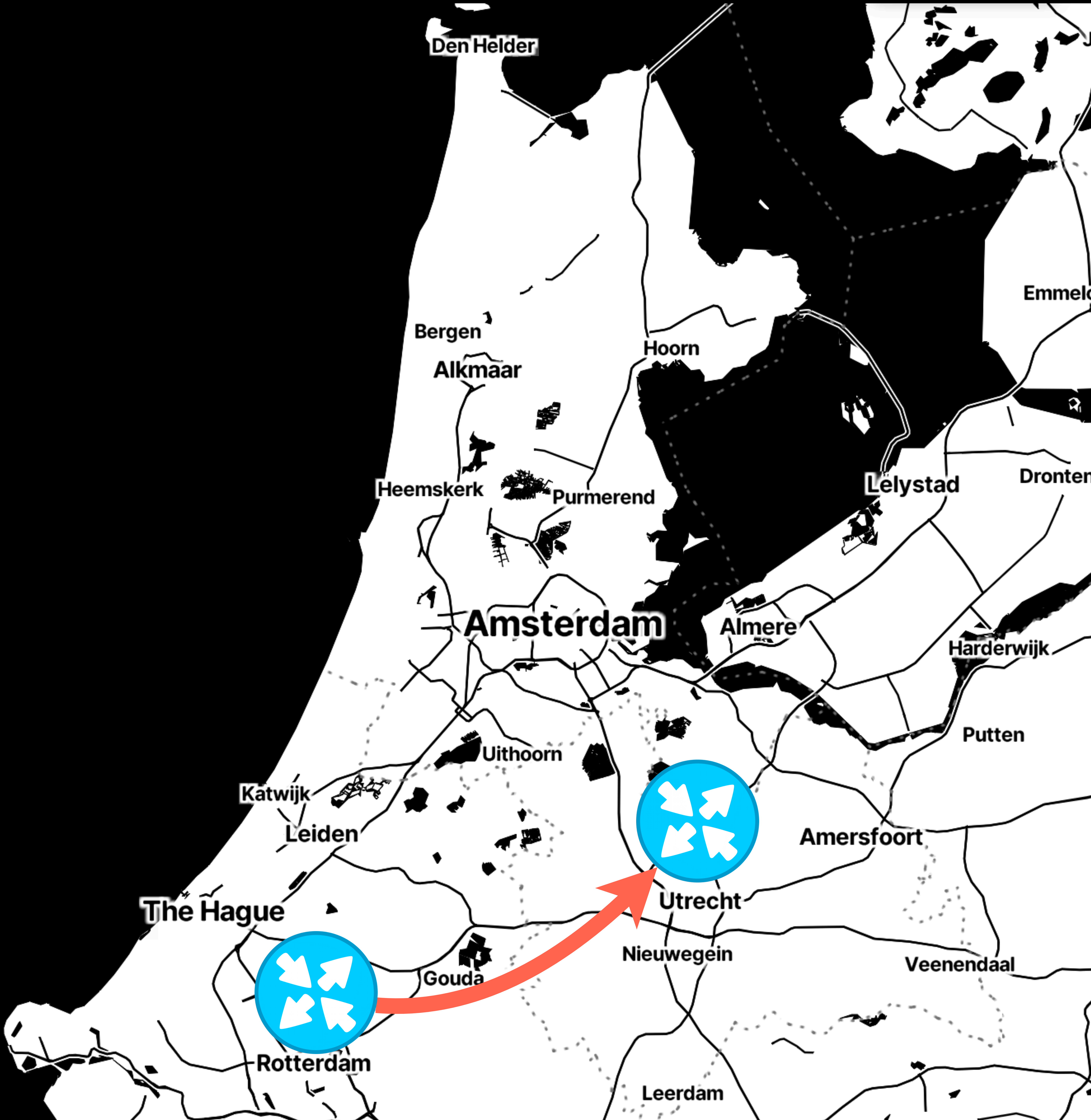
Early goal: routing.



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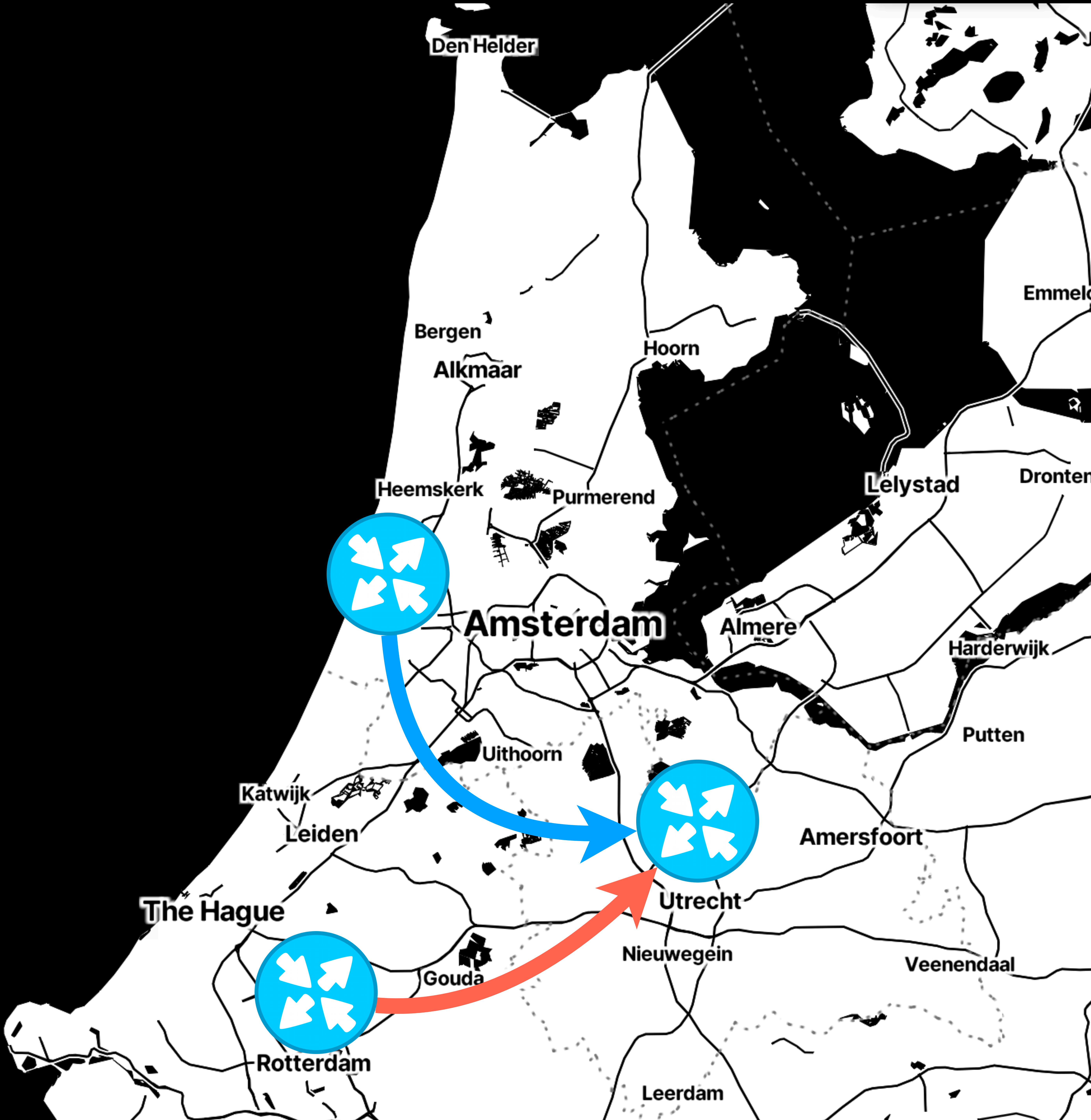

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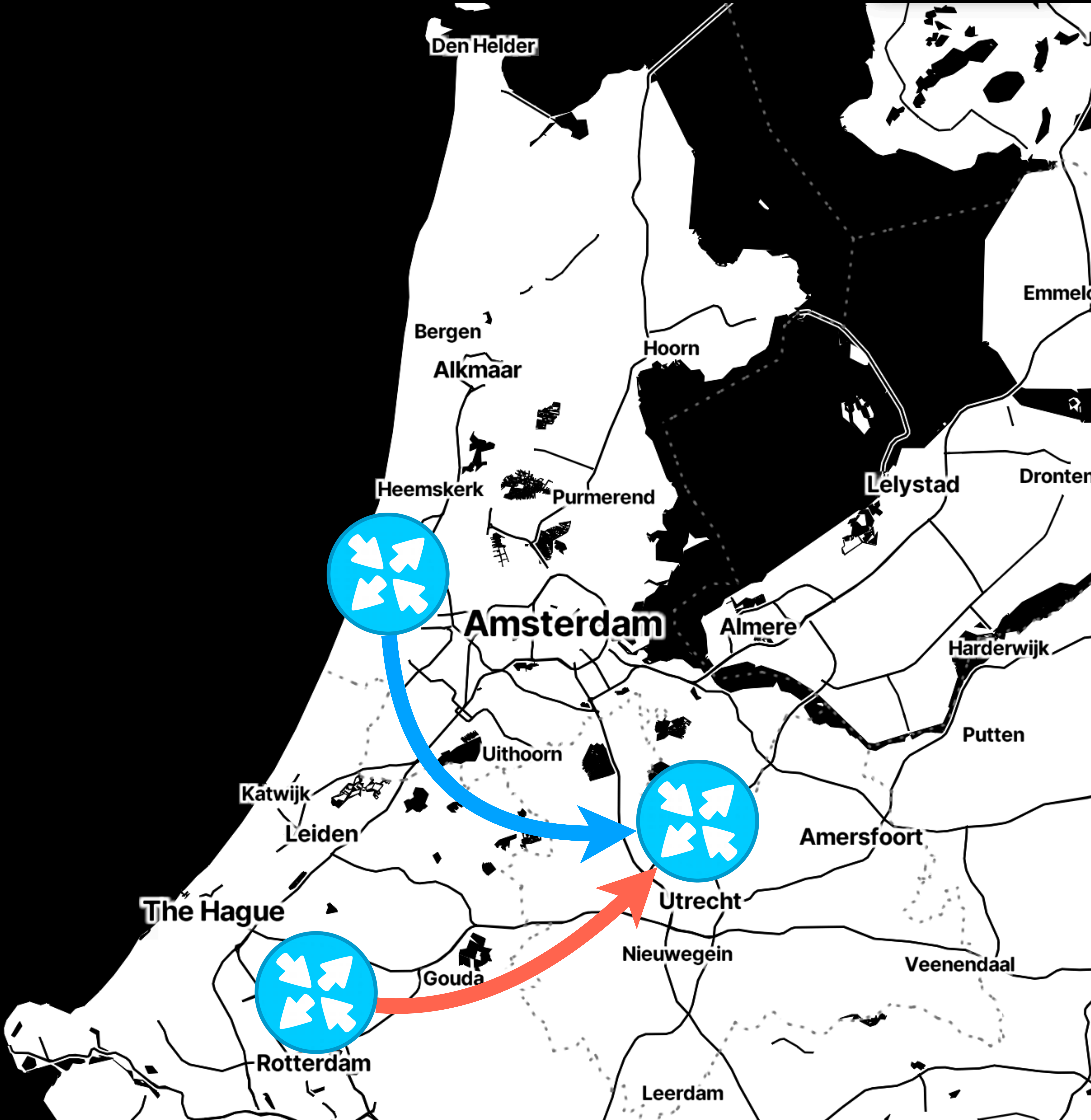

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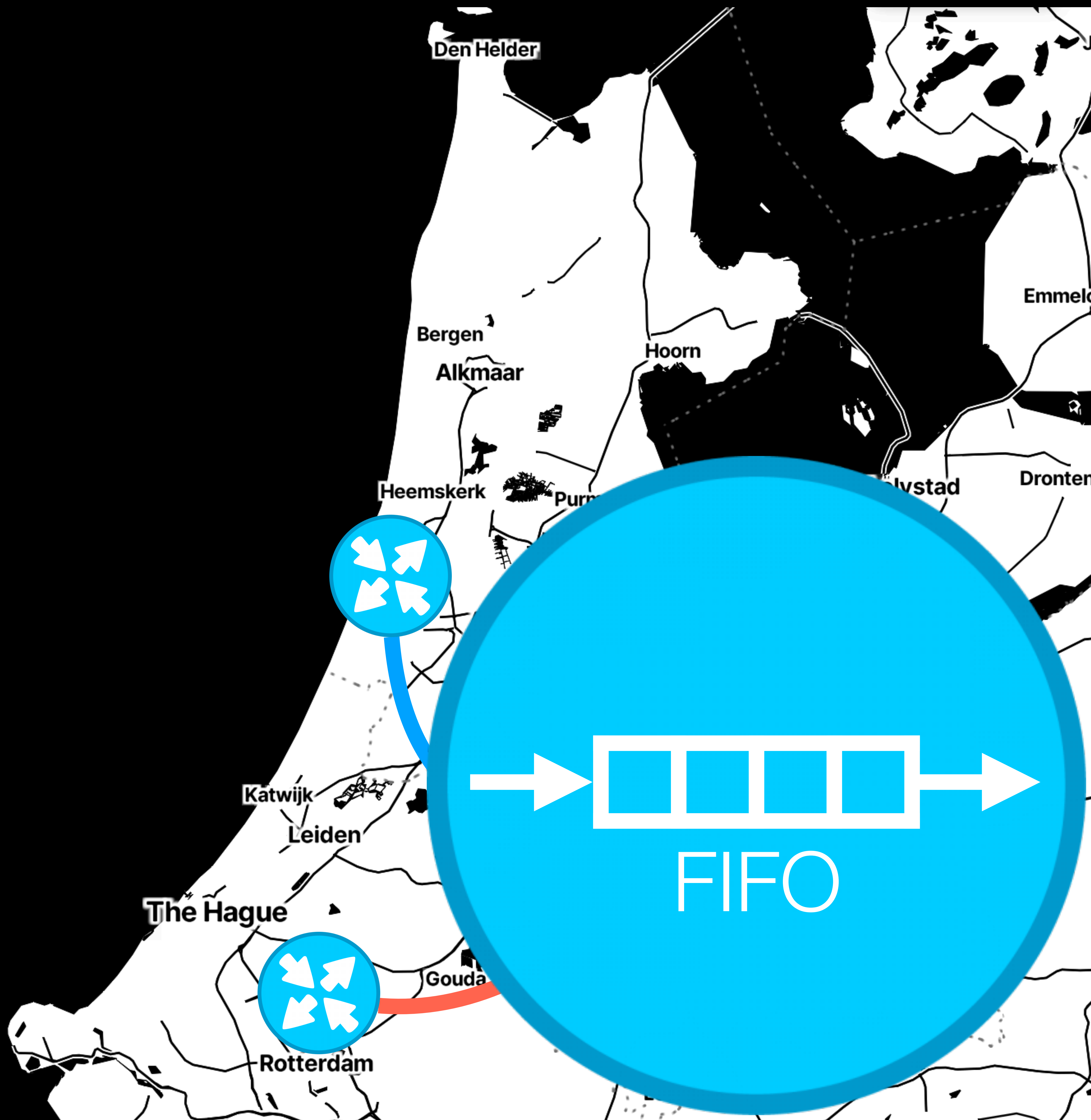

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PIFO

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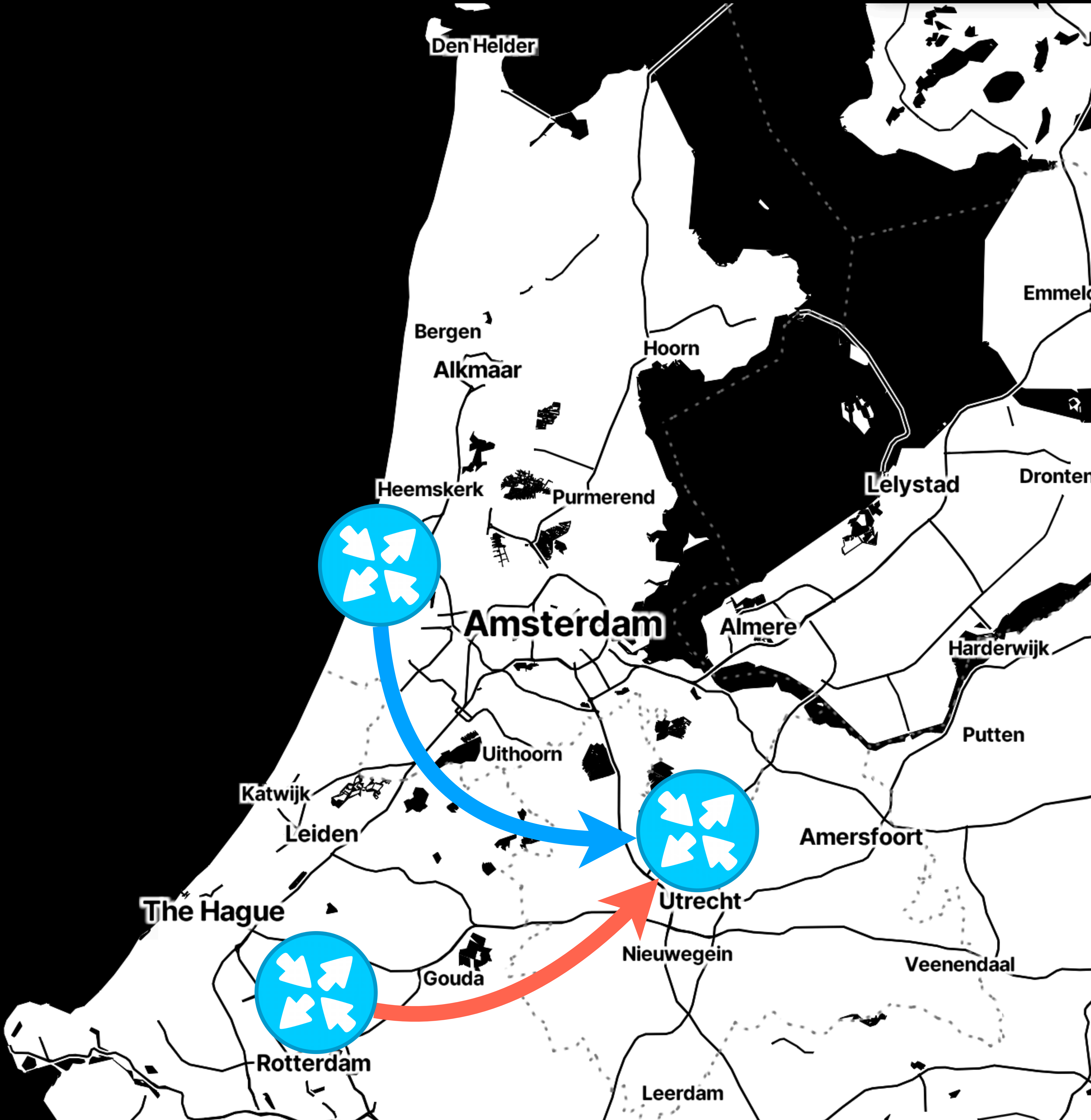

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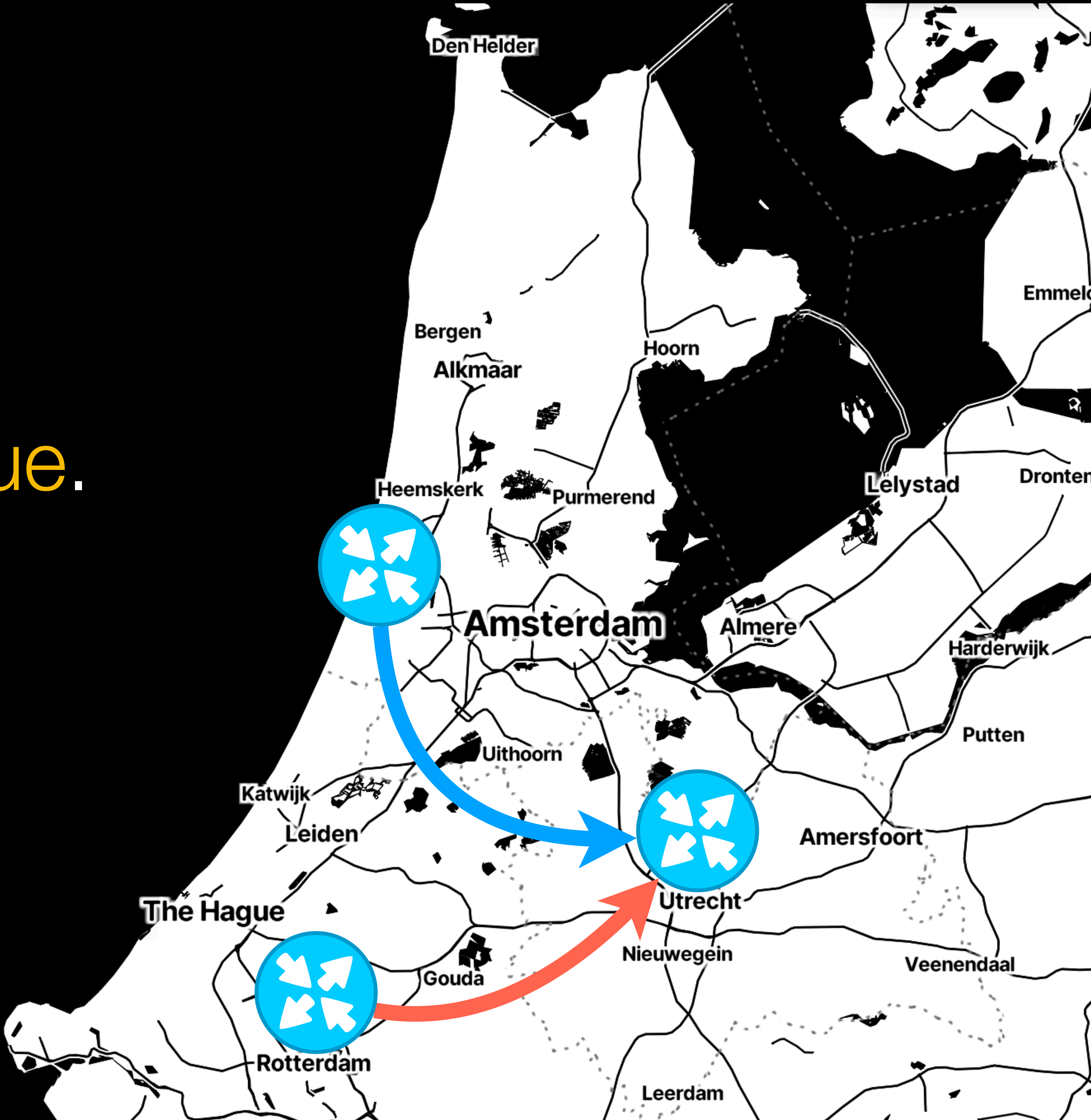

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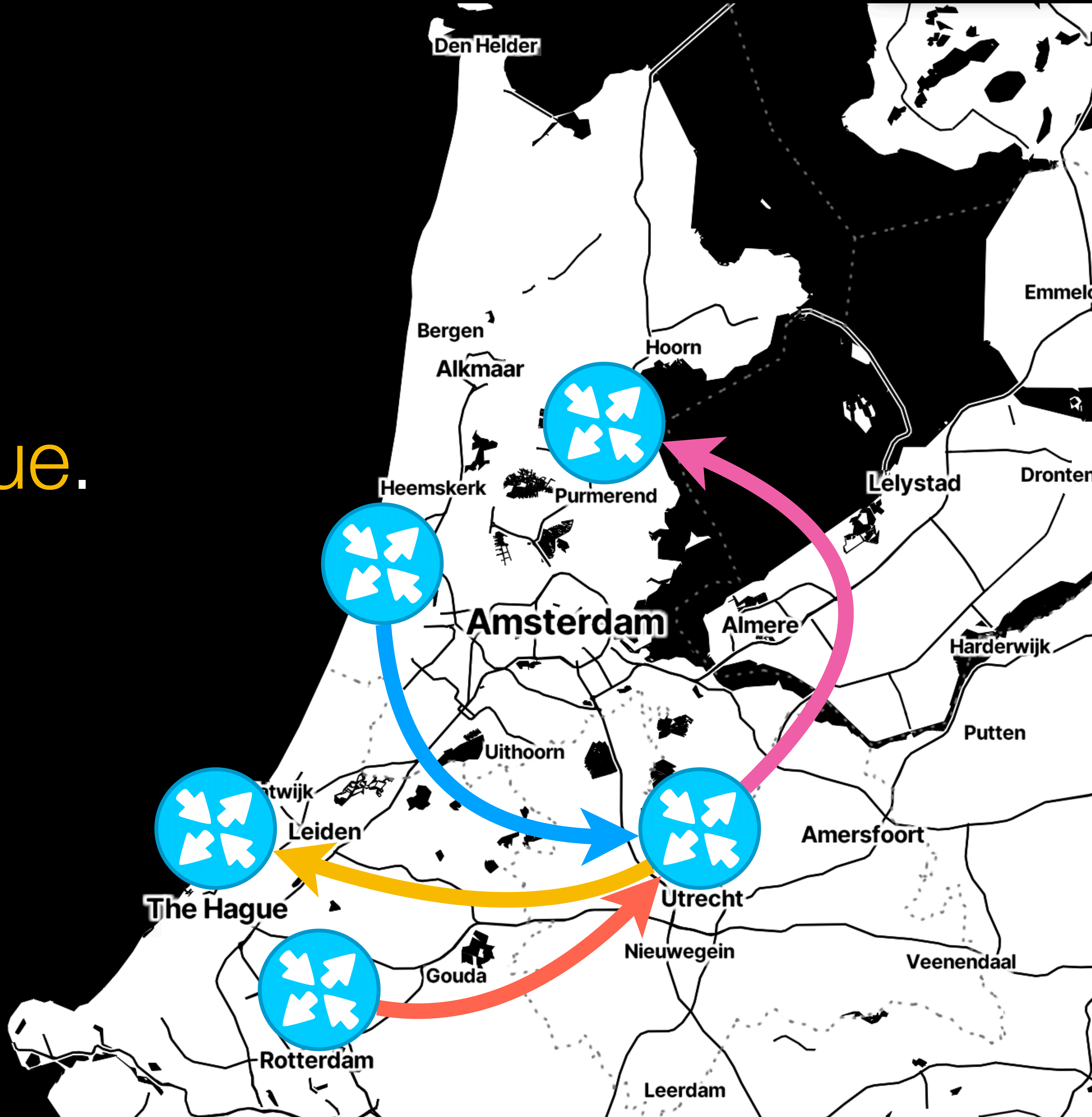

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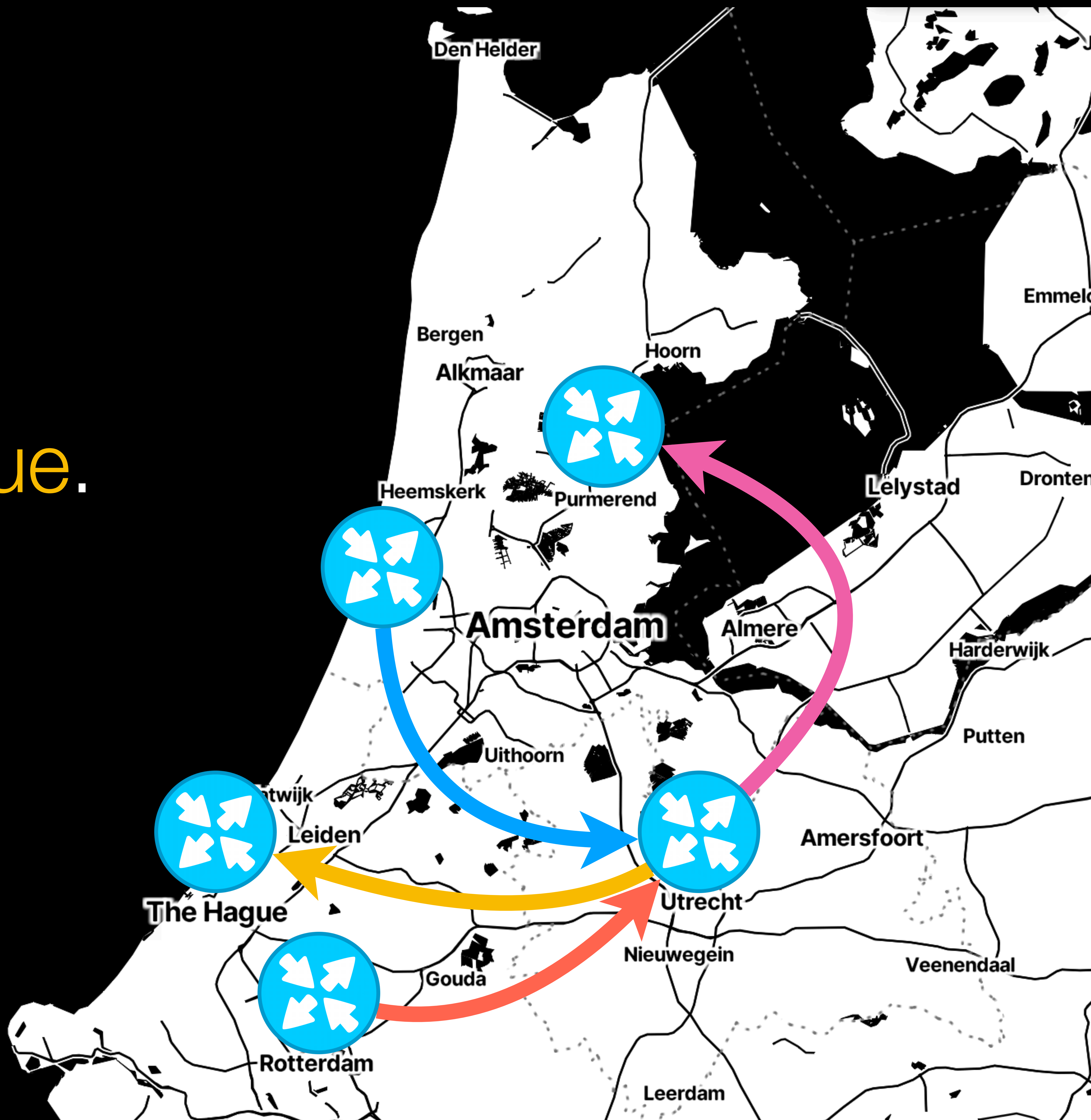

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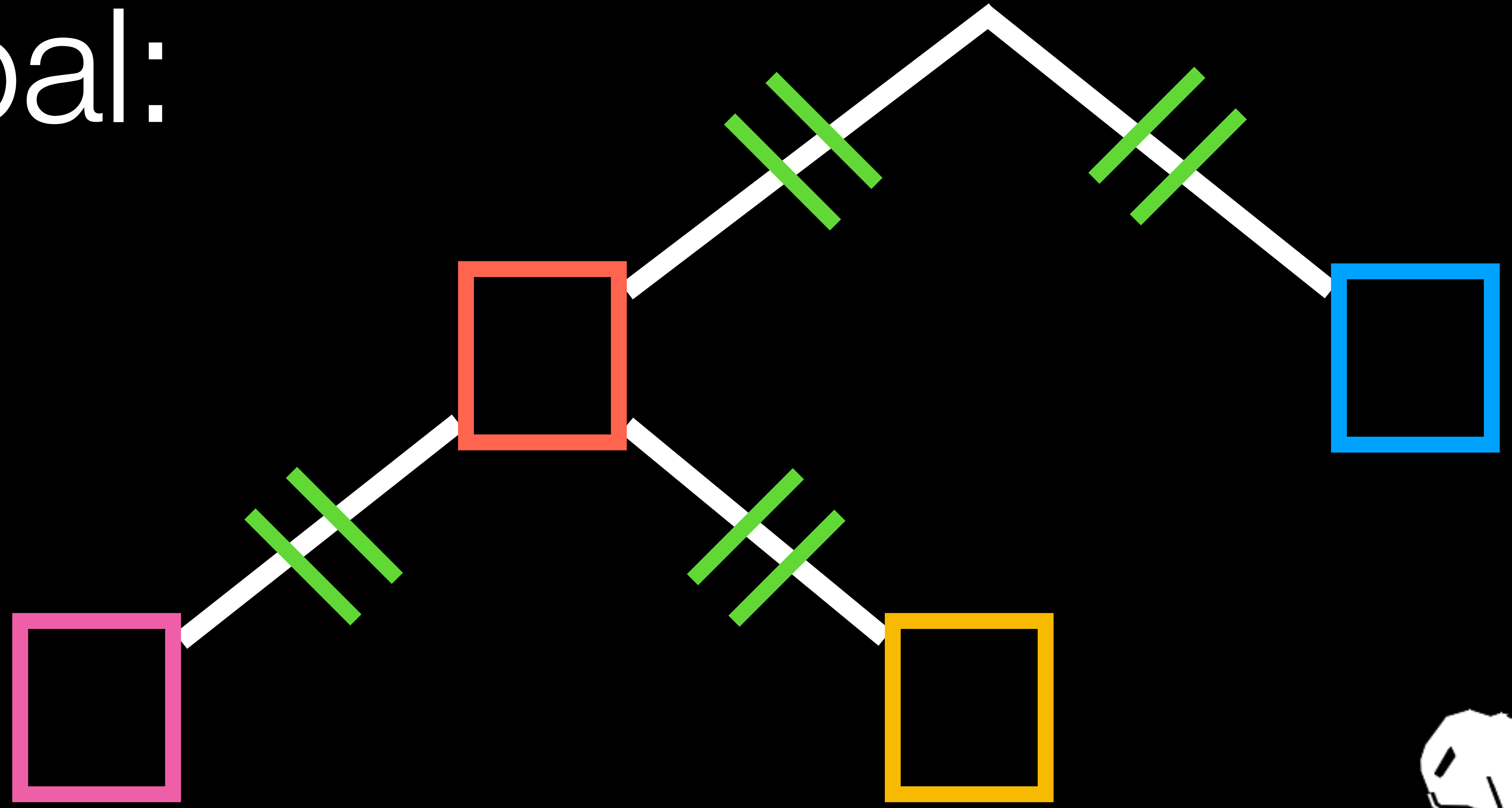

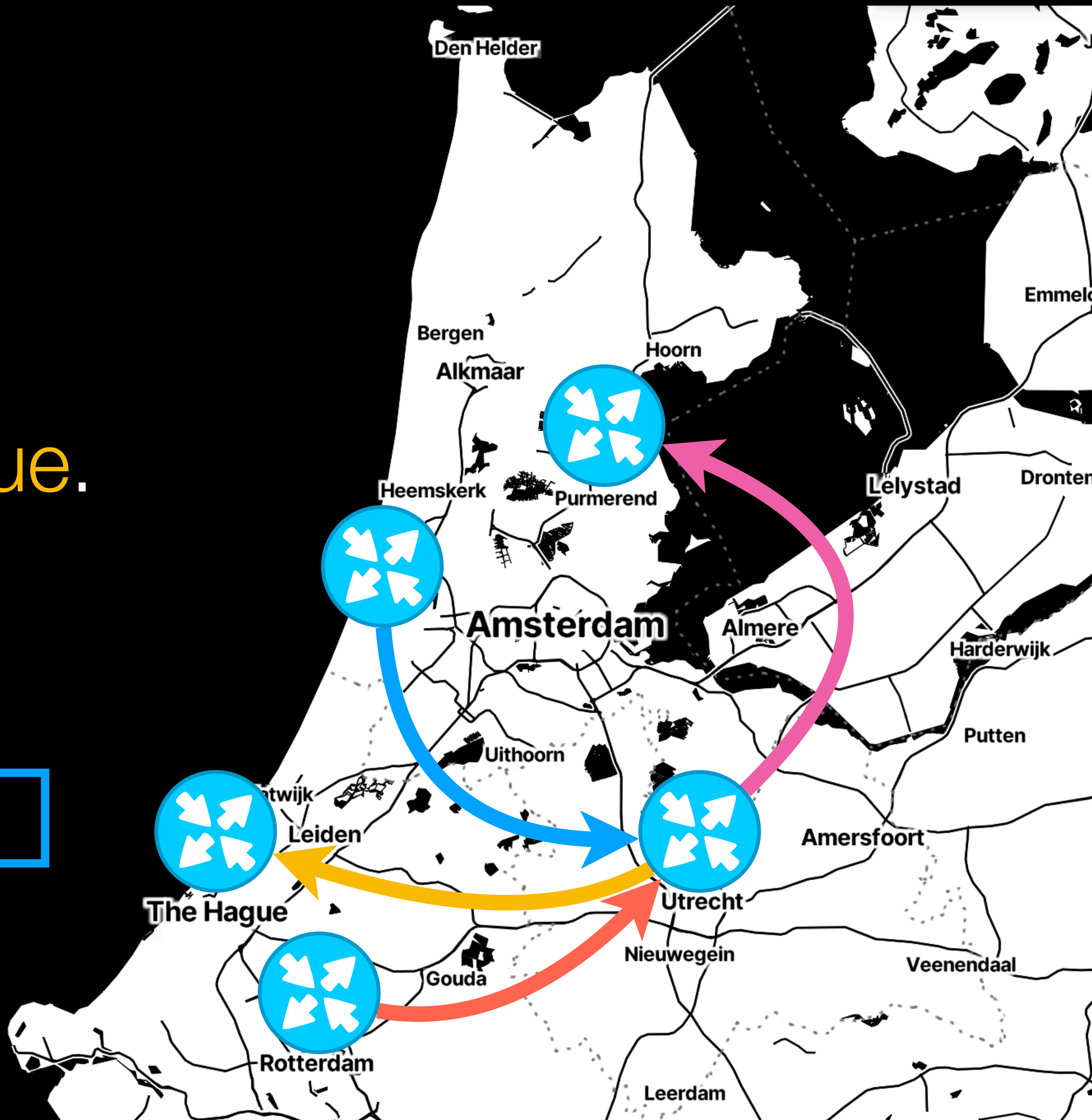

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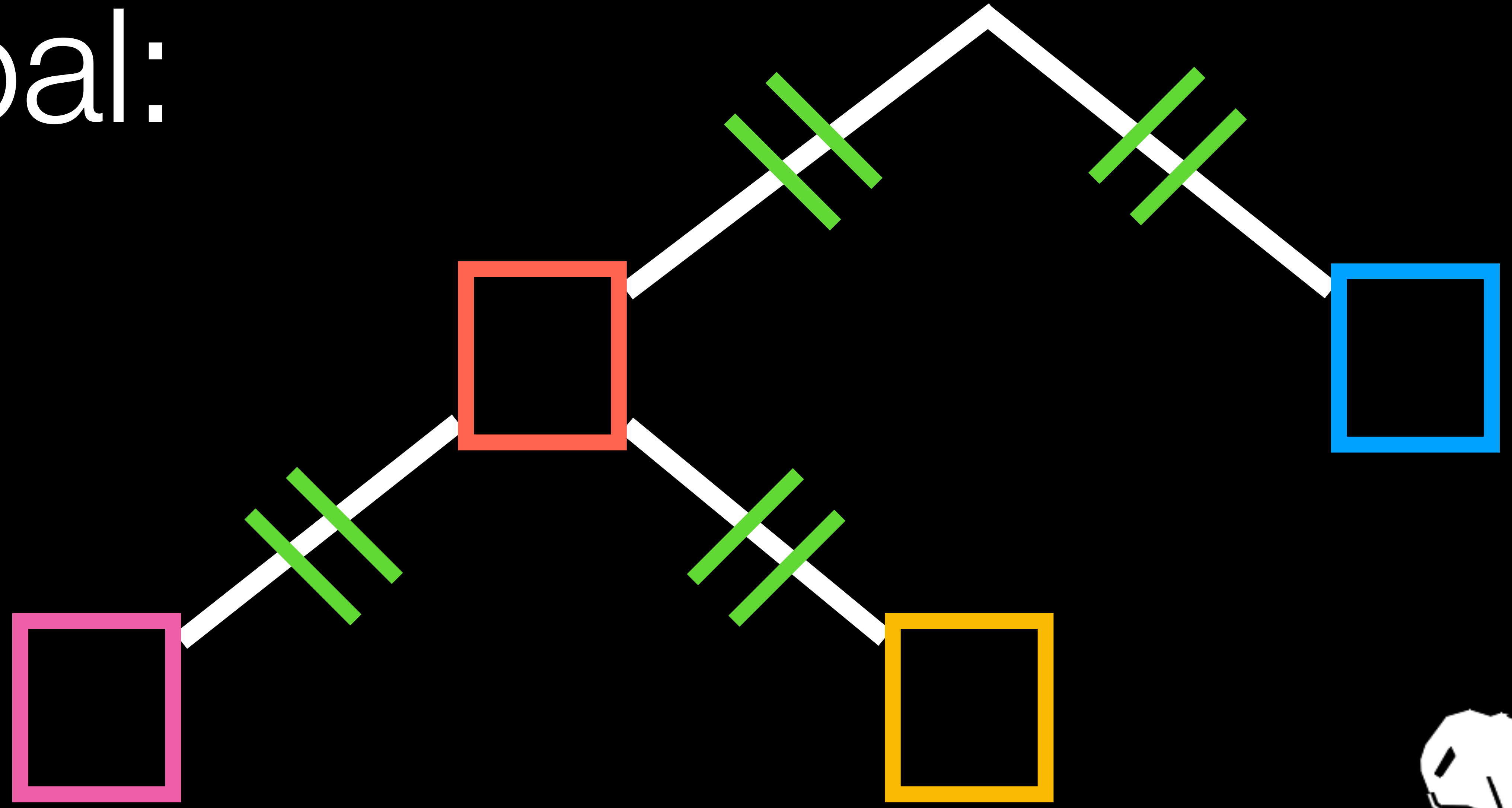

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New plan!

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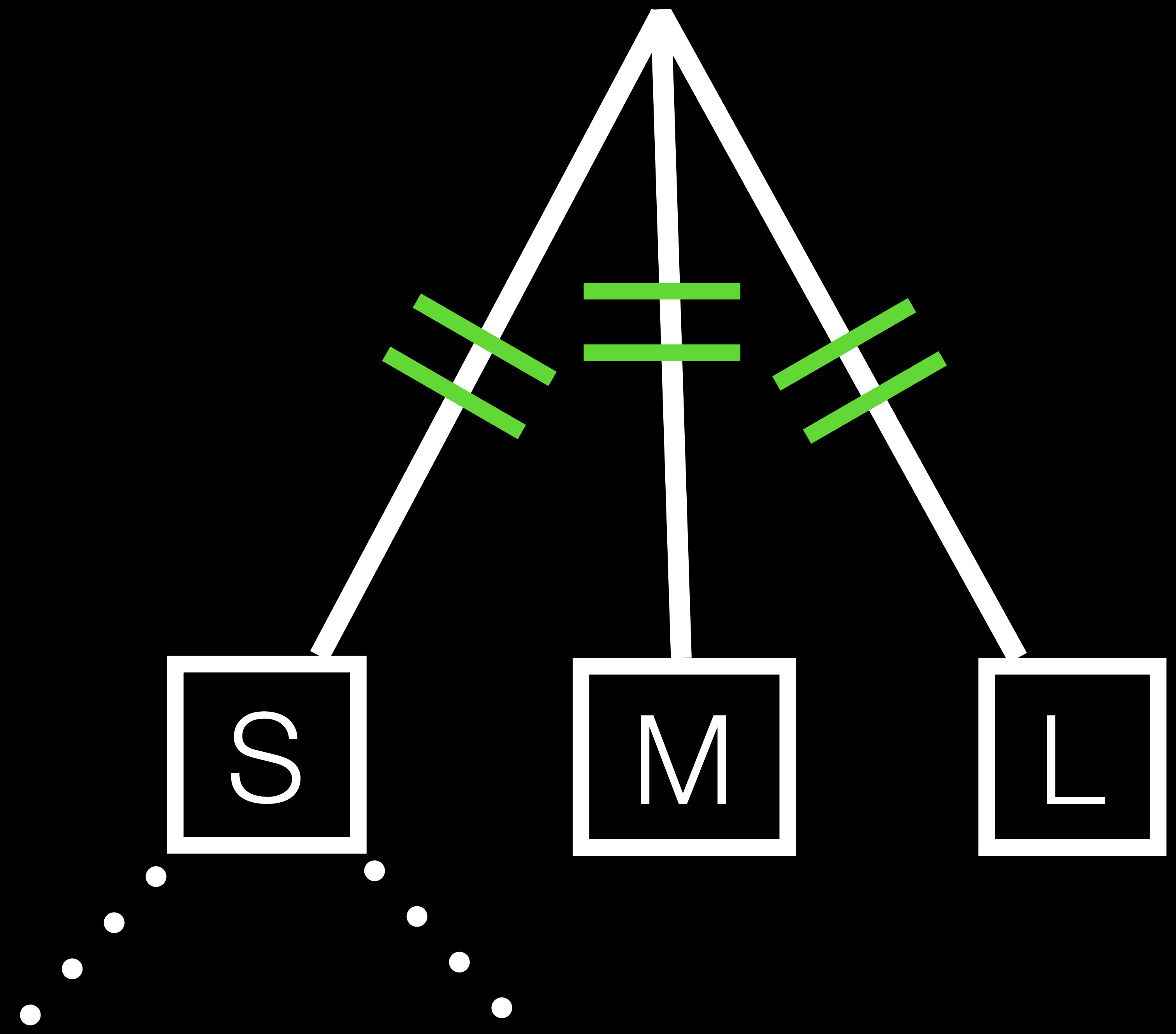

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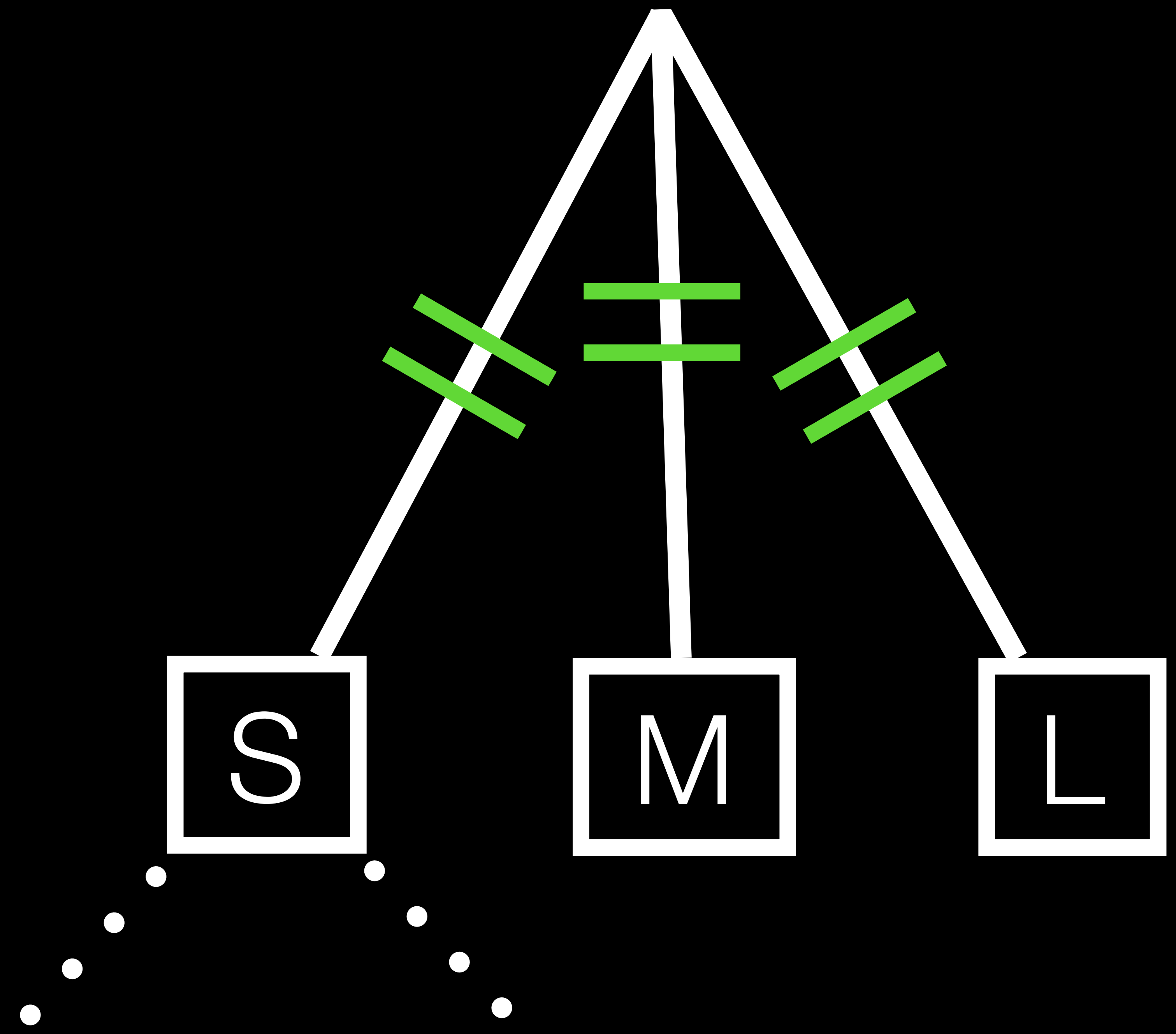

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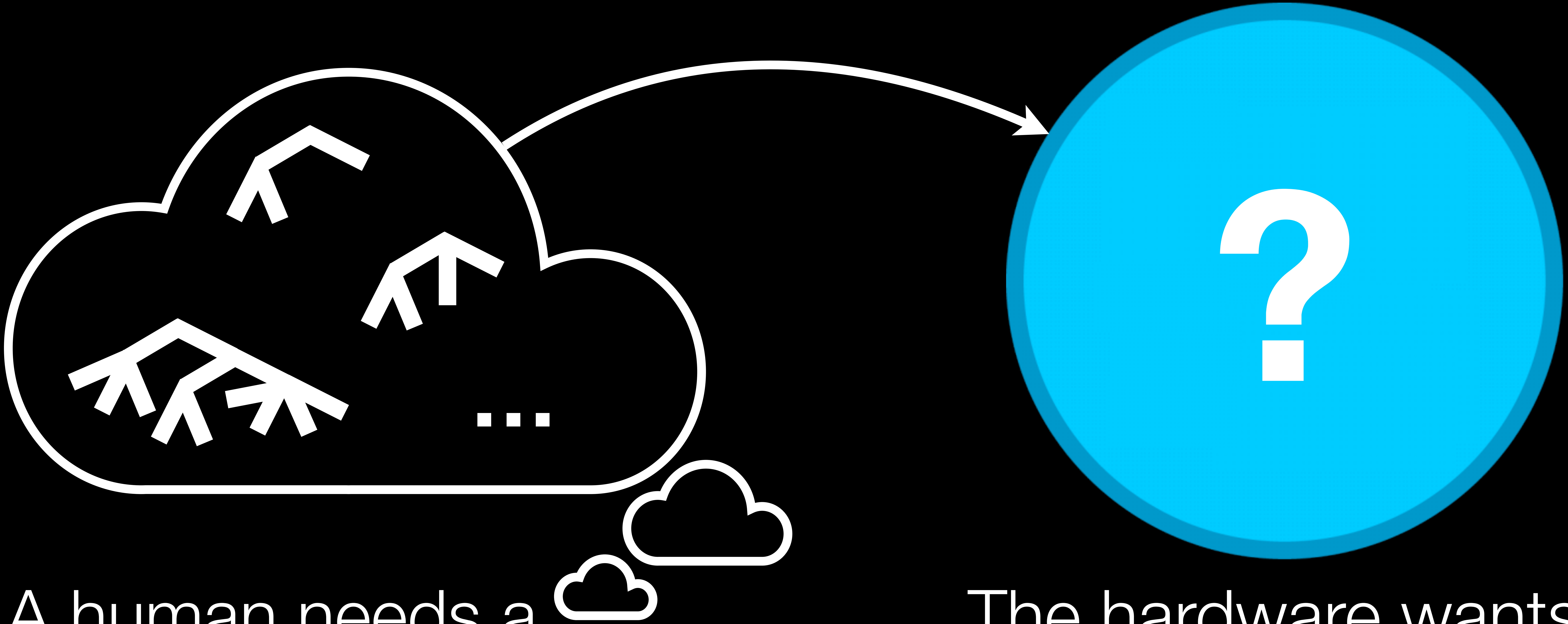

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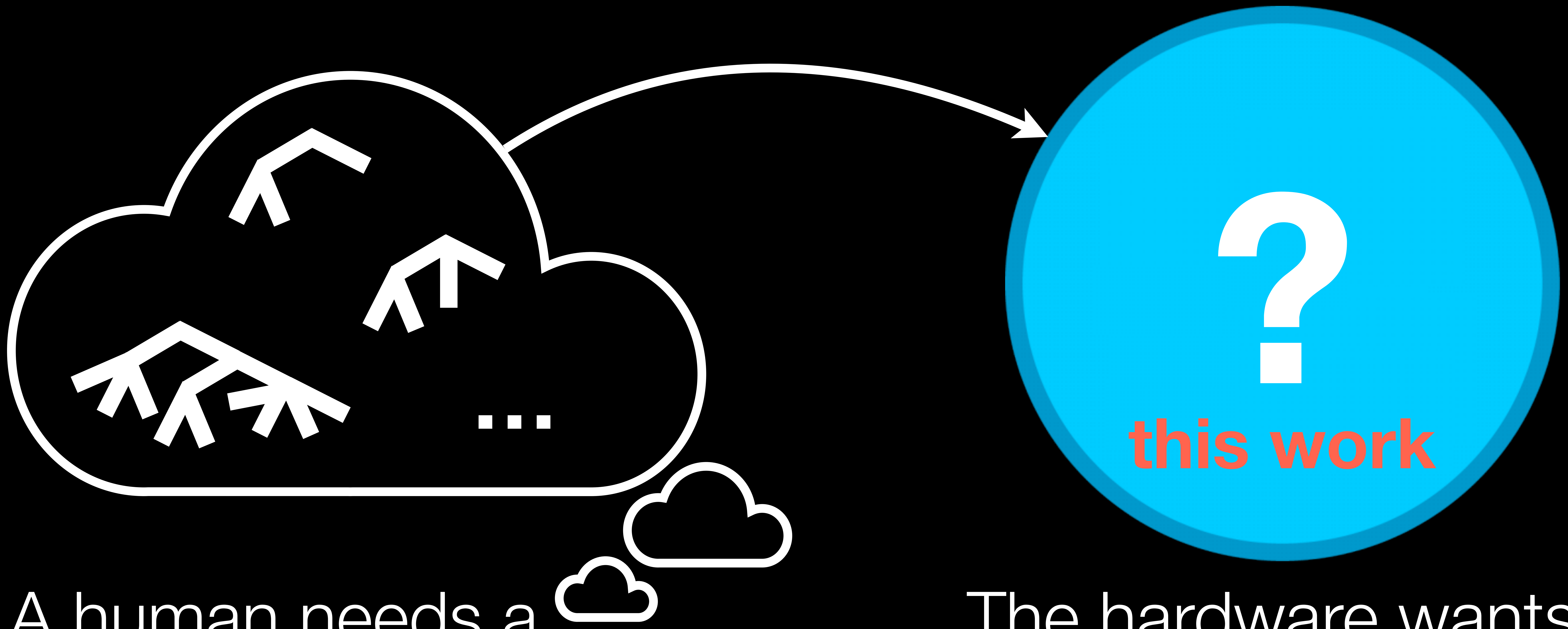

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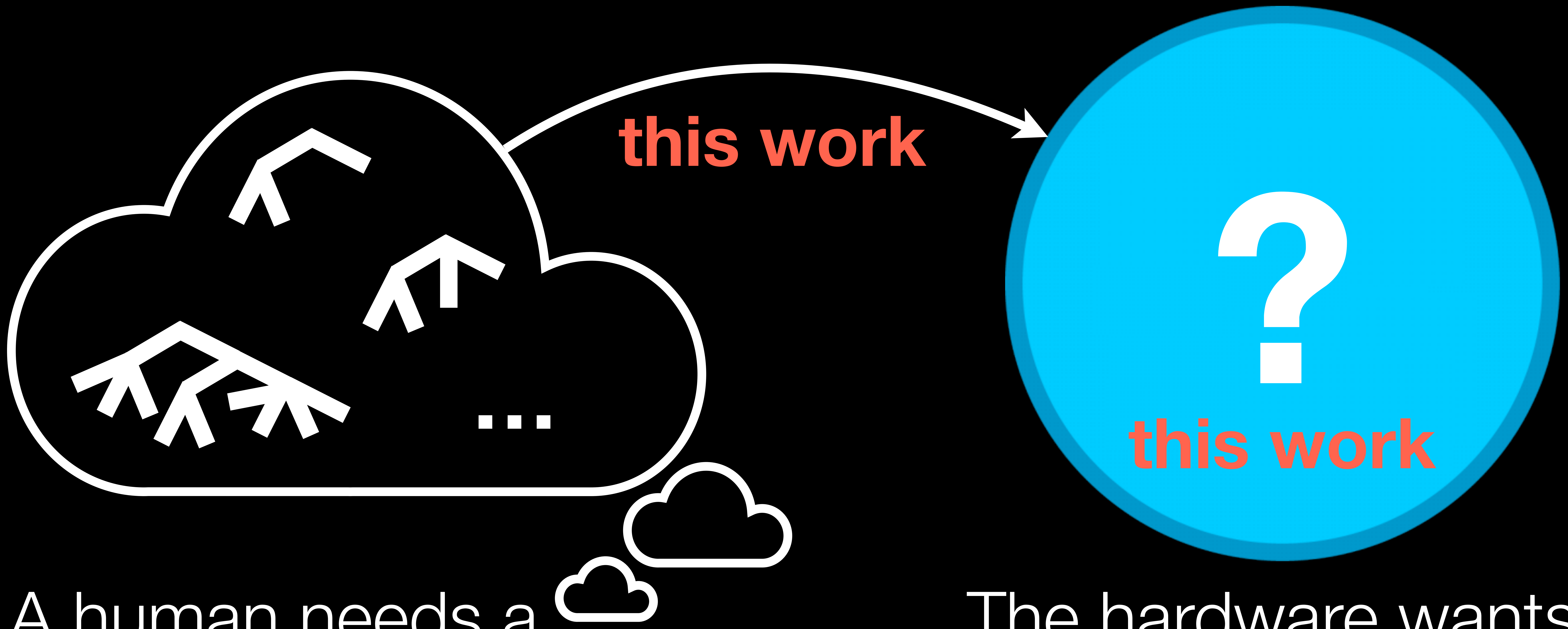

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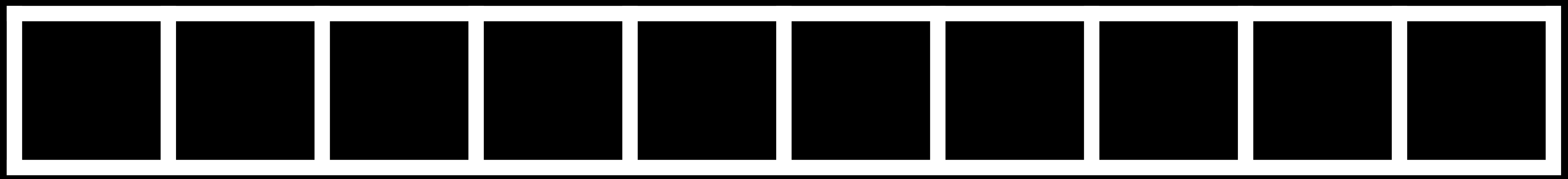
Aside: PIFO Trees

Sivaraman et al. at SIGCOMM ’16

Review: FIFO

Just an ordered collection.



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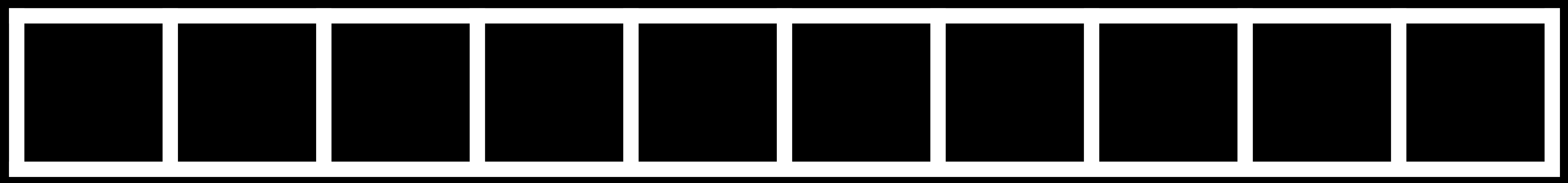

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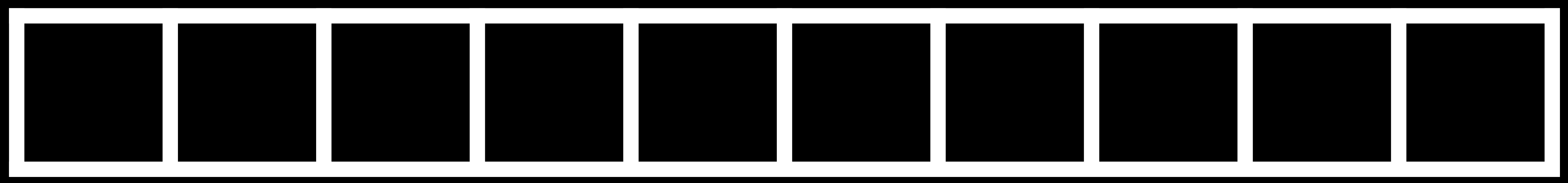

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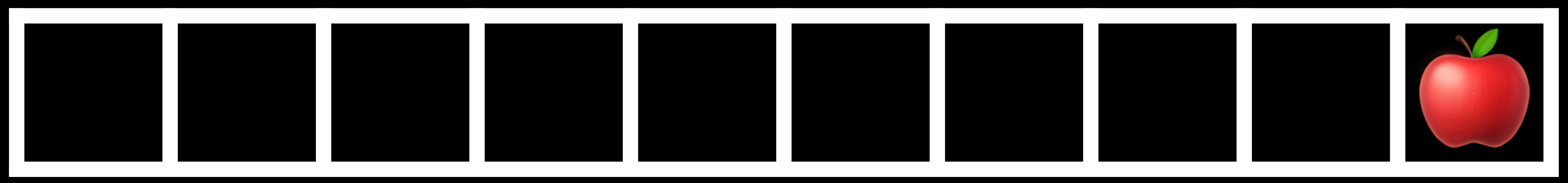

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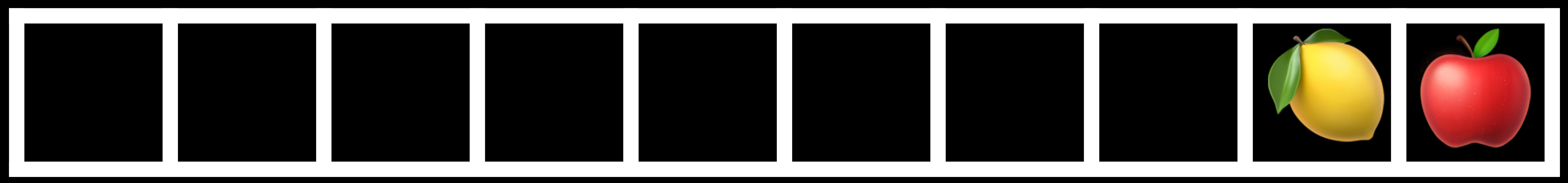

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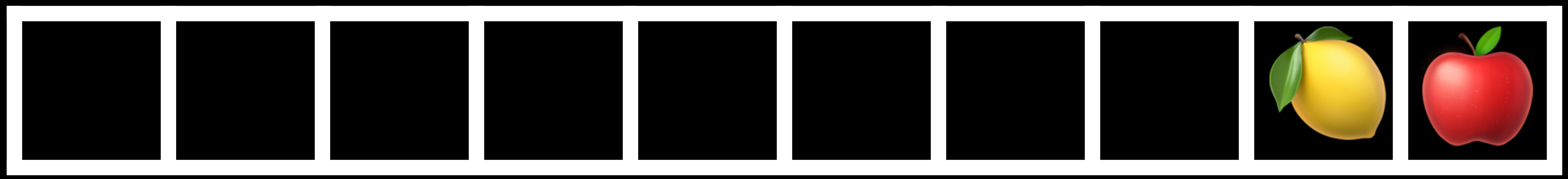

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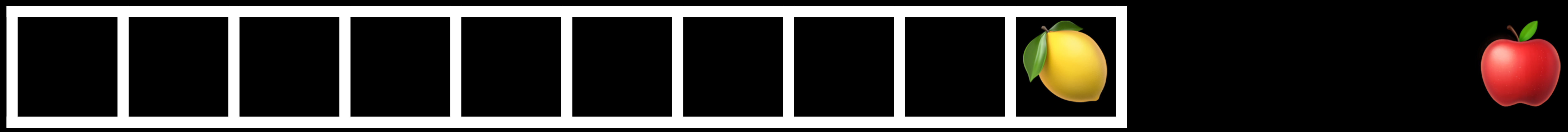

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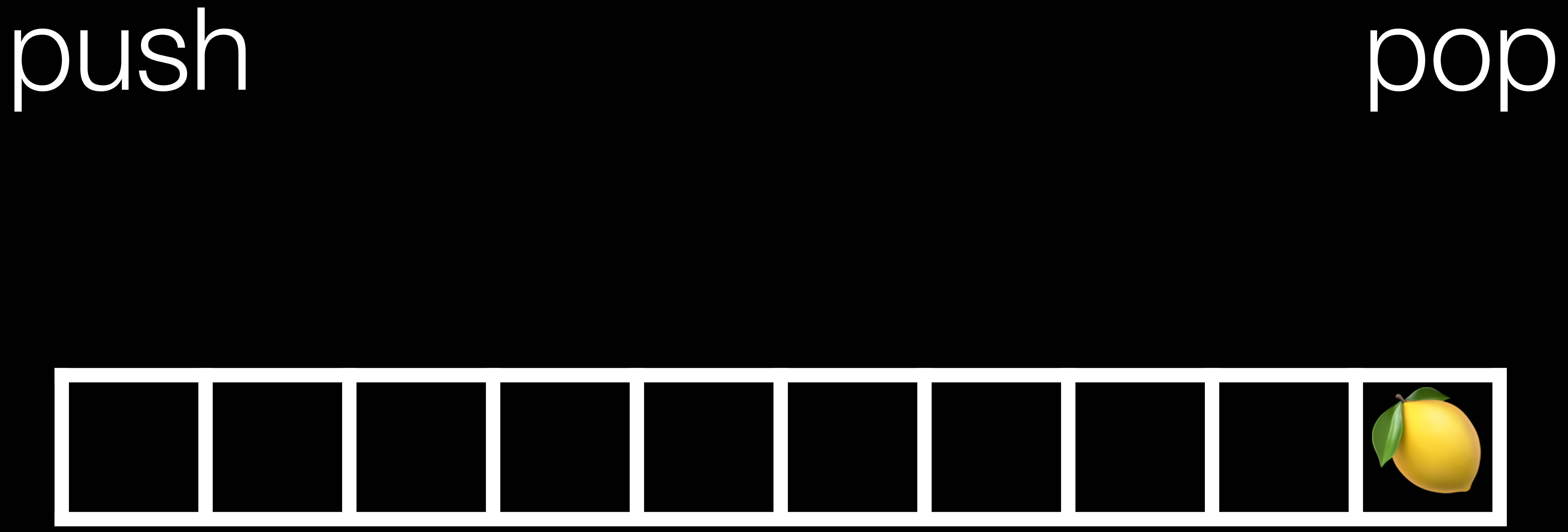

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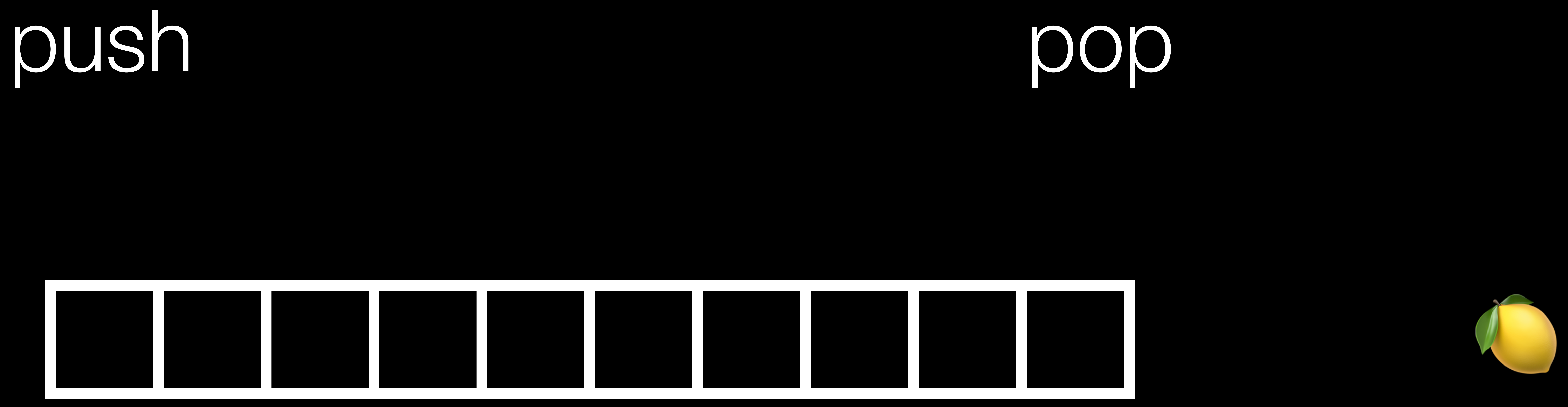

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Everything from before holds,
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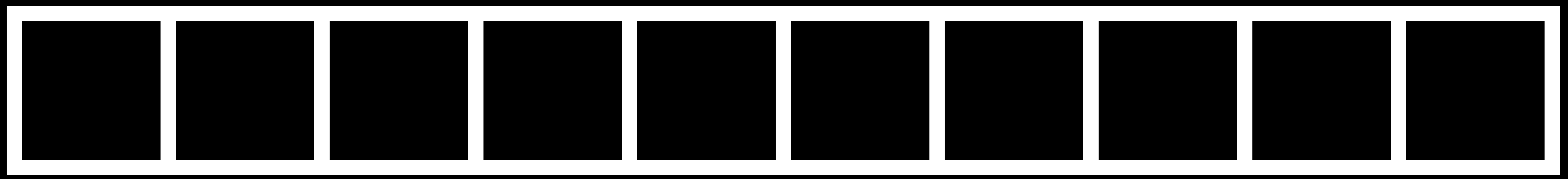

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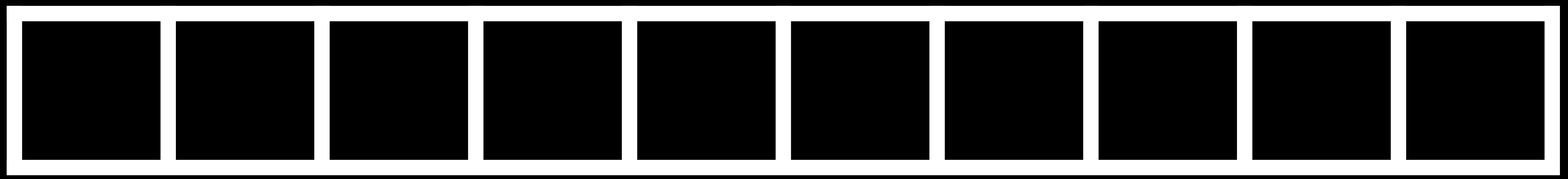

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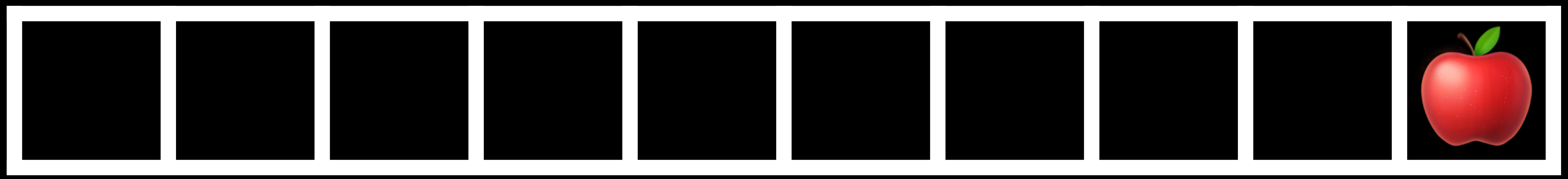

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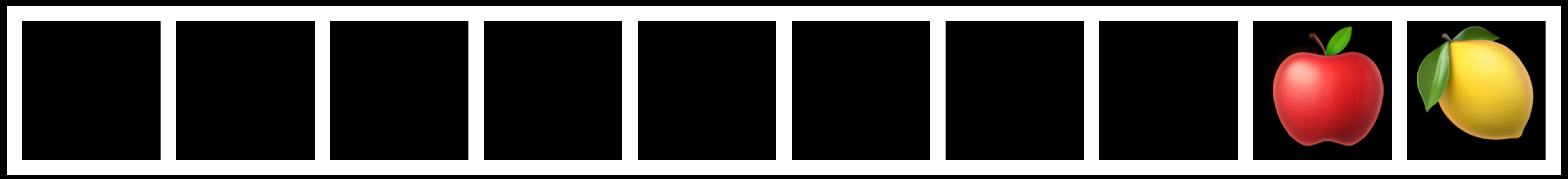

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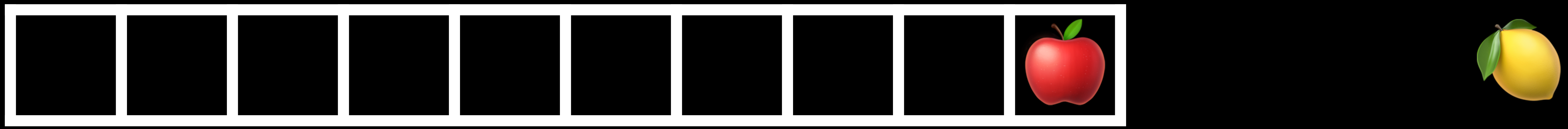

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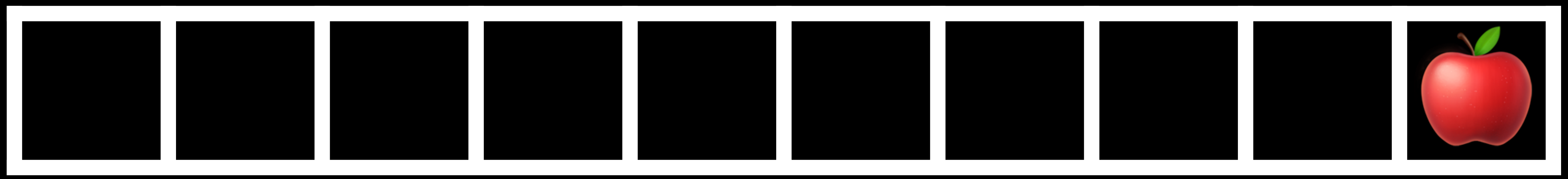

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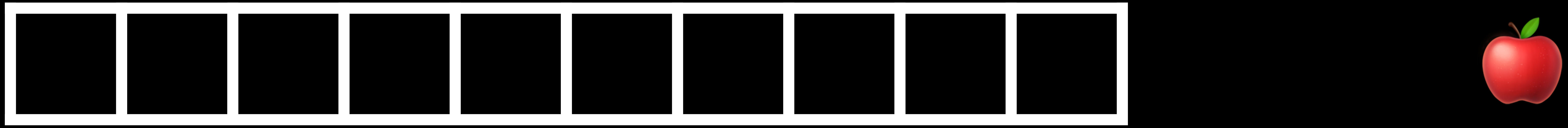

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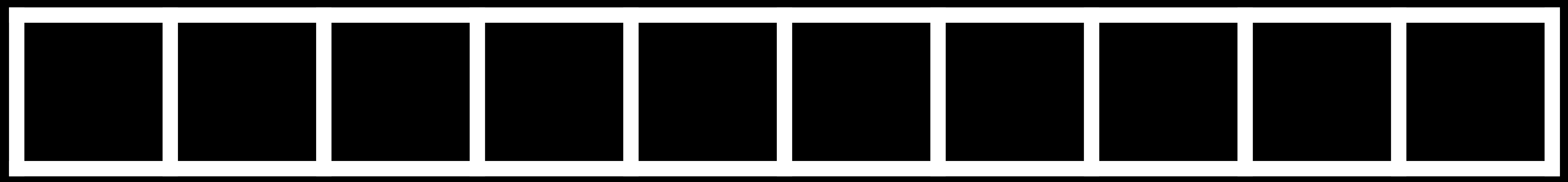
The priority need not be inherent to the item!



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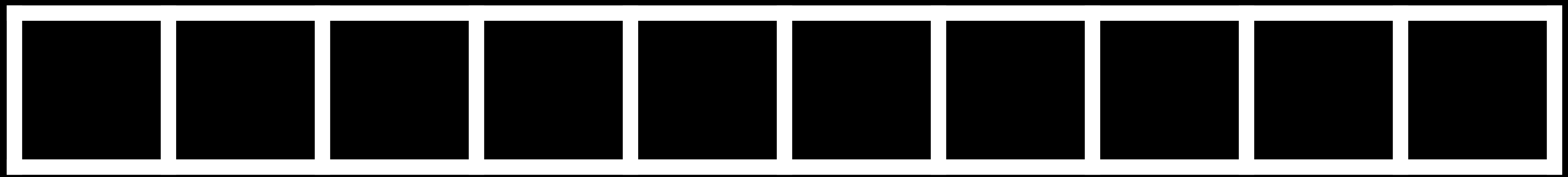

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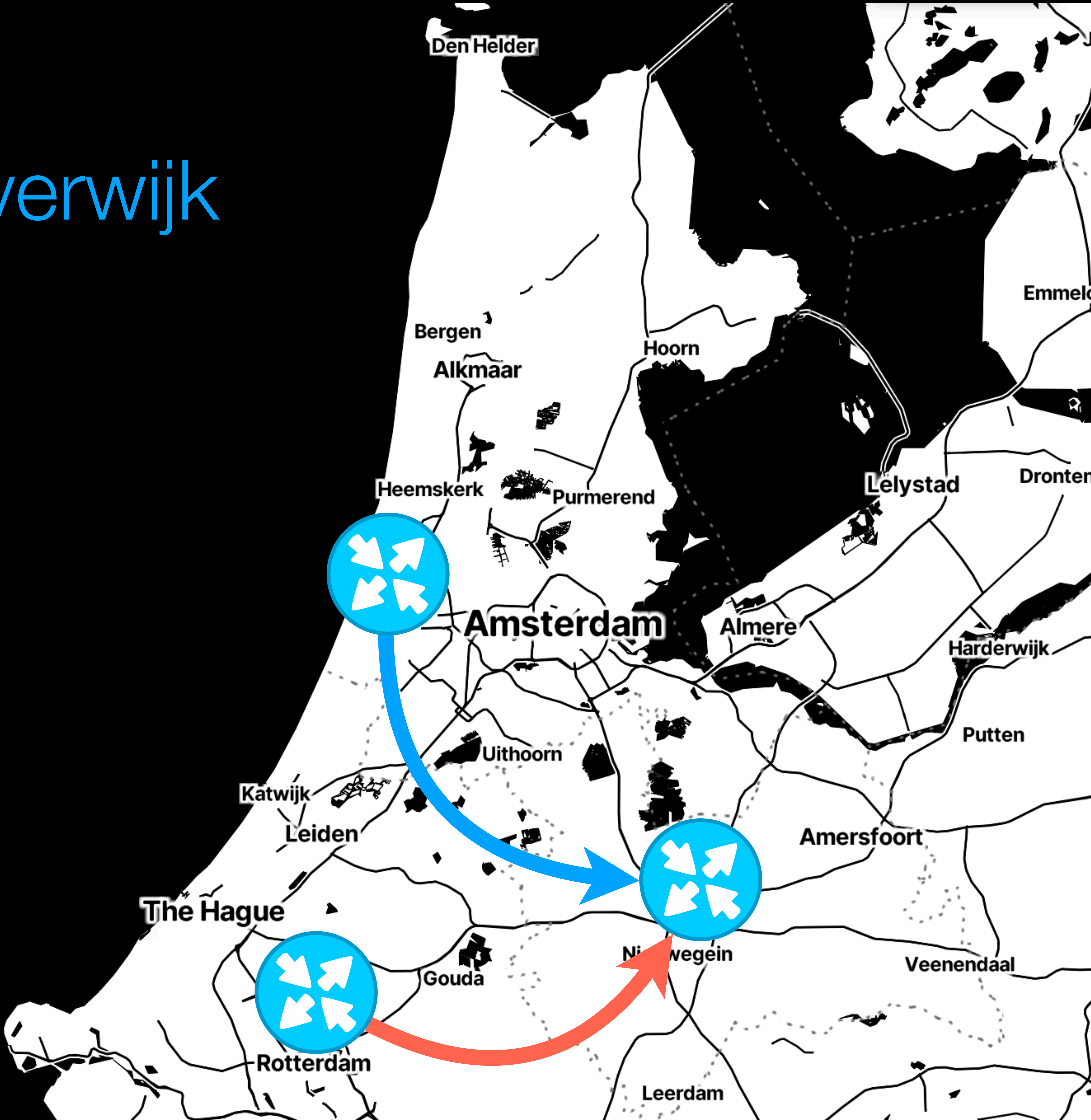
Introducing: PIFO

Just a PQ, with a ranking function,
but with *rank-ties* broken in FIFO order.

Traffic incoming from Rotterdam and Beverwijk



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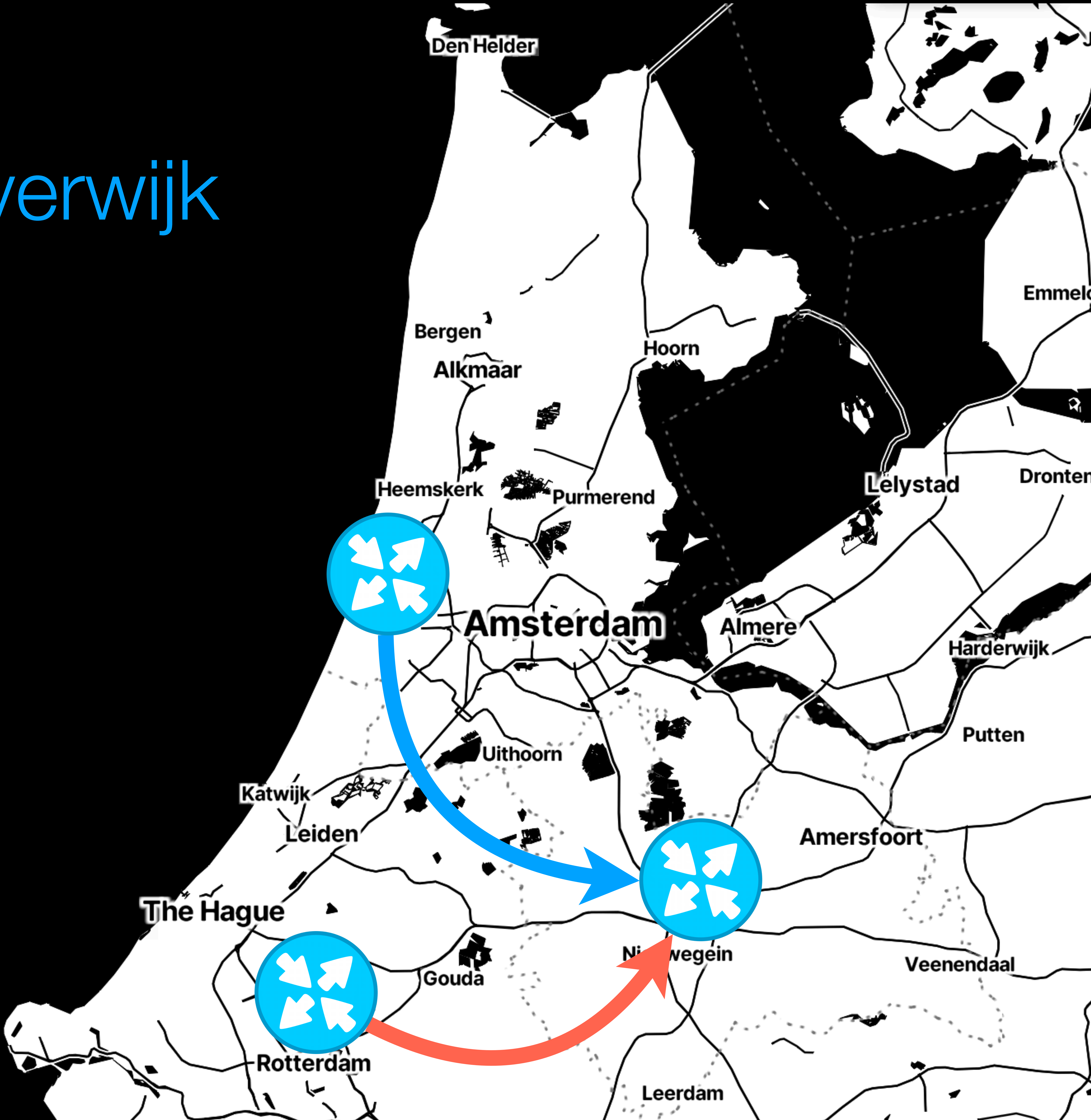

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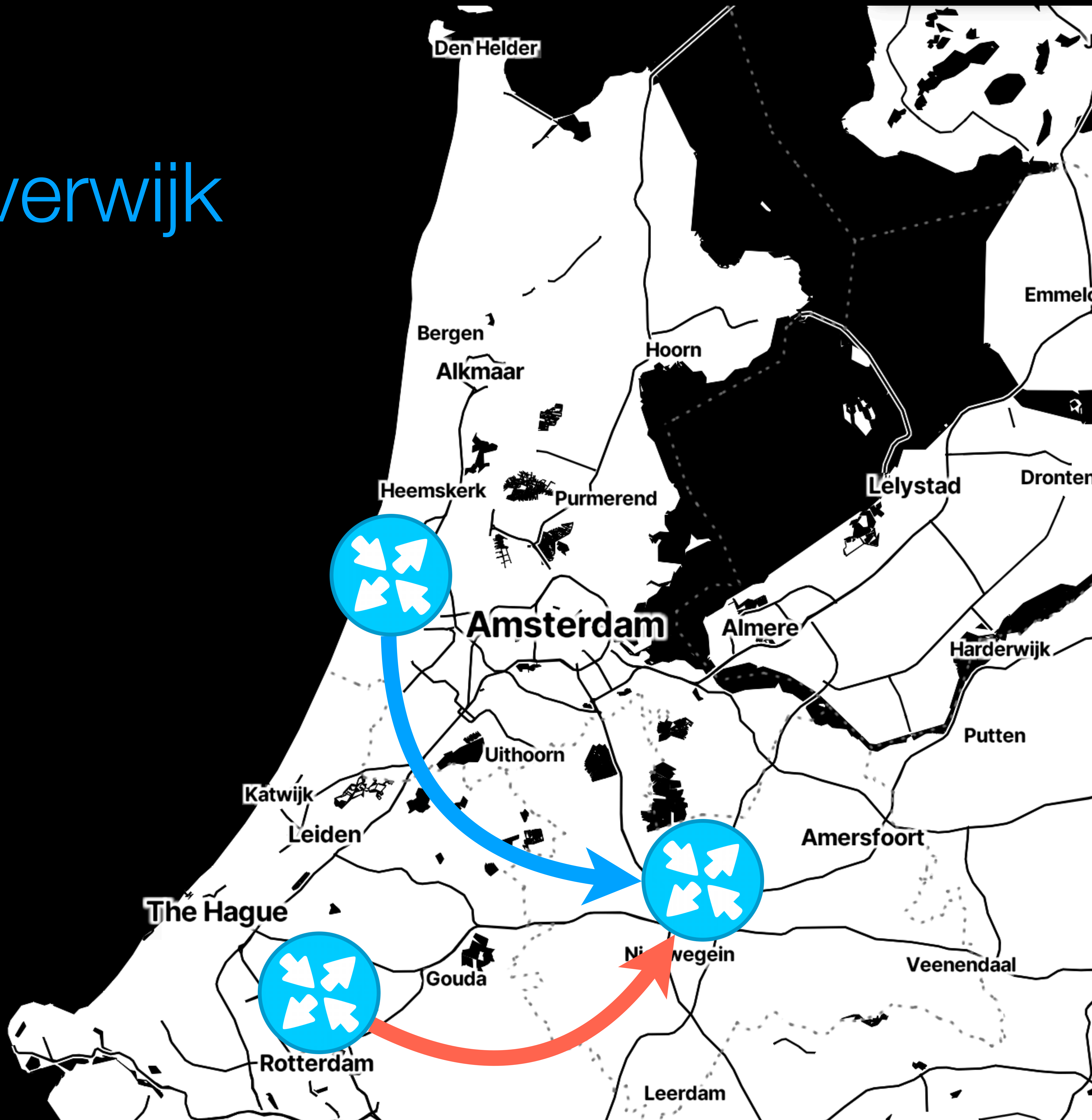

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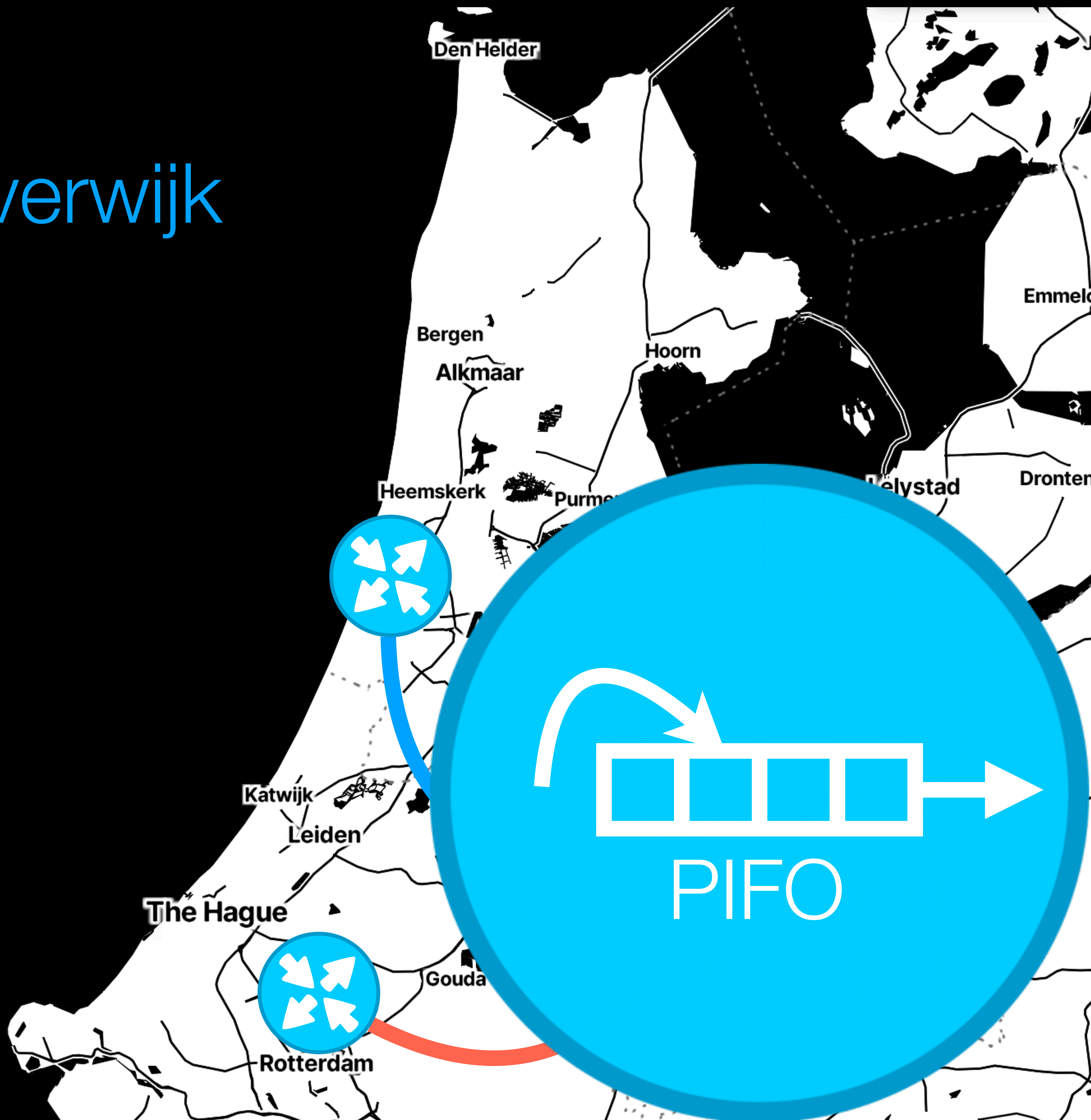

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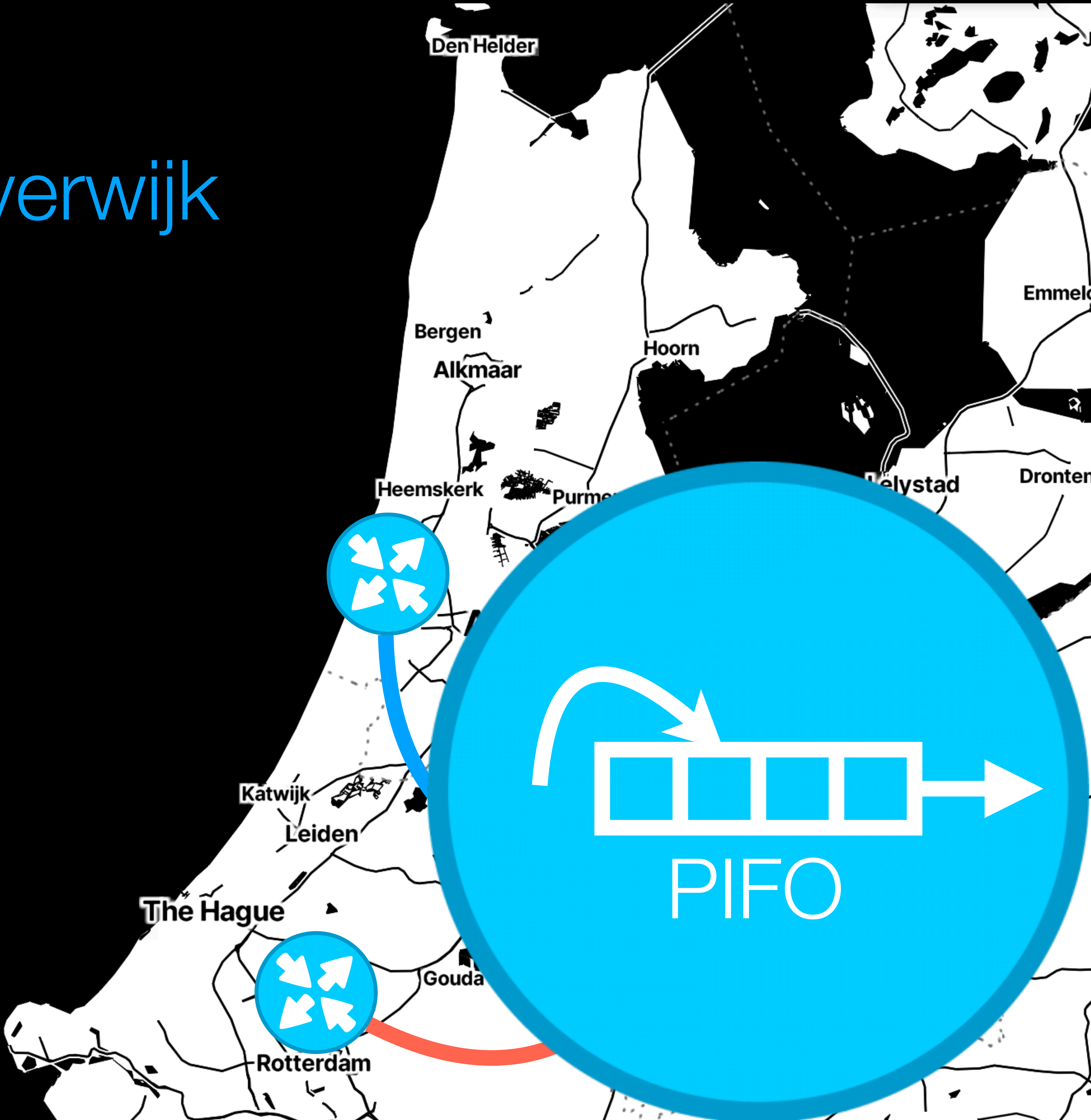

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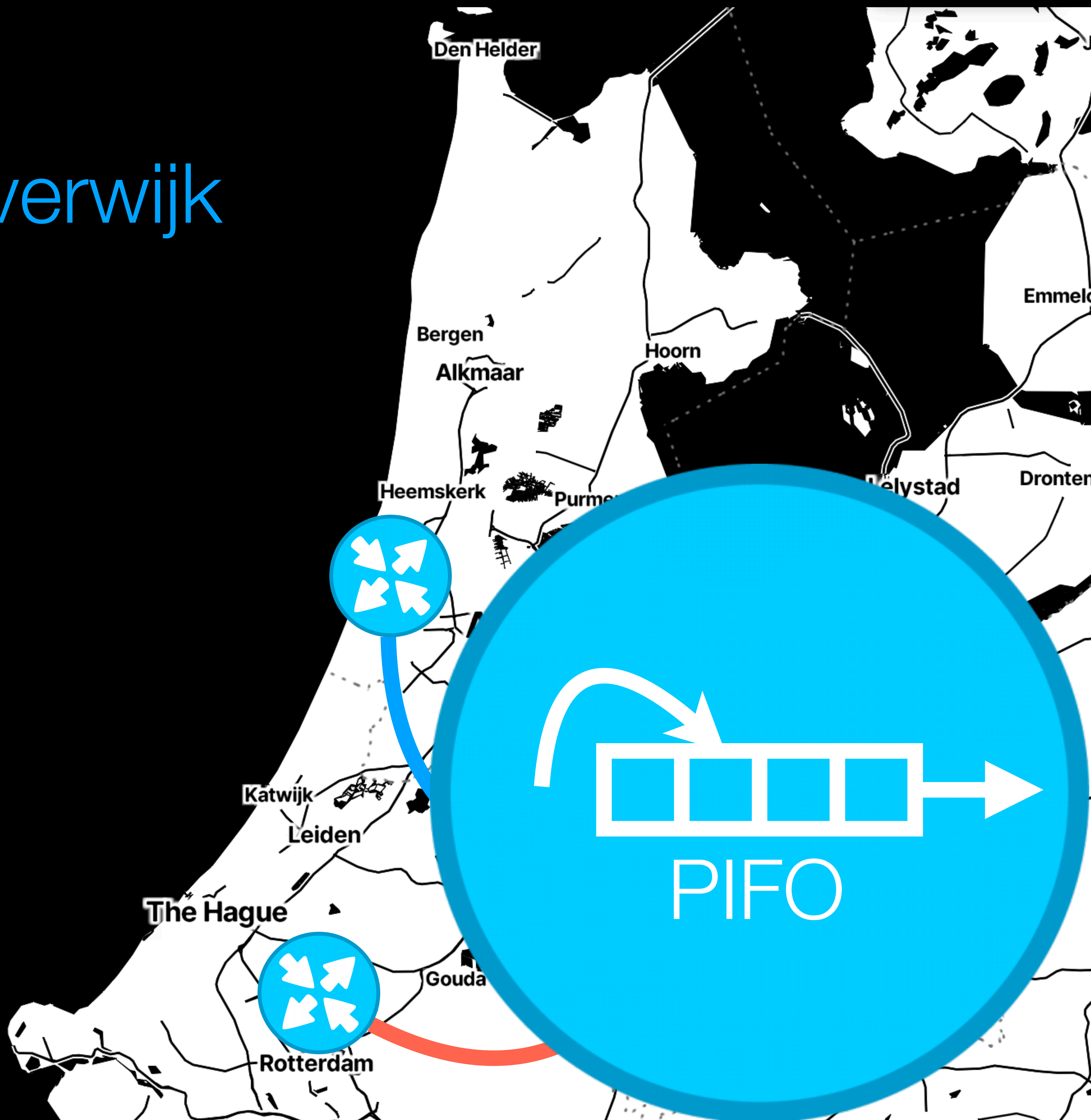
\end{tikzcd}$$



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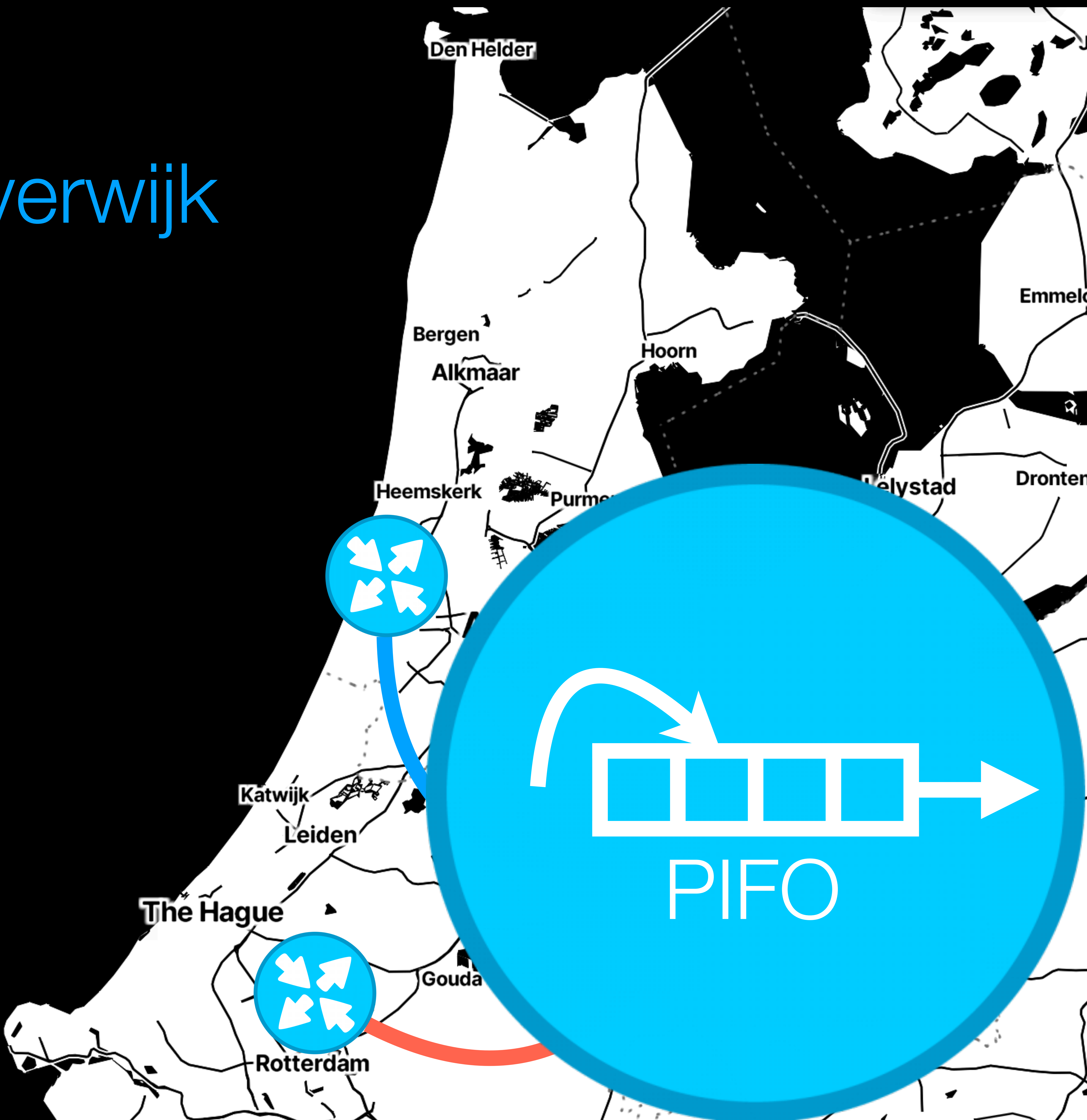

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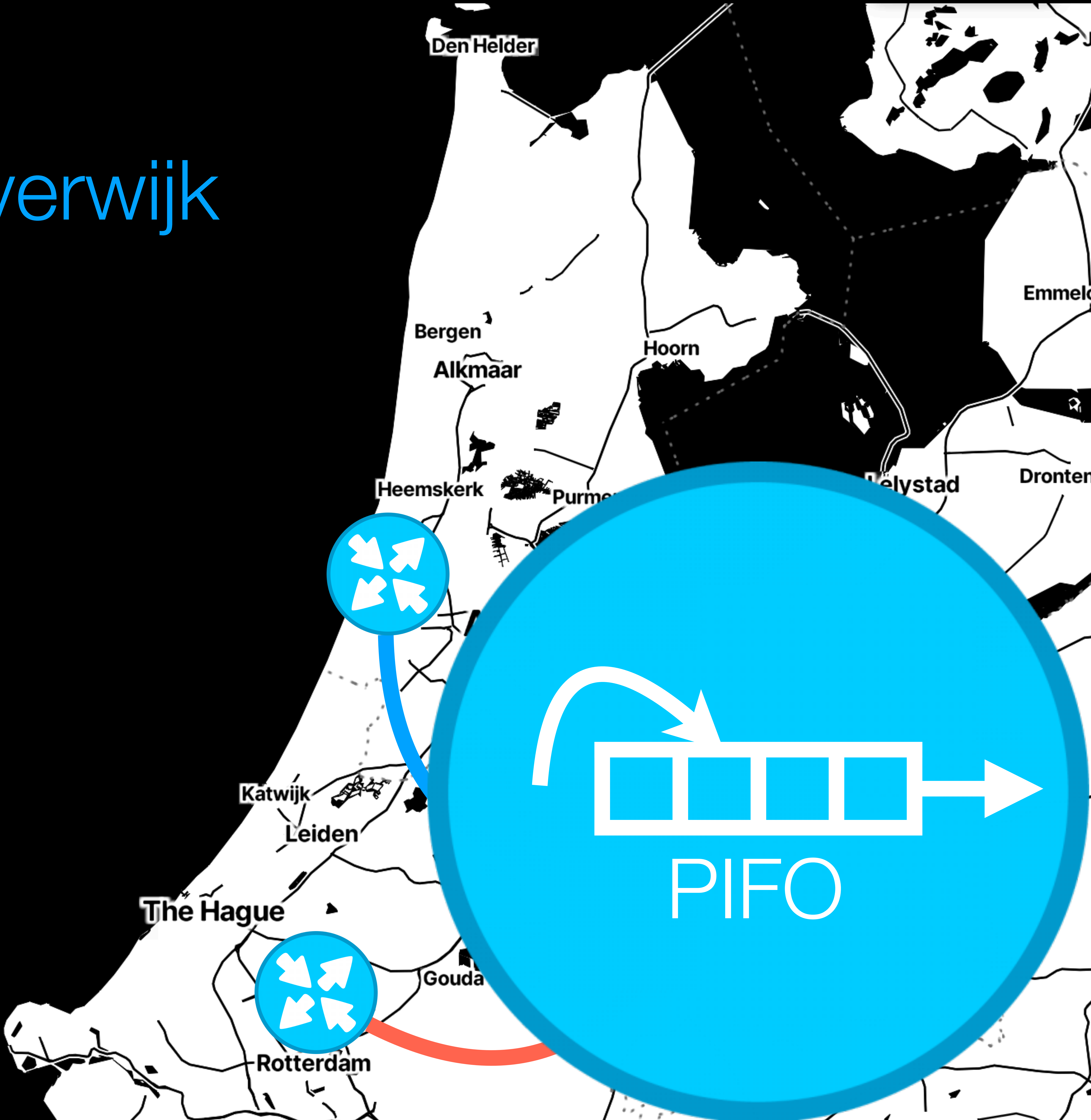

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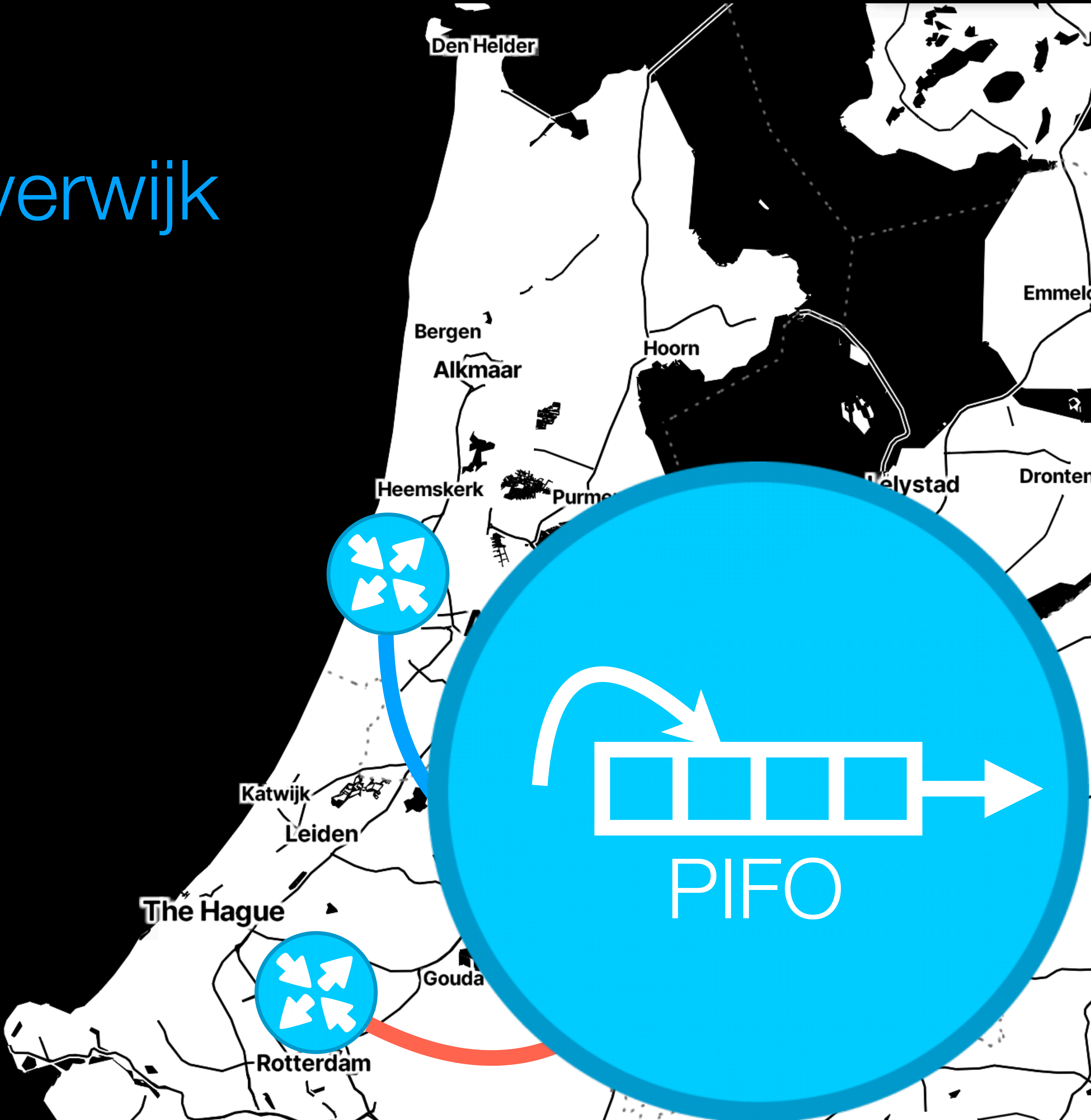

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(R,B)*

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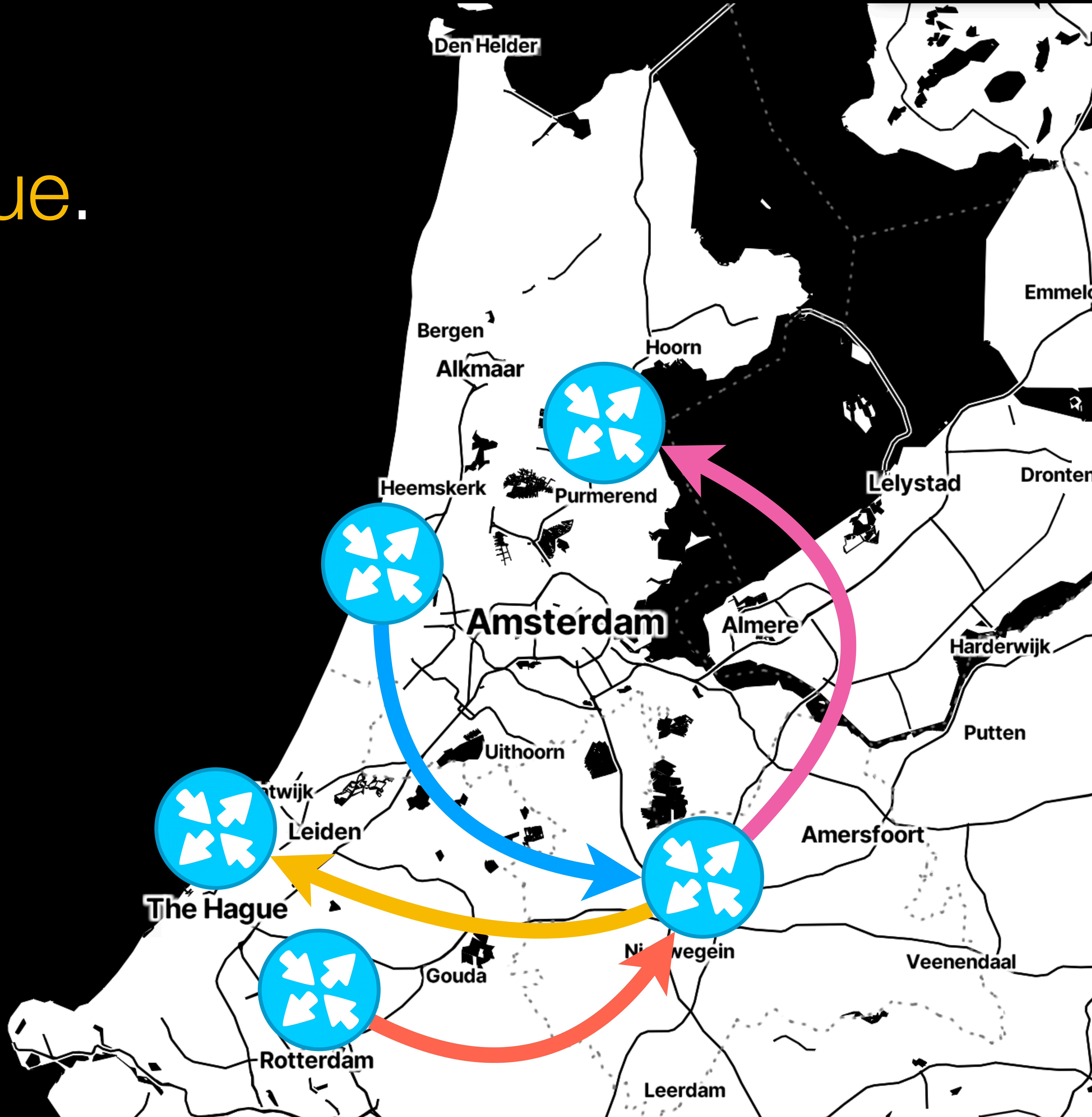

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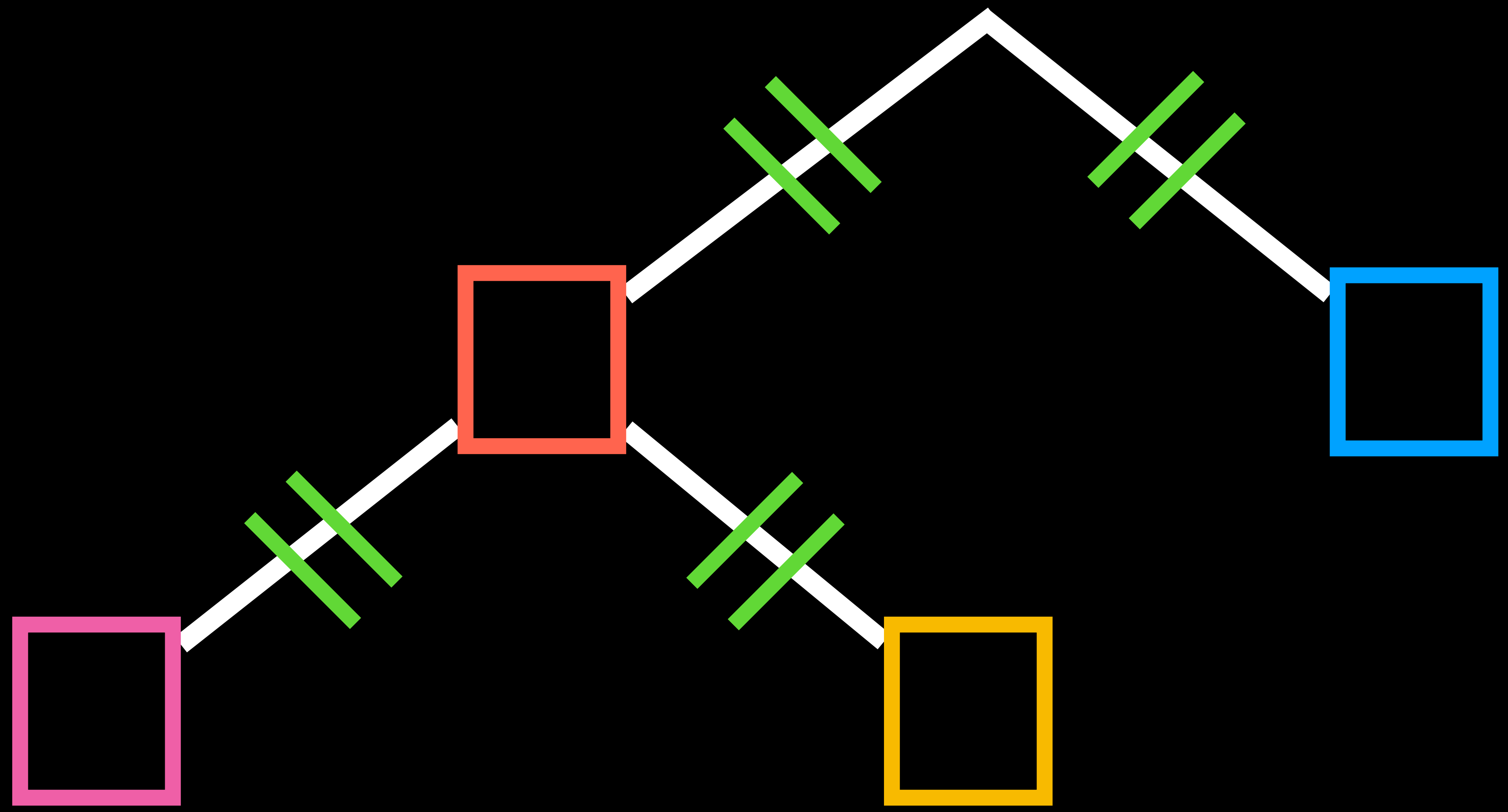

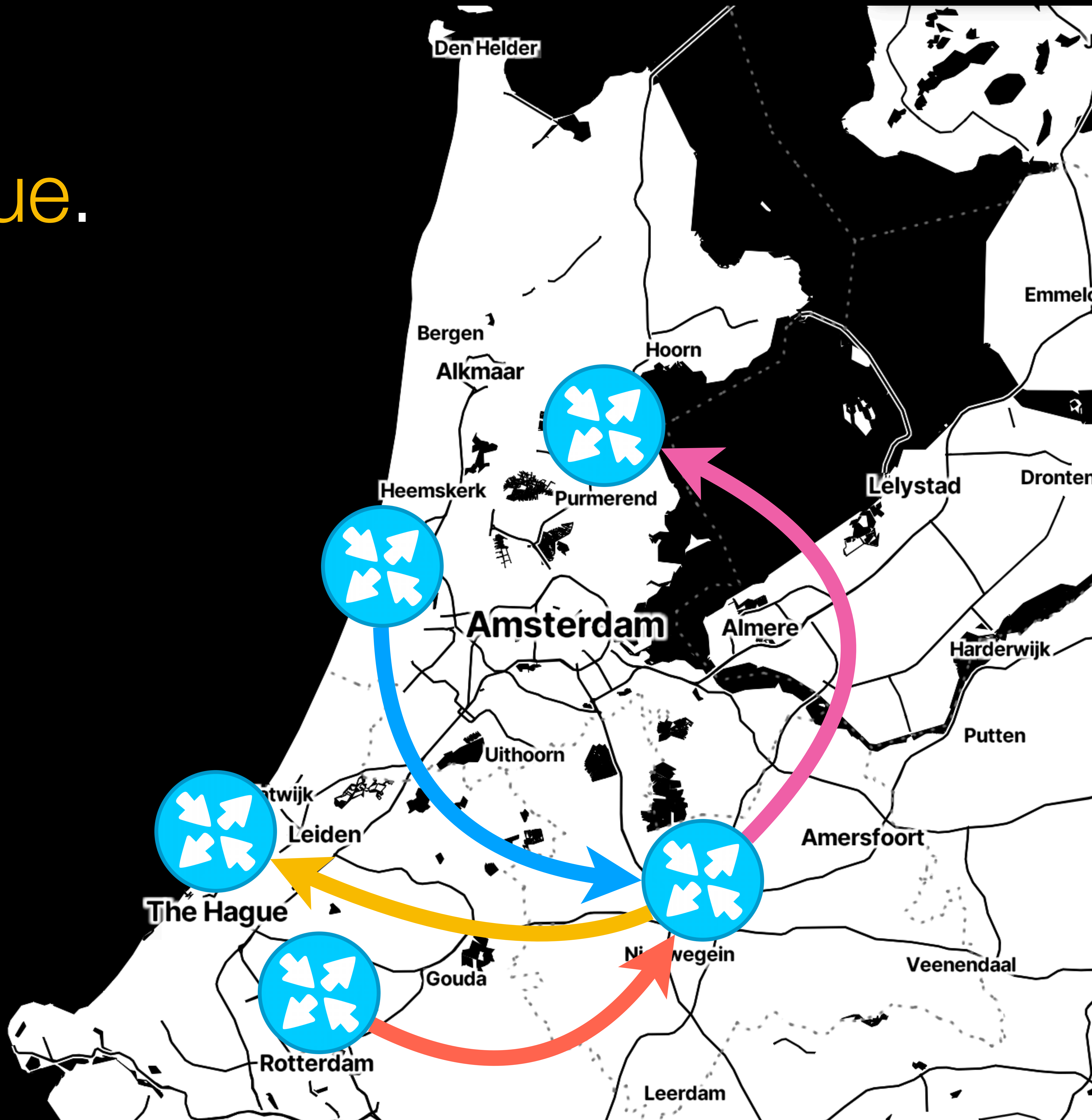

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~~$B_3, B_2, P_2, T_1, B_1, P_1$~~
~~$B_3, B_2, P_2, B_1, T_1, P_1$~~
~~$B_3, B_2, P_2, B_1, P_1, T_1$~~

$T_1 \longrightarrow B_3, B_2, P_2, B_1, P_1$

$B_3, P_2, B_2, T_1, B_1, P_1$
$B_3, P_2, B_2, P_1, B_1, T_1$

Dual goals:
interleave R and B;
interleave P and T.

~~B_3, B_2, P_2, T_1, B_1, P_1~~
~~B_3, B_2, P_2, B_1, T_1, P_1~~
~~B_3, B_2, P_2, B_1, P_1, T_1~~

T_1 → B_3, B_2, P_2, B_1, P_1

B_3, 🟥, B_2, 🟥, B_1, 🟥
B_3, 🟥, B_2, 🟥, B_1, 🟥

Dual goals:
interleave R and B;
interleave P and T.

~~$B_3, B_2, P_2, T_1, B_1, P_1$~~
~~$B_3, B_2, P_2, B_1, T_1, P_1$~~
~~$B_3, B_2, P_2, B_1, P_1, T_1$~~

$T_1 \longrightarrow B_3, B_2, P_2, B_1, P_1$

$B_3, P_2, B_2, T_1, B_1, P_1$
$B_3, P_2, B_2, P_1, B_1, T_1$

Dual goals:
interleave R and B;
interleave P and T.

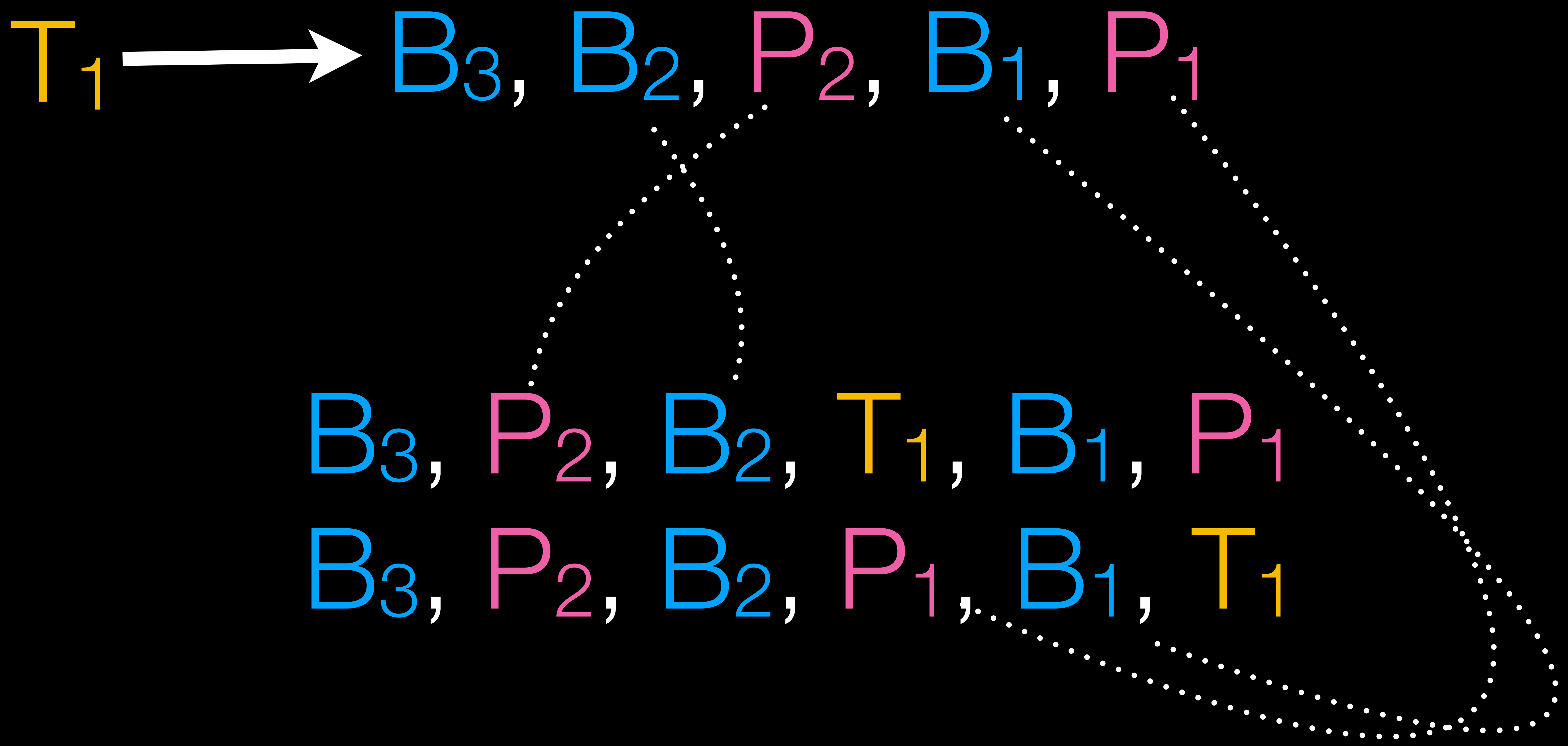
~~$B_3, B_2, P_2, T_1, B_1, P_1$~~
~~$B_3, B_2, P_2, B_1, T_1, P_1$~~
~~$B_3, B_2, P_2, B_1, P_1, T_1$~~

$T_1 \longrightarrow B_3, B_2, P_2, B_1, P_1$

$B_3, P_2, B_2, T_1, B_1, P_1$
$B_3, P_2, B_2, P_1, B_1, T_1$

Dual goals:
interleave R and B;
interleave P and T.

~~B_3, B_2, P_2, T_1, B_1, P_1~~
~~B_3, B_2, P_2, B_1, T_1, P_1~~
~~B_3, B_2, P_2, B_1, P_1, T_1~~



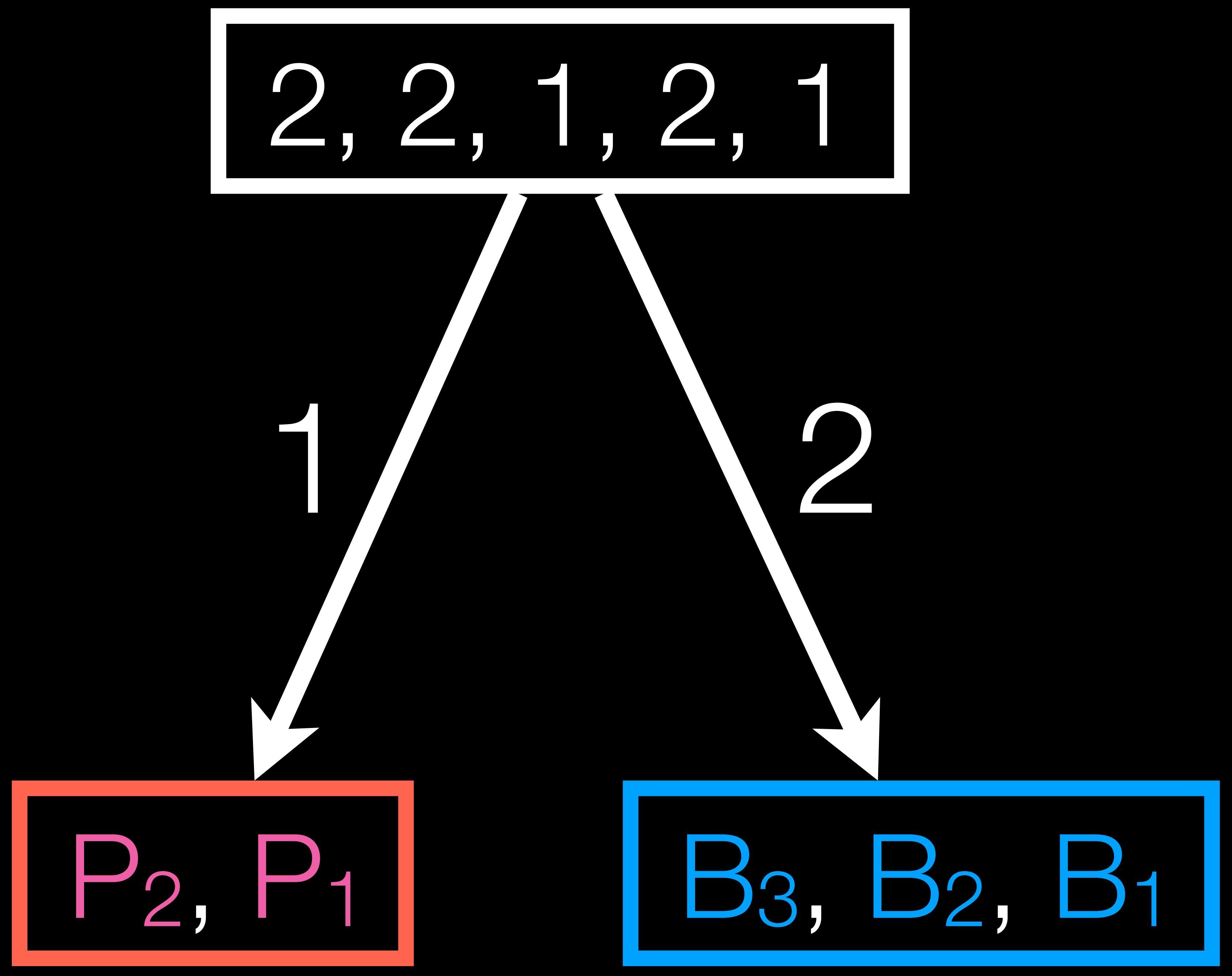
T_1 → B_3, B_2, P_2, B_1, P_1

B_3, P_2, B_2, T_1, B_1, P_1
B_3, P_2, B_2, P_1, B_1, T_1

Enqueueing a packet can require the reordering of buffered packets.

No PIFO can do this.

Introducing: PIFO trees



2, 2, 1, 2, 1

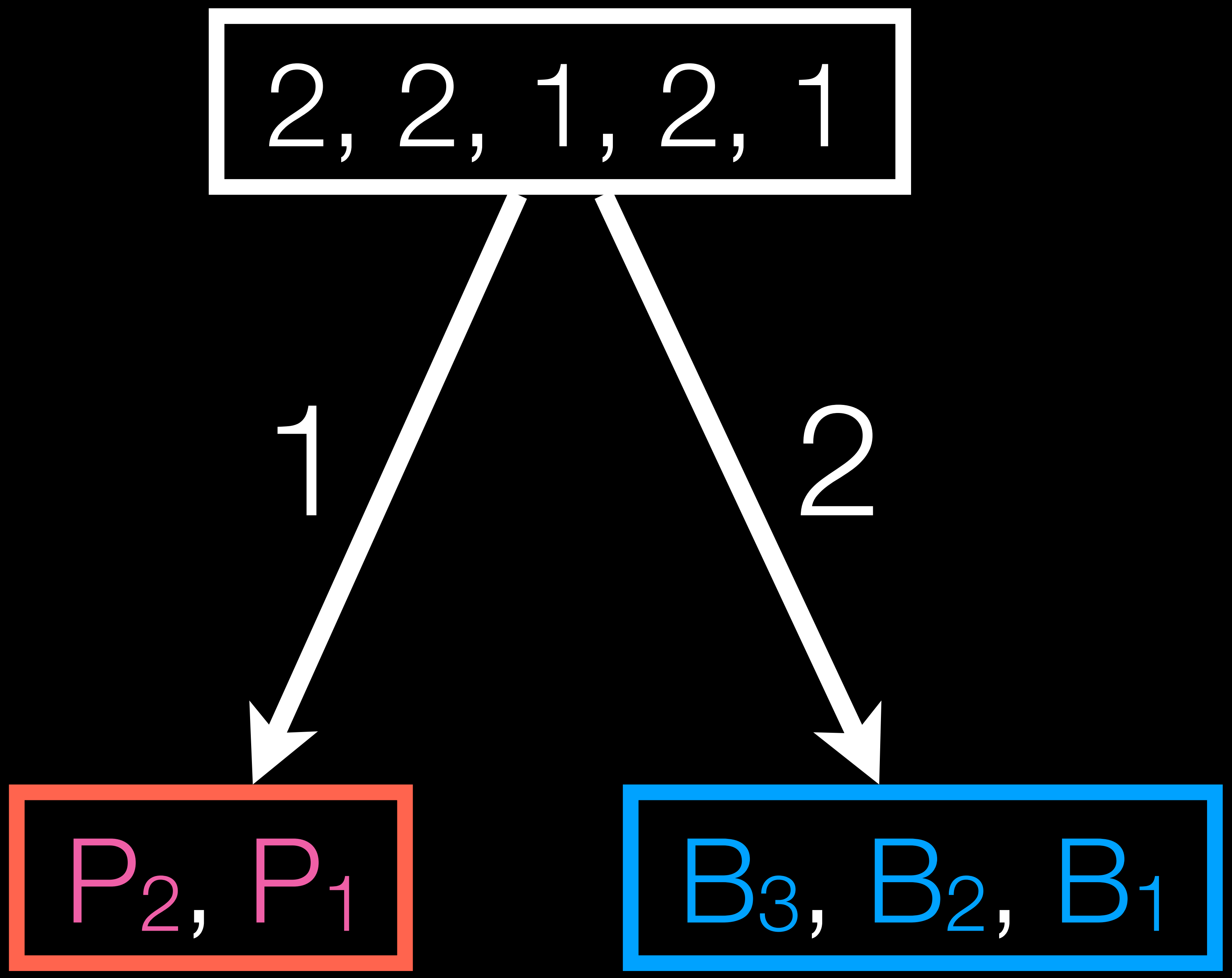
1

2

P_2, P_1

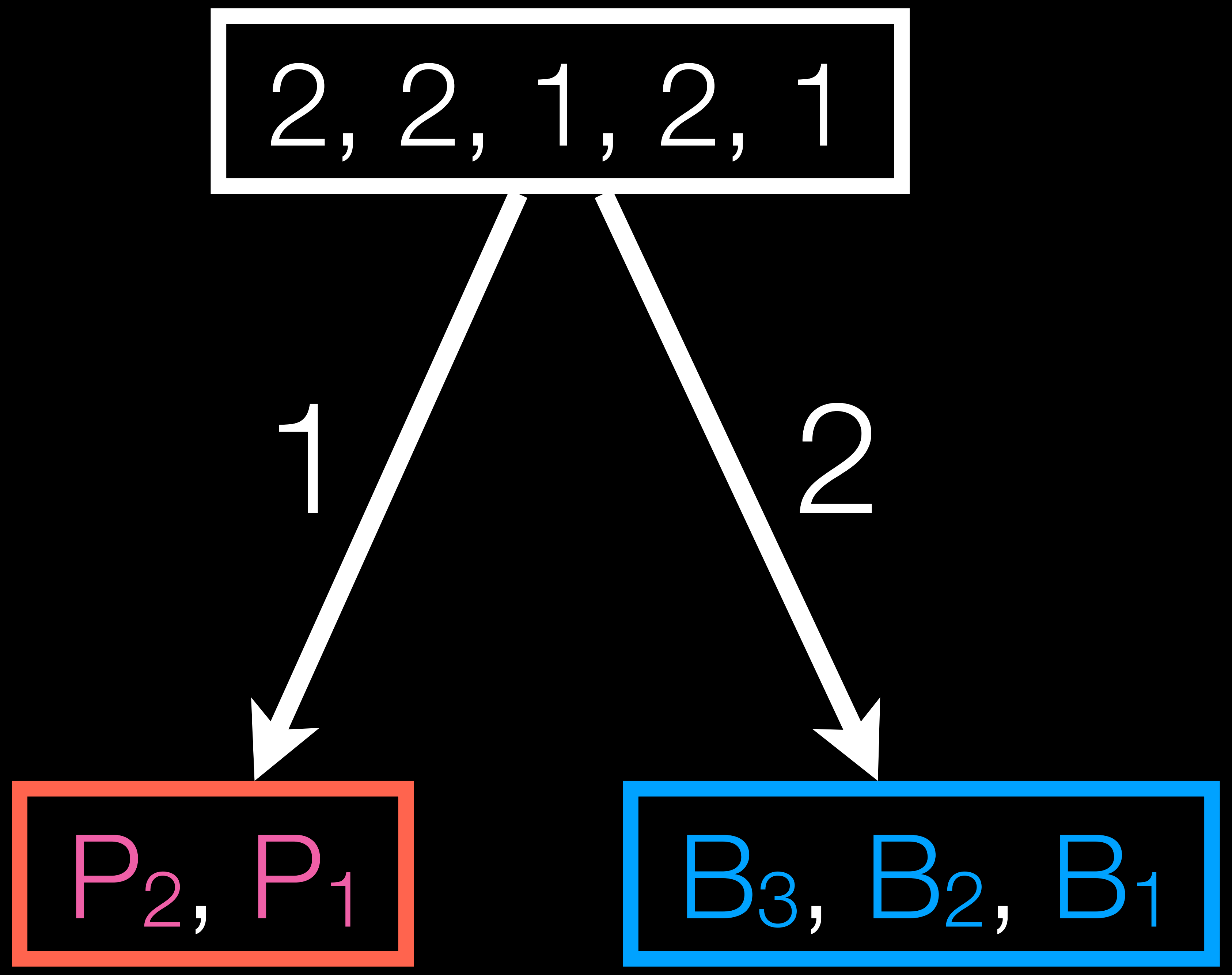
B_3, B_2, B_1

Introducing: PIFO trees



This behaves like a queue!

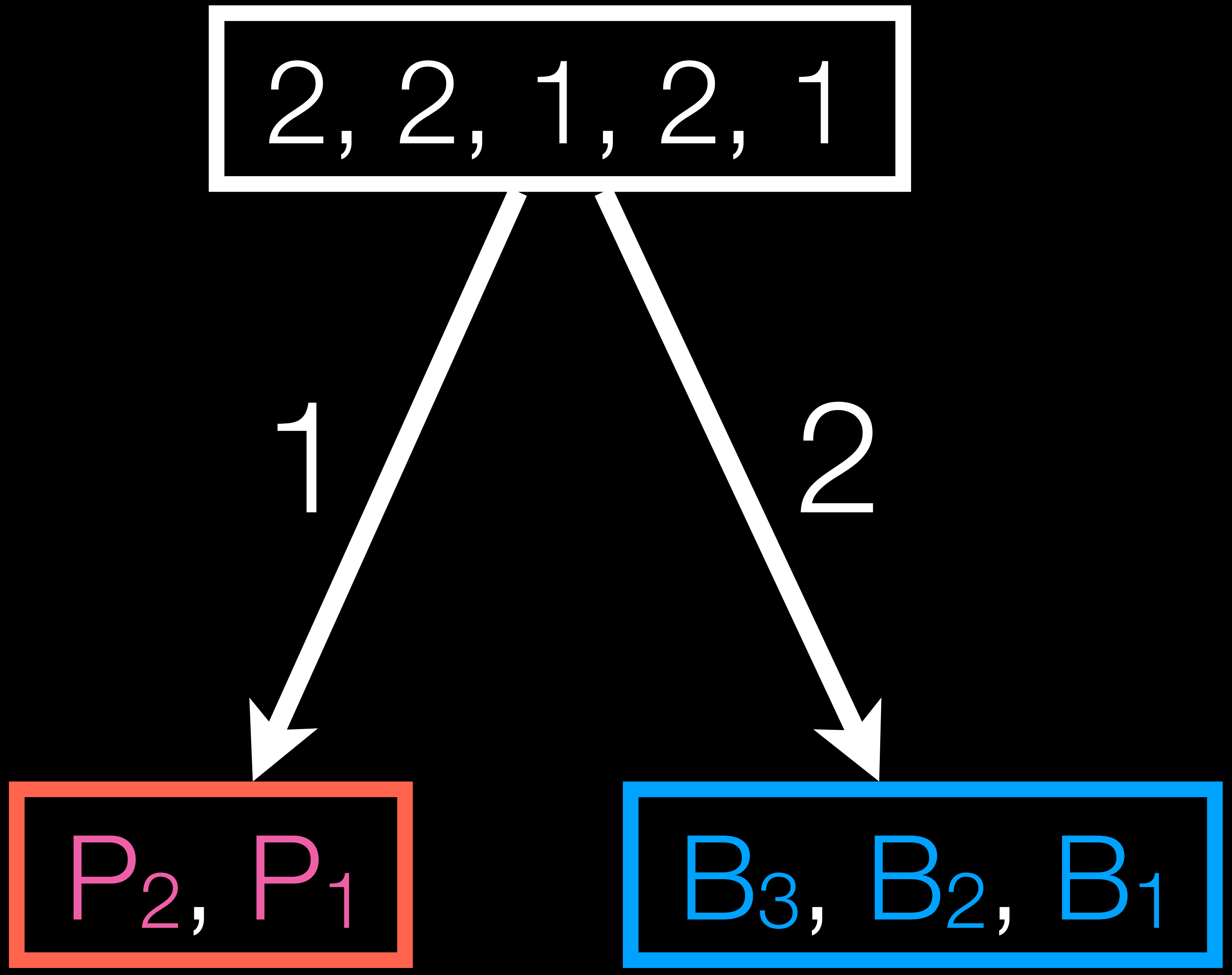
Introducing: PIFO trees



This behaves like a queue!

How do we pop it?

Introducing: PIFO trees



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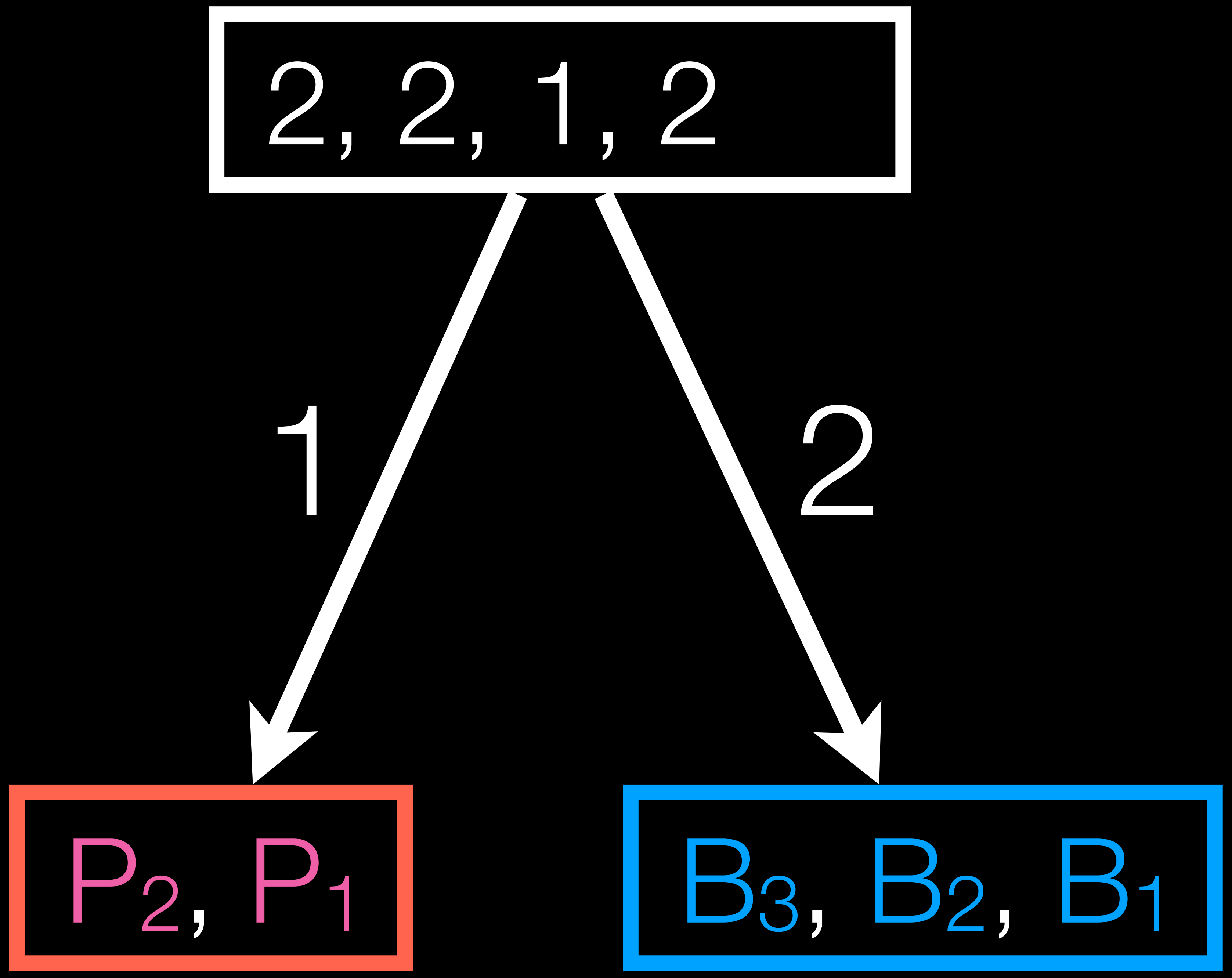

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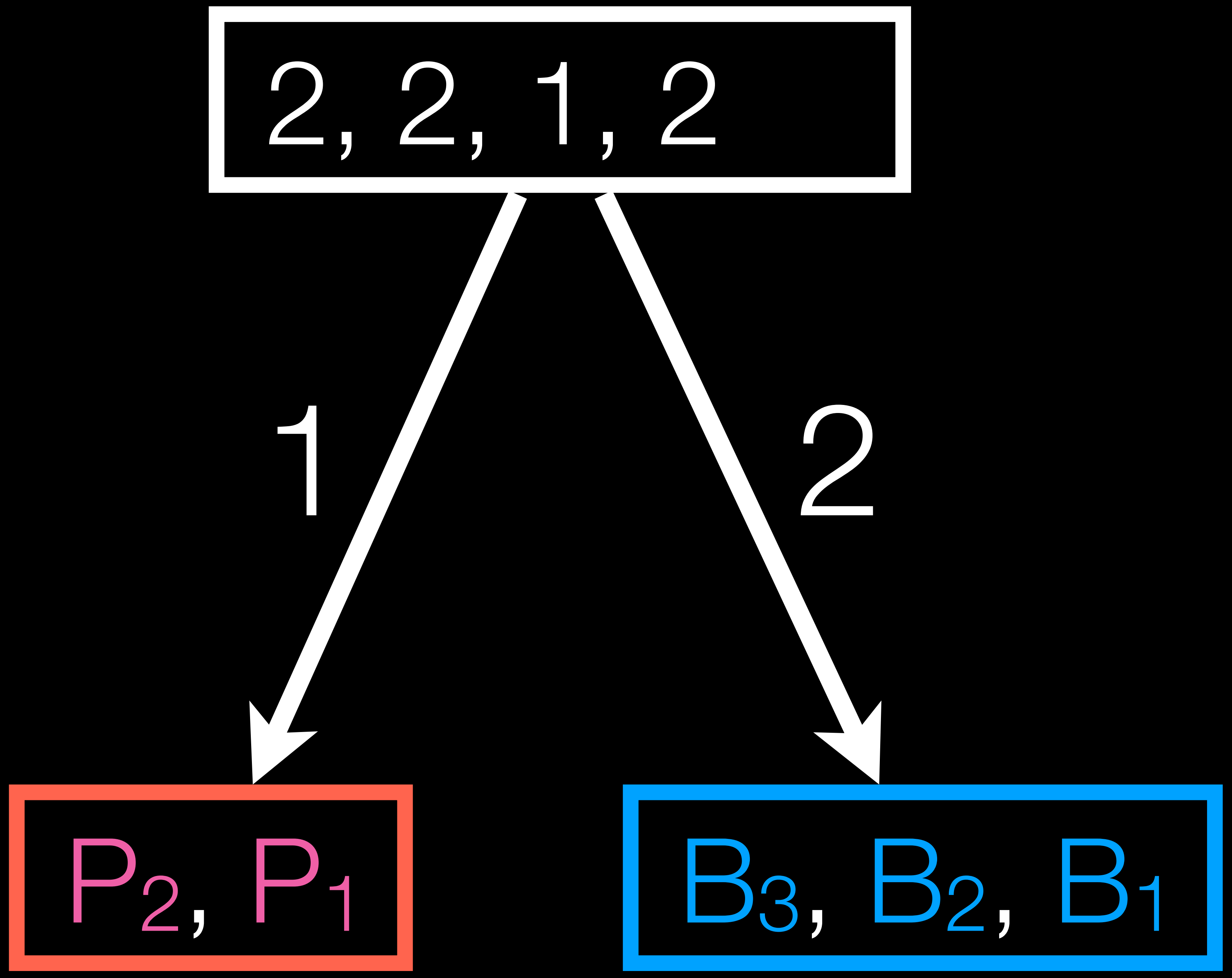

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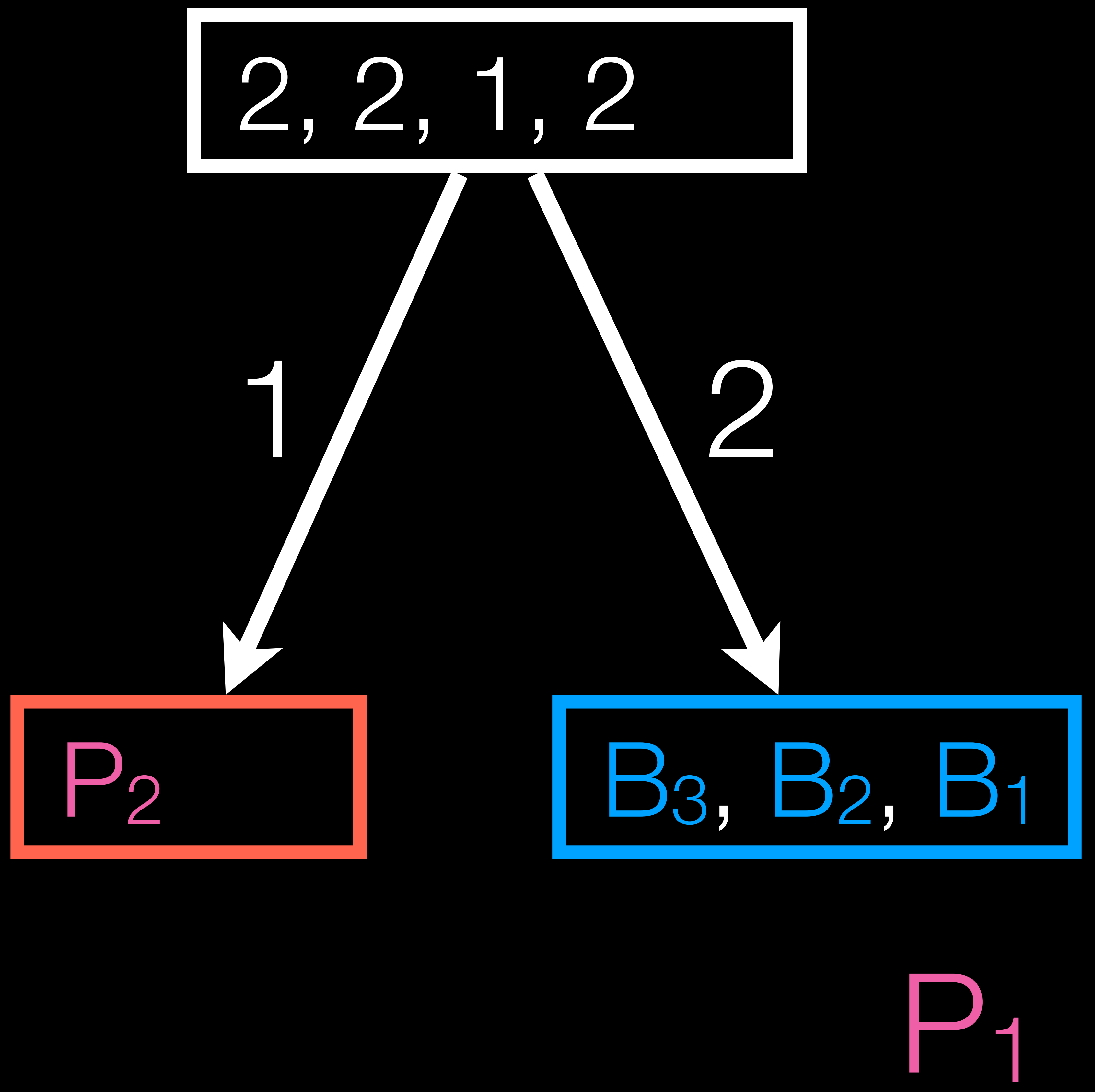

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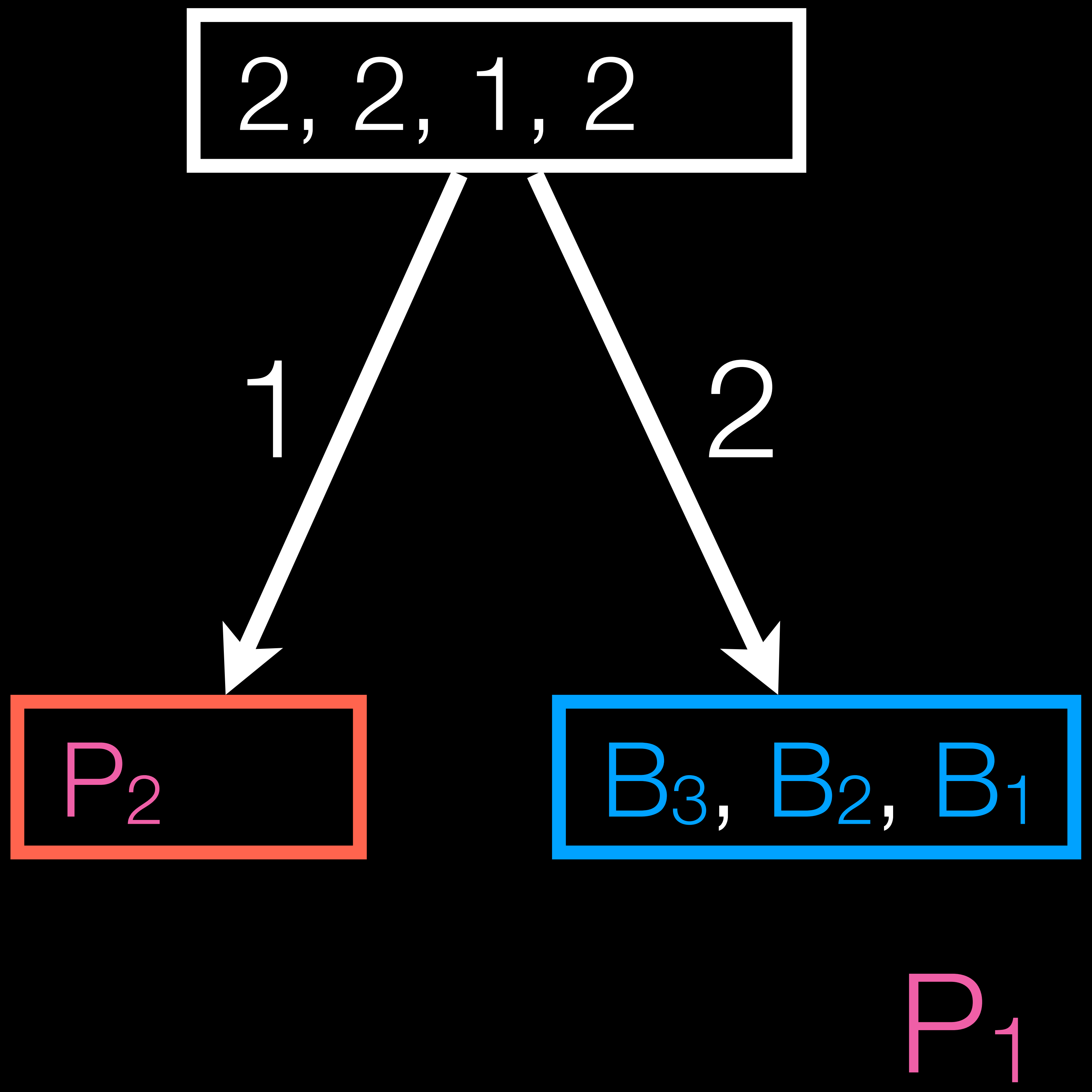
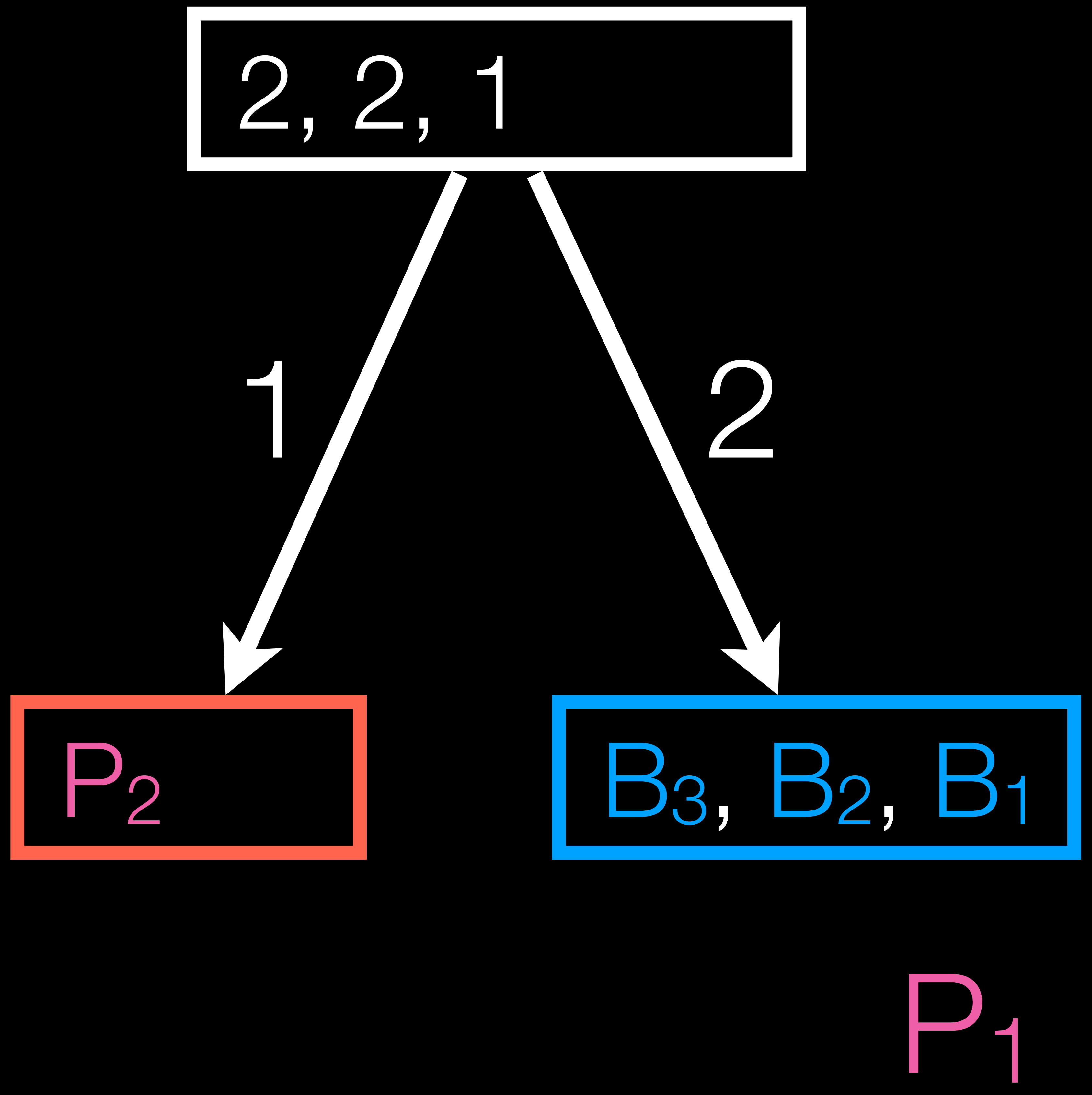
Introducing: PIFO trees



This behaves like a queue!

How do we pop it?

Introducing: PIFO trees



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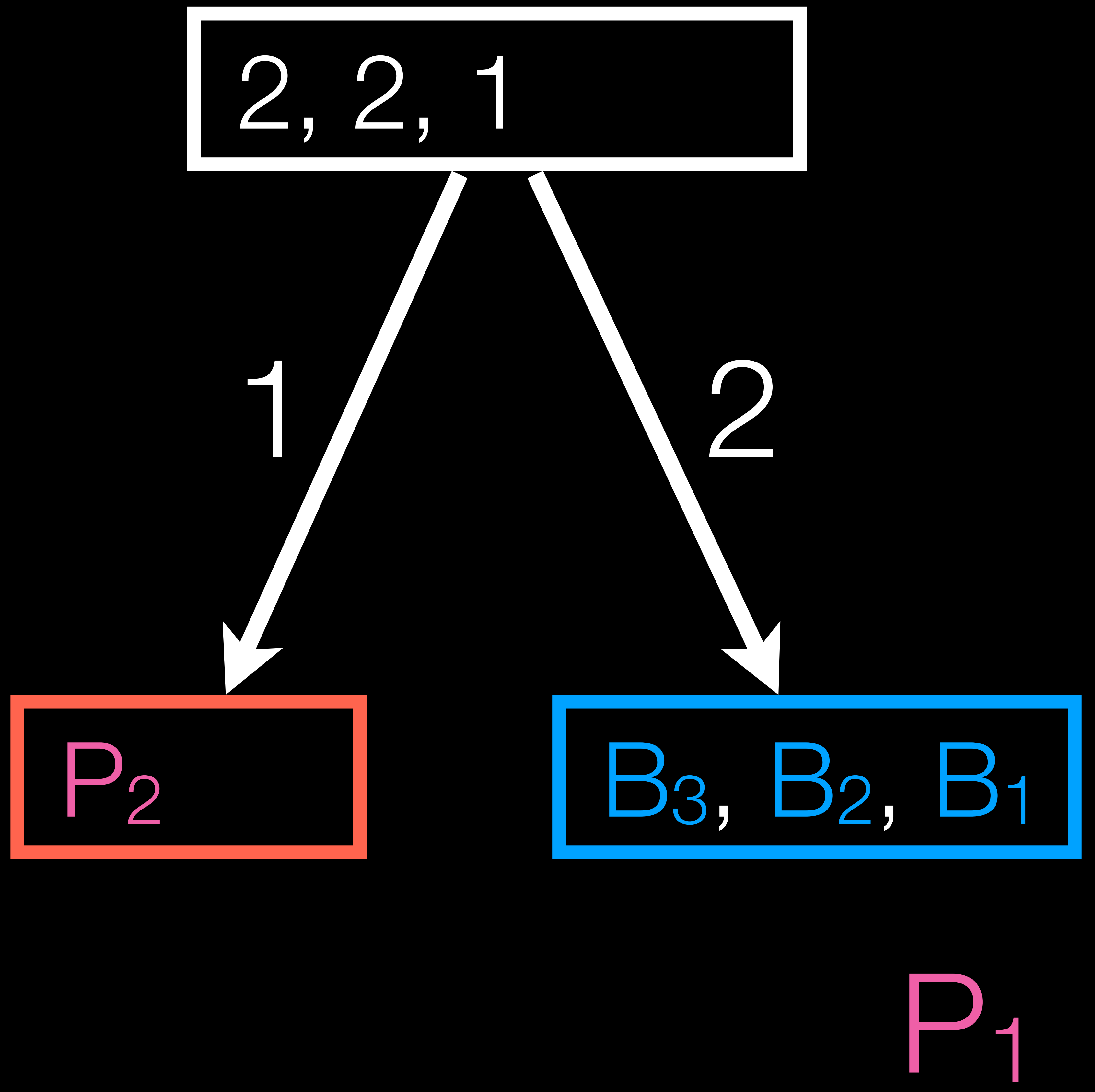
Introducing: PIFO trees



This behaves like a queue!

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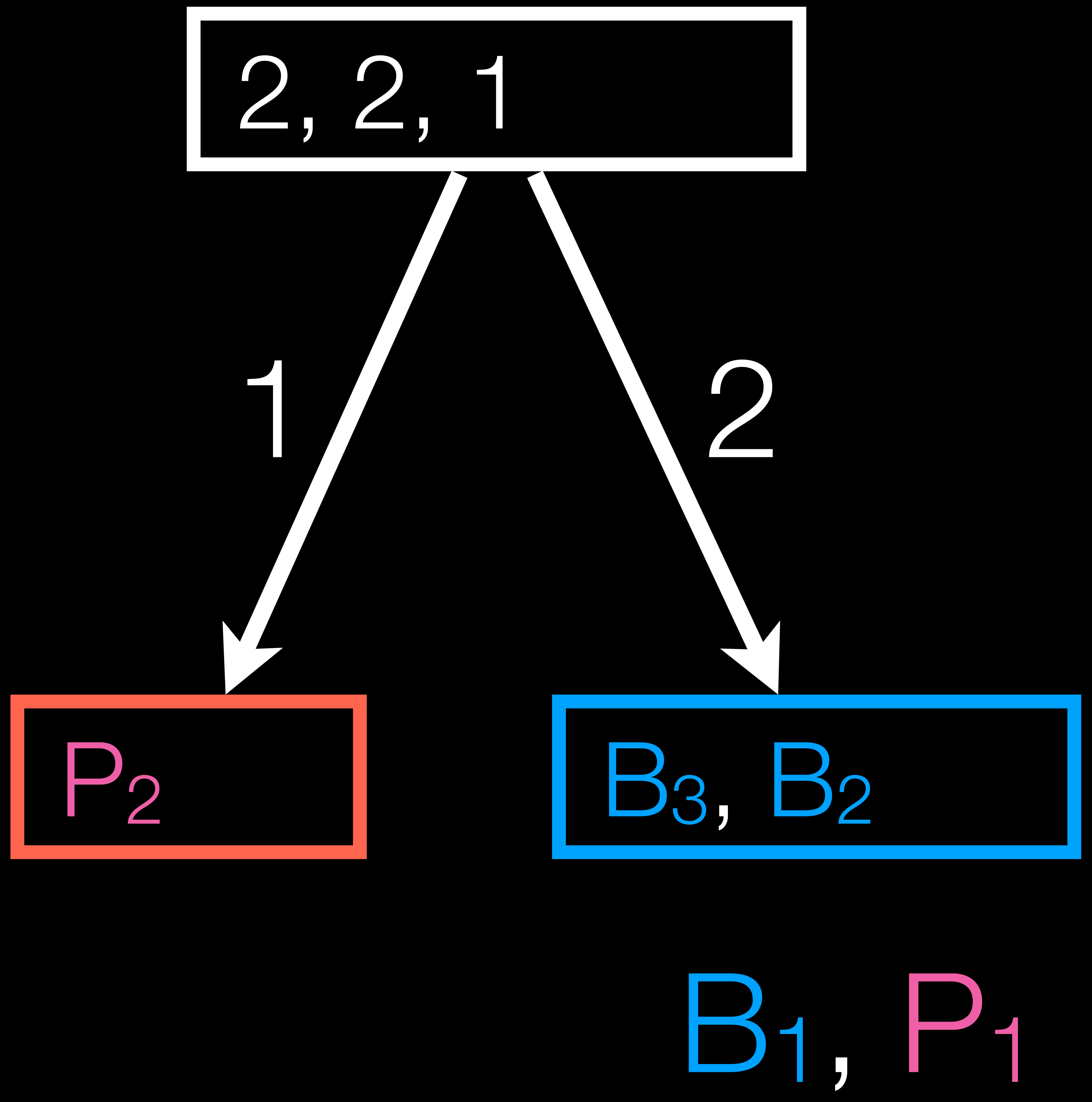
Introducing: PIFO trees



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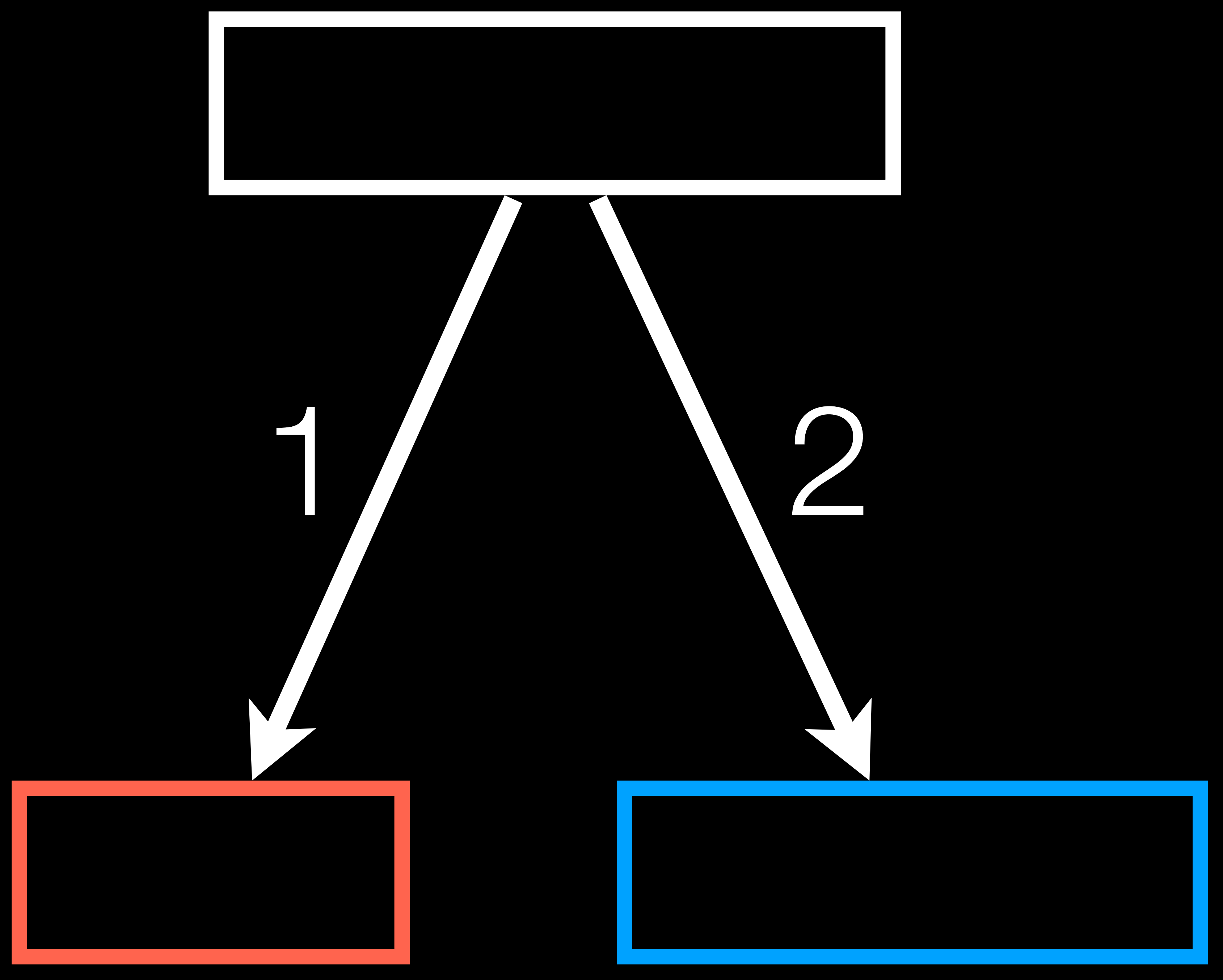
Introducing: PIFO trees

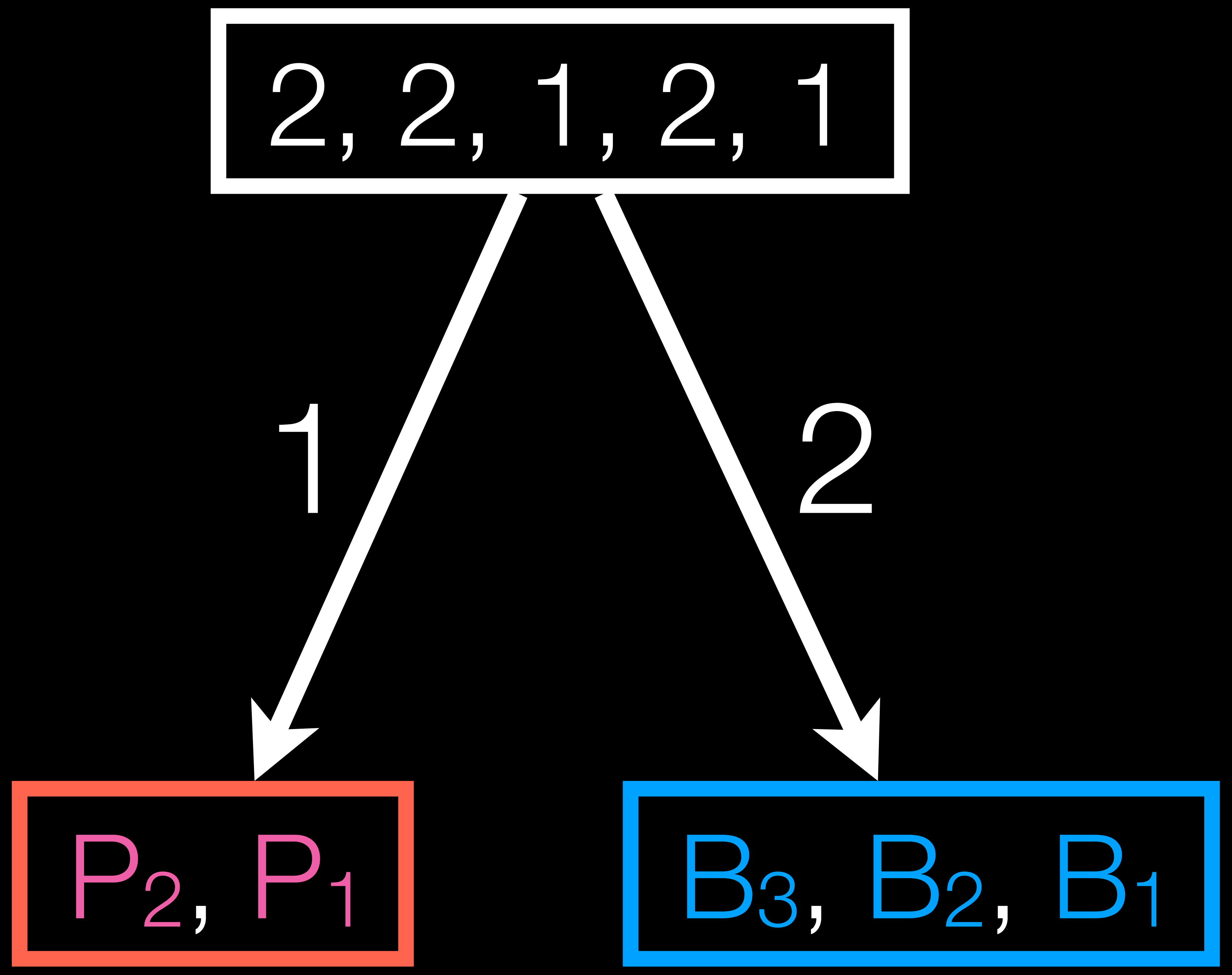


B_3, B_2, P_2, B_1, P_1

This behaves like a queue!

How do we pop it?

Introducing: PIFO trees



B_3, B_2, P_2, B_1, P_1

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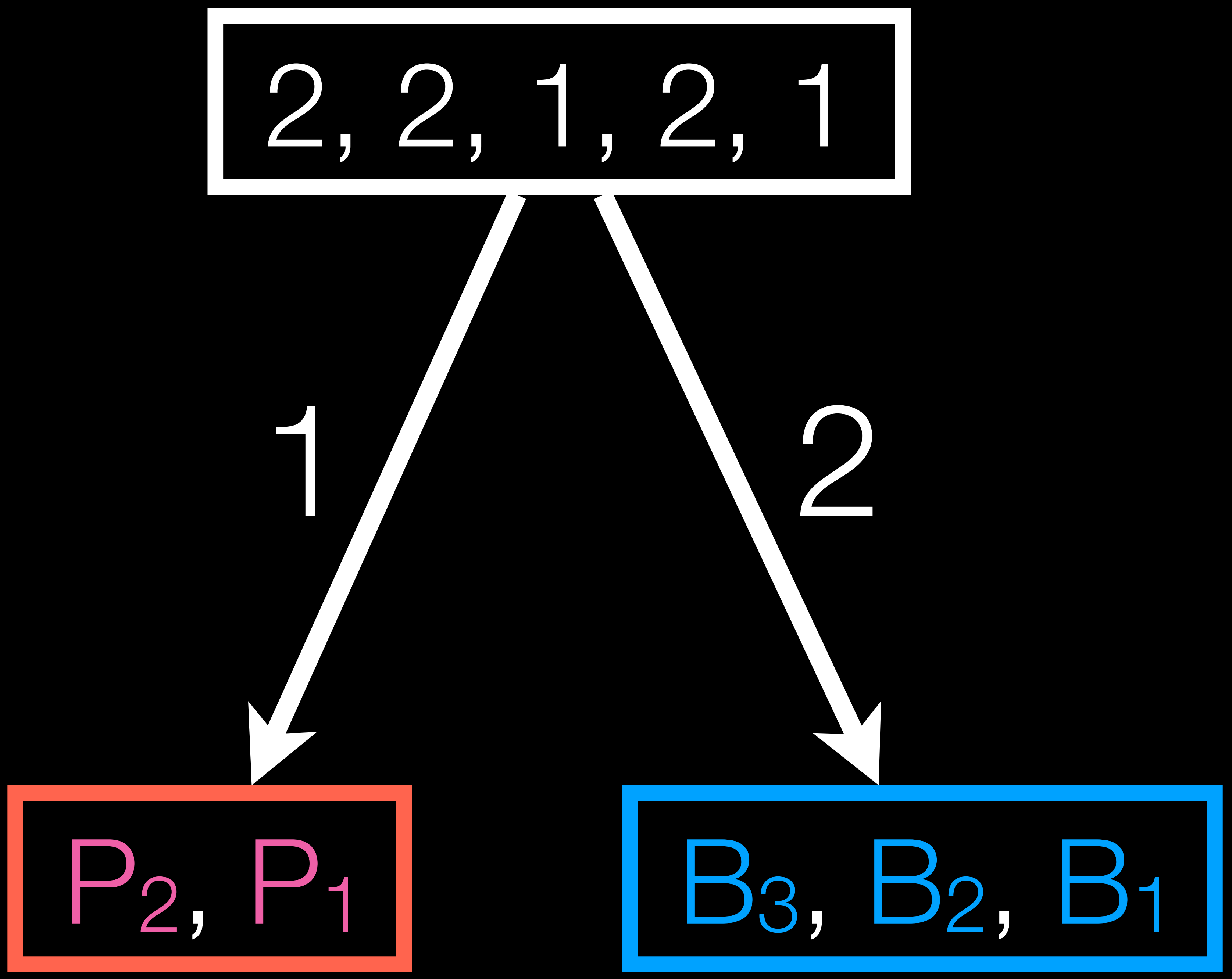
How do we pop it?

Introducing: PIFO trees



2, 2, 1, 2, 1

1

2

P_2, P_1

B_3, B_2, B_1

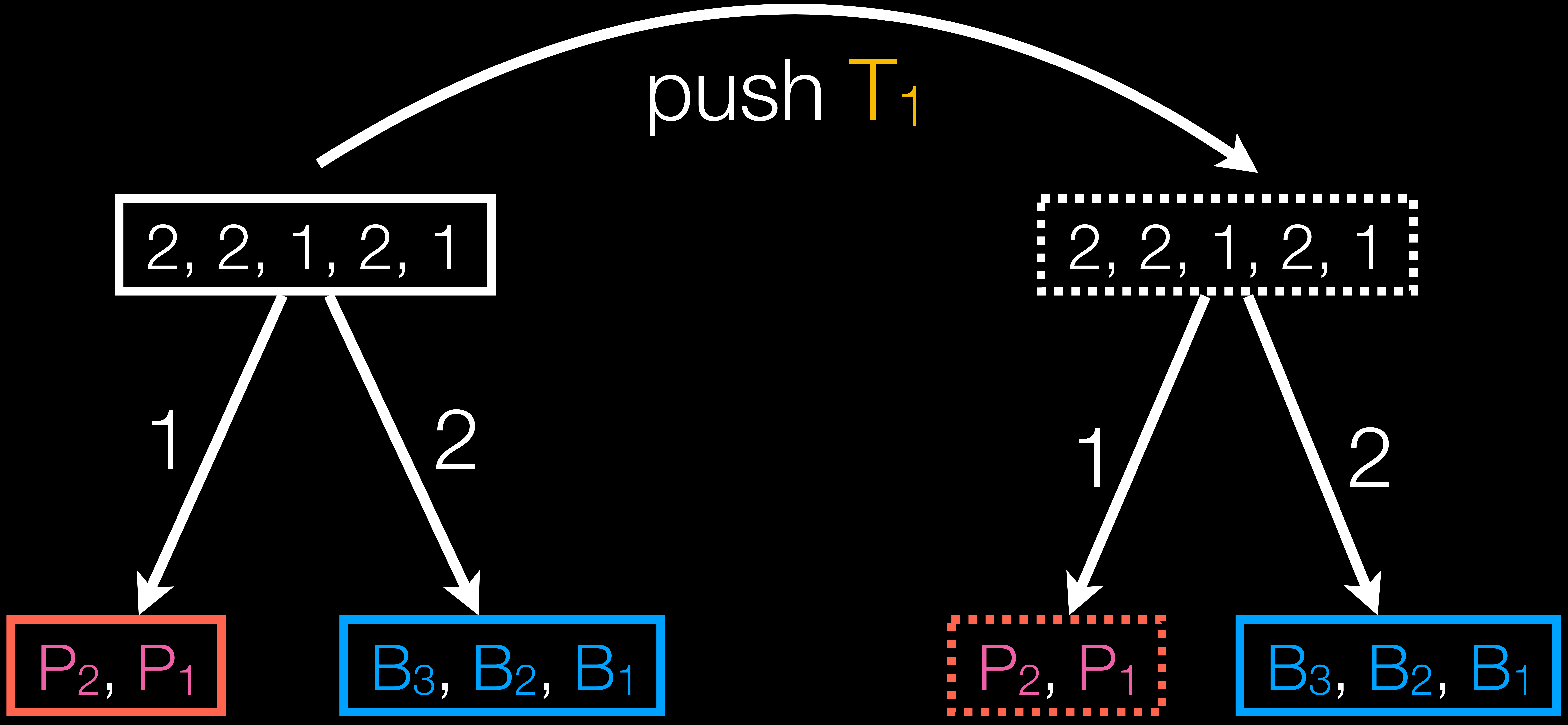
B_3, B_2, P_2, B_1, P_1

This behaves like a queue!

How do we pop it?

How do we push into it?

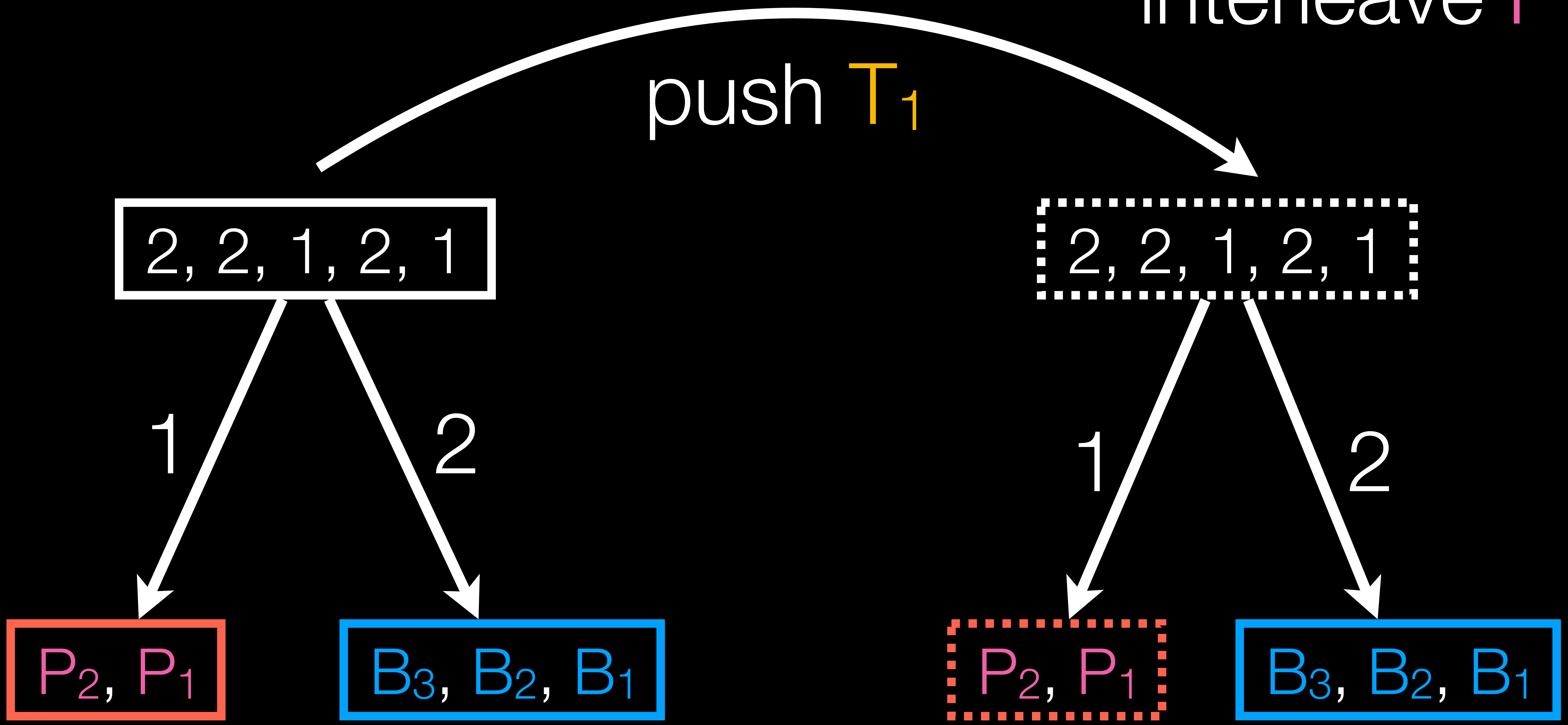
Introducing: PIFO trees



B_3, B_2, P_2, B_1, P_1

Introducing: PIFO trees

interleave R and B;
interleave P and T.



push T_1

2, 2, 1, 2, 1

1

2

P_2, P_1

B_3, B_2, B_1

2, 2, 1, 2, 1

1

2

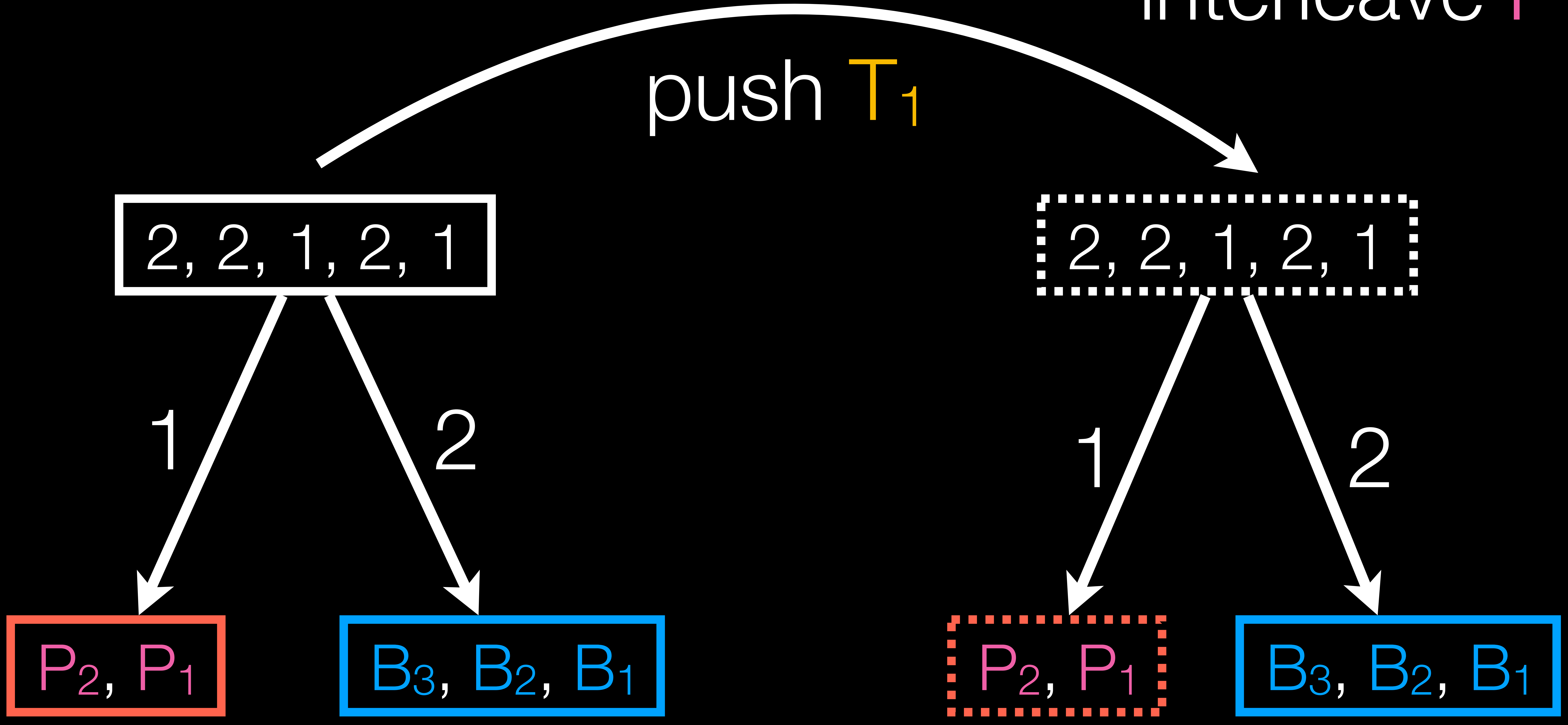
P_2, P_1

B_3, B_2, B_1

B_3, B_2, P_2, B_1, P_1

Introducing: PIFO trees

interleave R and B;
interleave P and T.



B_3, B_2, P_2, B_1, P_1

Introducing: PIFO trees

interleave R and B;
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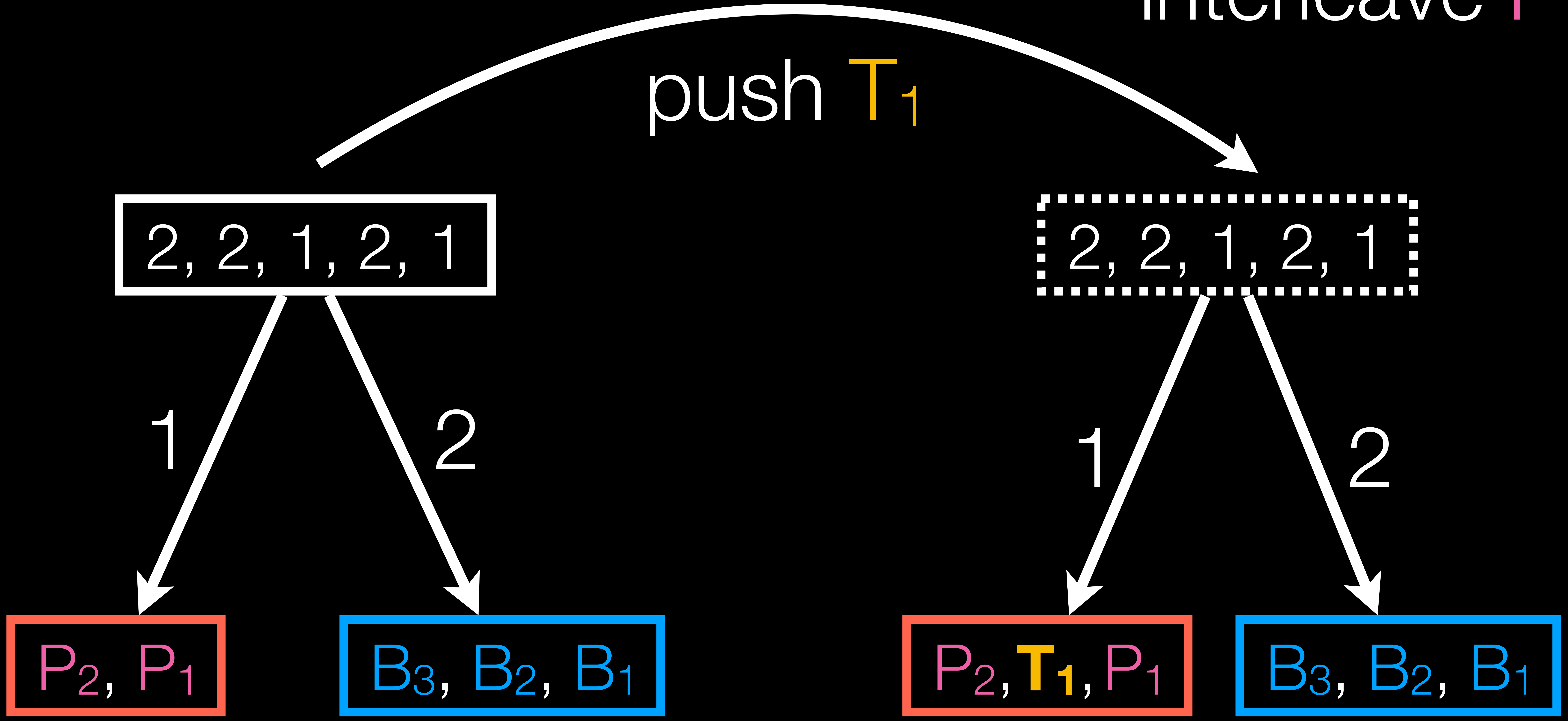

B_3, B_2, P_2, B_1, P_1

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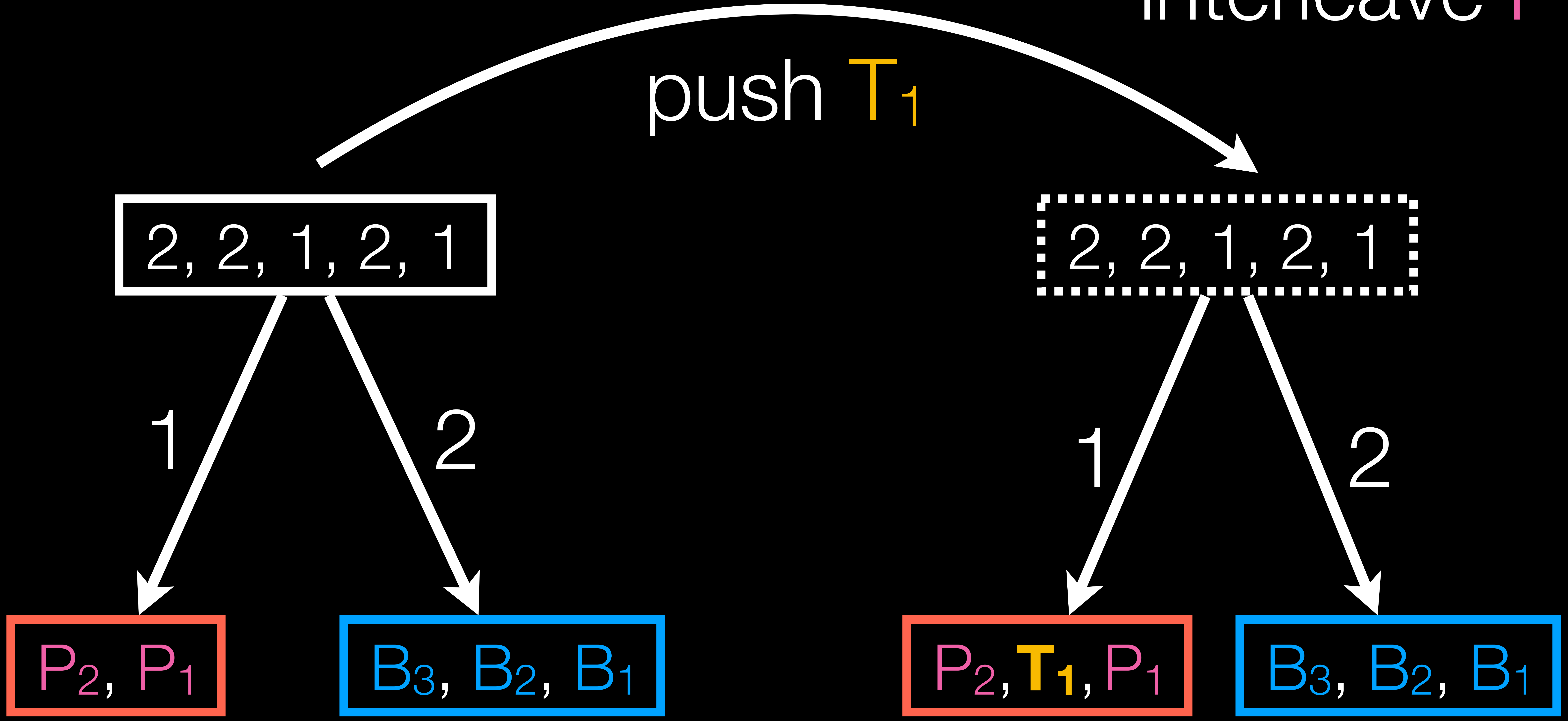
interleave R and B;
interleave P and T.



B_3, B_2, P_2, B_1, P_1

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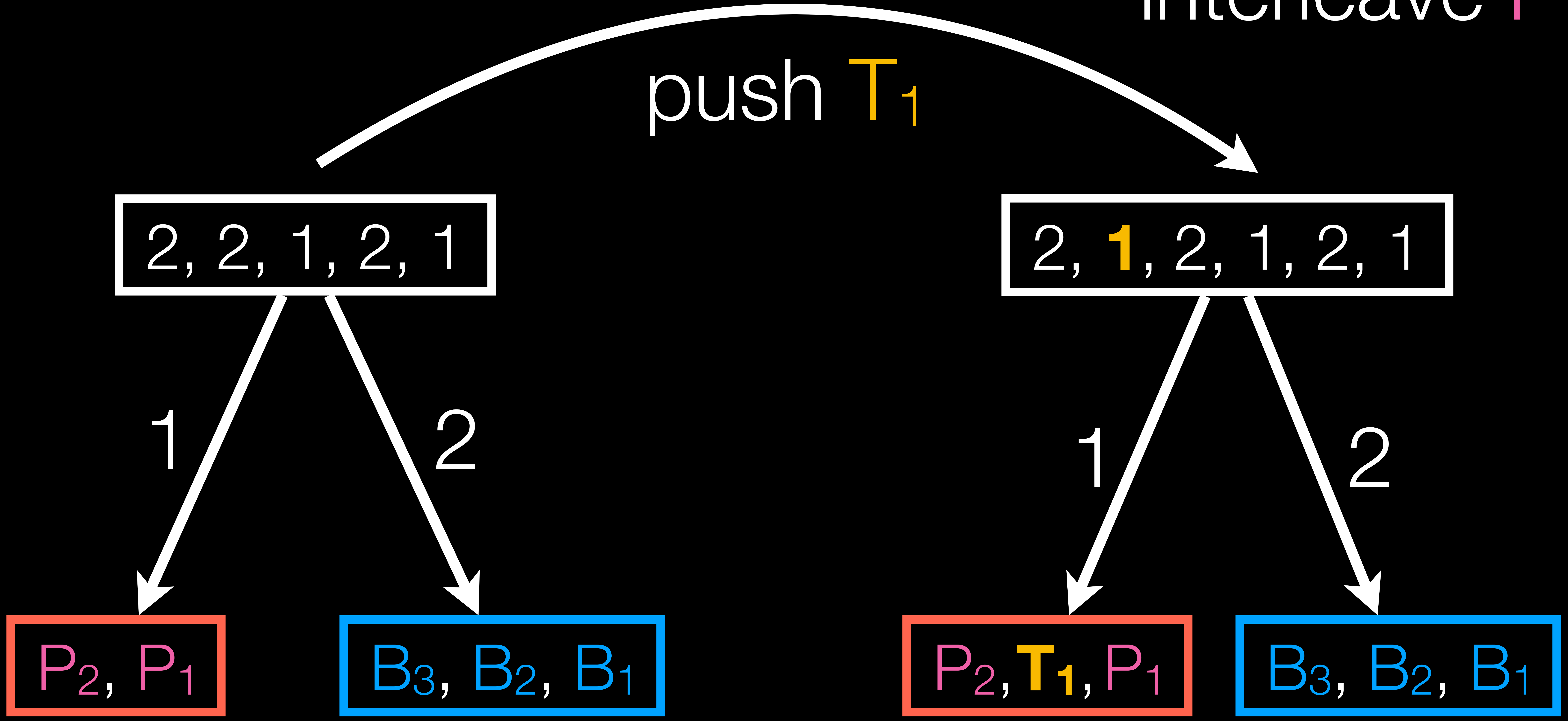


push T_1

2, 2, 1, 2, 1

1

2

P_2, P_1

B_3, B_2, B_1

2, **1**, 2, 1, 2, 1

1

2

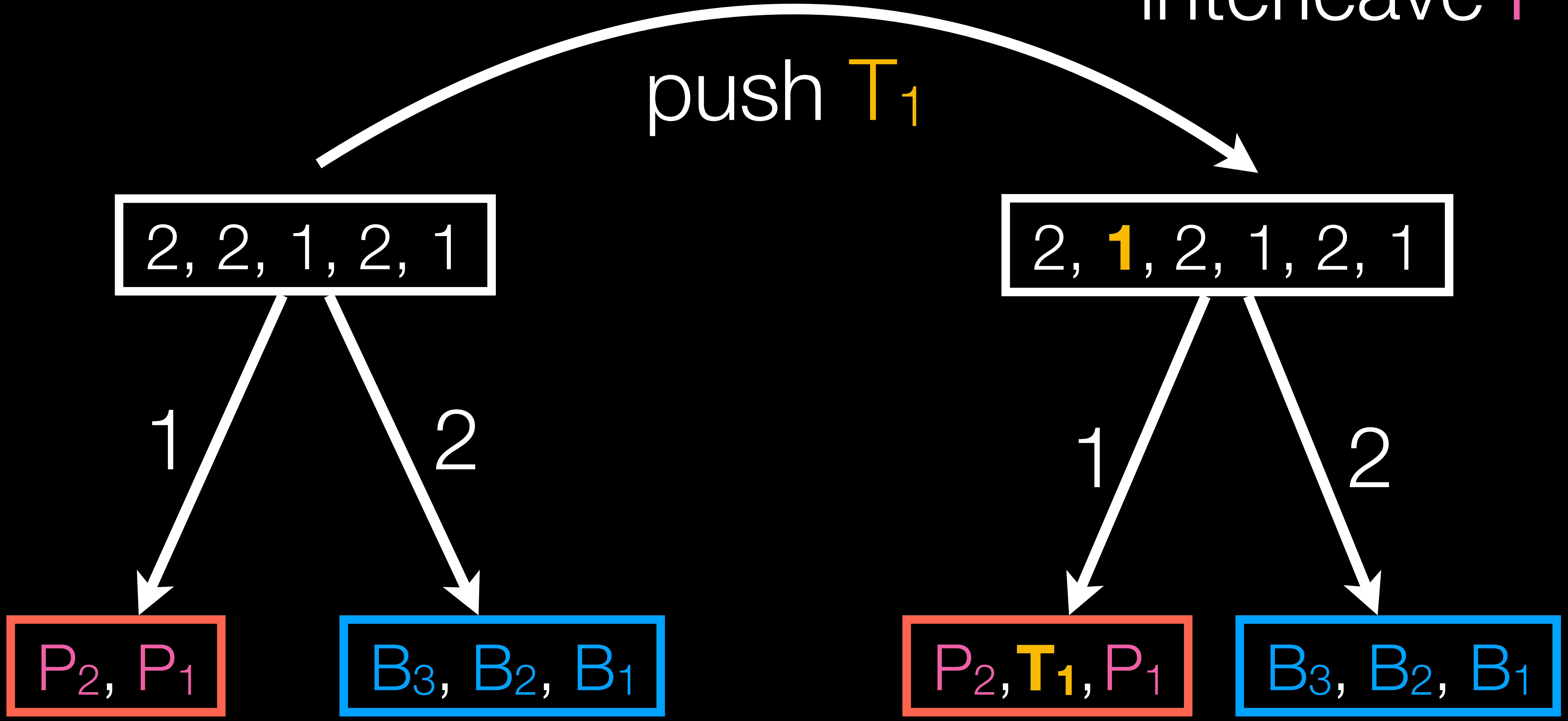
P_2, $\mathbf{T_1}$, P_1

B_3, B_2, B_1

B_3, B_2, P_2, B_1, P_1

Introducing: PIFO trees

interleave R and B;
interleave P and T.



B_3, B_2, P_2, B_1, P_1

B_3, P_2, B_2, $\mathbf{T_1}$, B_1, P_1

Introducing: PIFO trees

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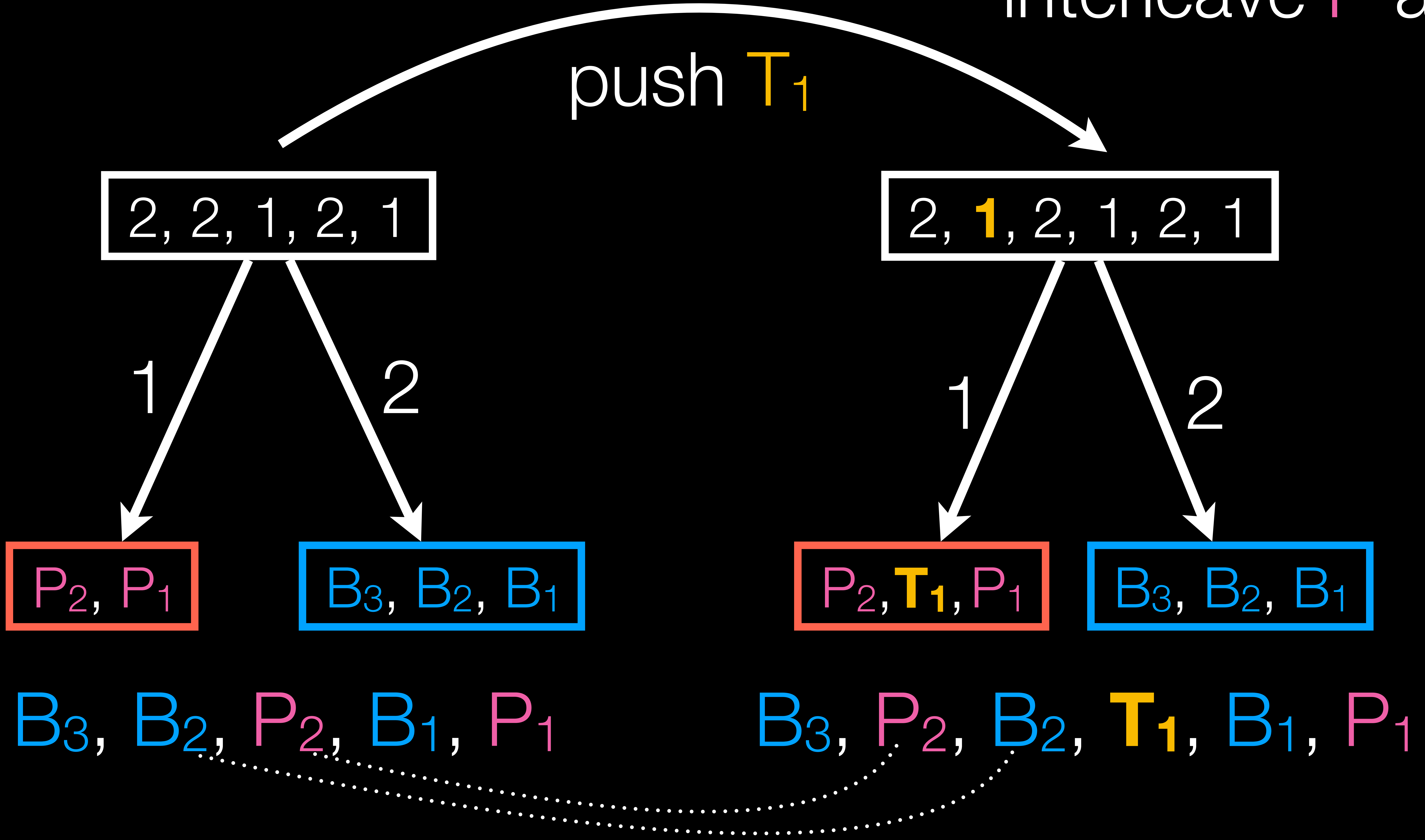
interleave P and T.



push T_1

2, 2, 1, 2, 1

1

2

P_2, P_1

B_3, B_2, B_1

B_3, B_2, P_2, B_1, P_1

2, **1**, 2, 1, 2, 1

1

2

P_2, $\mathbf{T_1}$, P_1

B_3, B_2, B_1

B_3, P_2, B_2, $\mathbf{T_1}$, B_1, P_1

Introducing: PIFO trees

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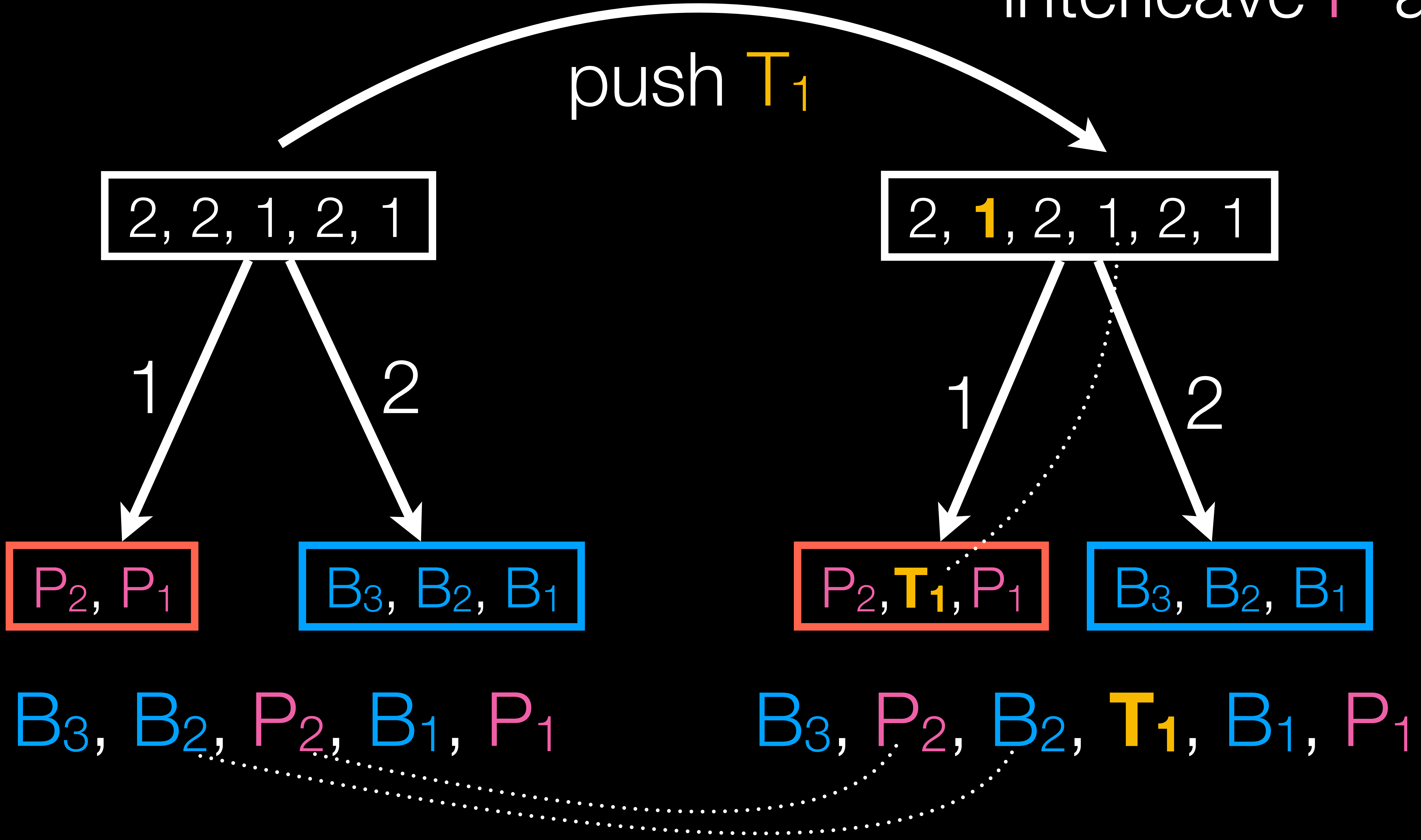

push T_1

2, 2, 1, 2, 1

1

2

P_2, P_1

B_3, B_2, B_1

B_3, B_2, P_2, B_1, P_1

2, **1**, 2, 1, 2, 1

1

2

P_2, $\mathbf{T_1}$, P_1

B_3, B_2, B_1

B_3, P_2, B_2, $\mathbf{T_1}$, B_1, P_1

Introducing: PIFO trees

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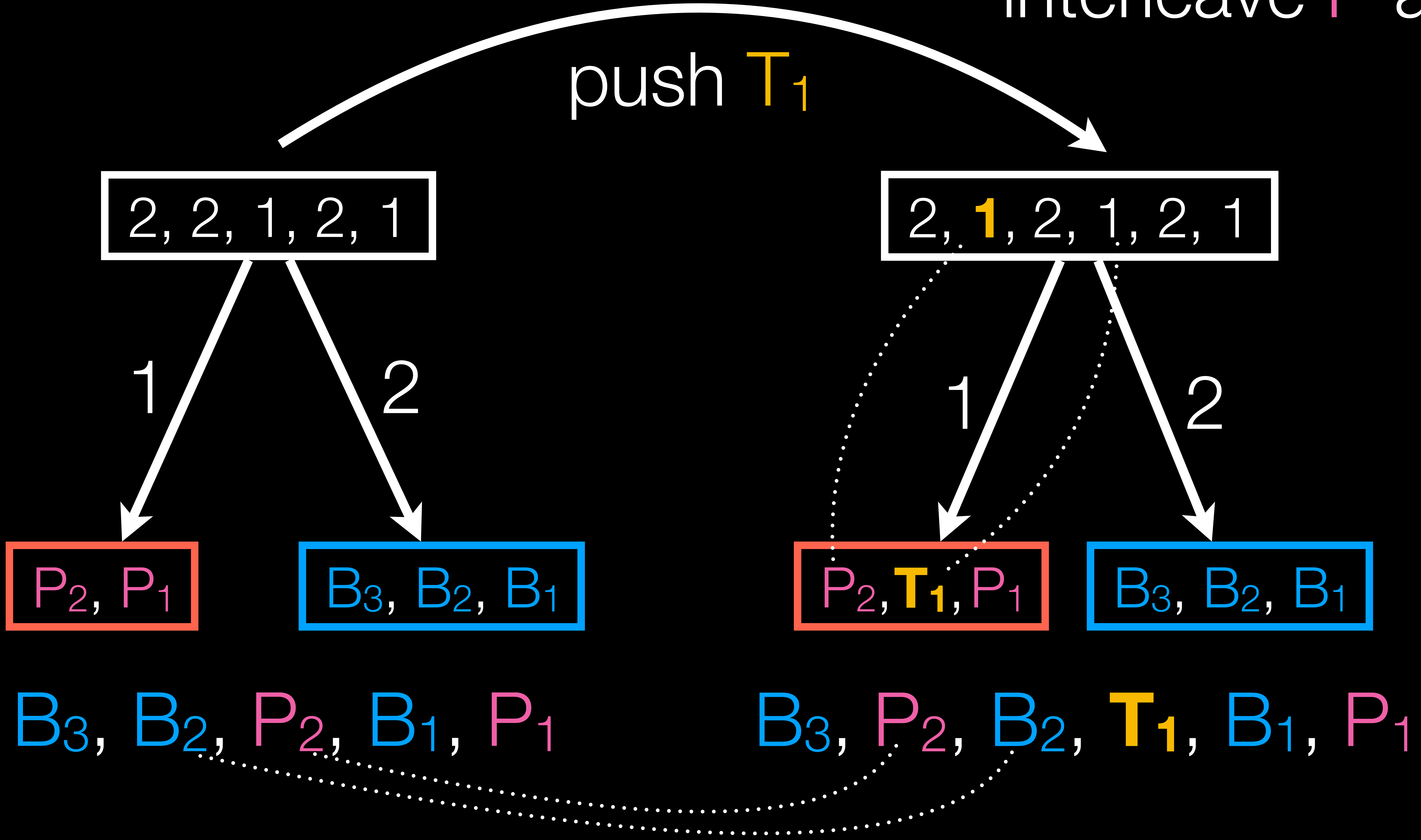
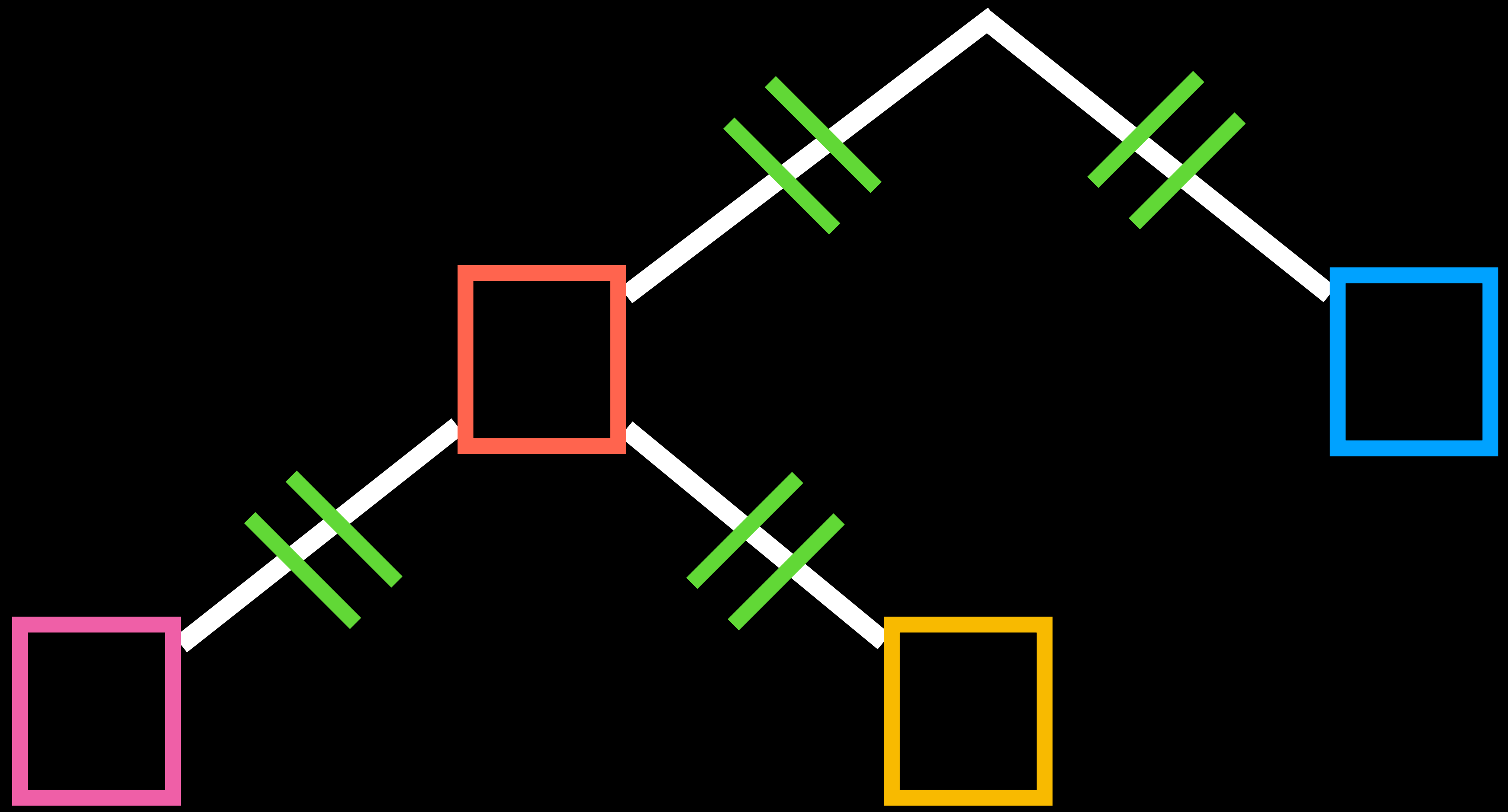
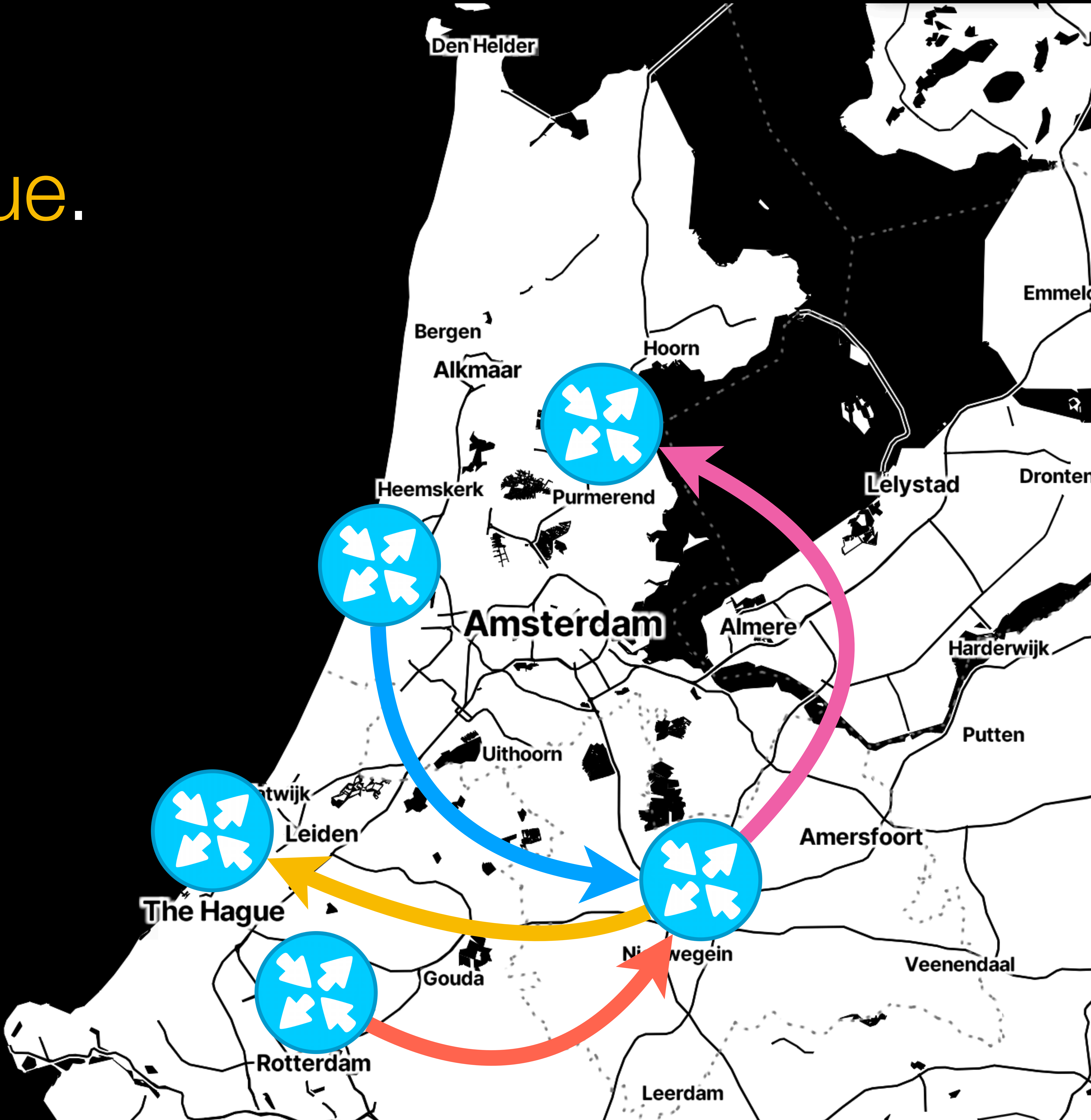

push T_1
2, 2, 1, 2, 1
1
2
P_2, P_1
B_3, B_2, B_1
2, **1**, 2, 1, 2, 1
1
2
P_2, $\mathbf{T_1}$, P_1
B_3, B_2, B_1
B_3, B_2, P_2, B_1, P_1
B_3, P_2, B_2, $\mathbf{T_1}$, B_1, P_1

R traffic goes to either Purmerend or The Hague.

Goal:

Interleave R and B;
interleave P and T.





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Goal:

Interleave R and B;
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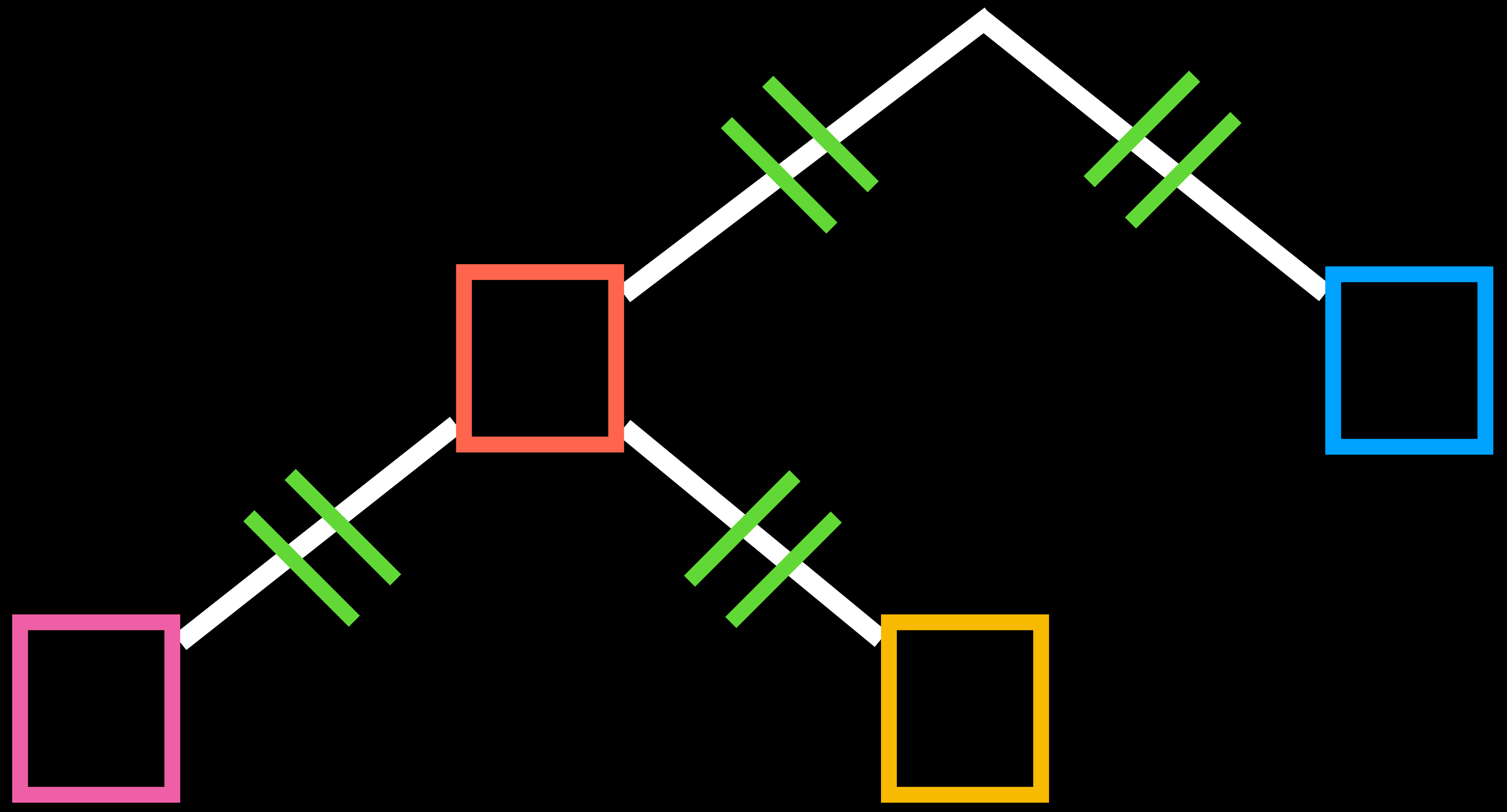
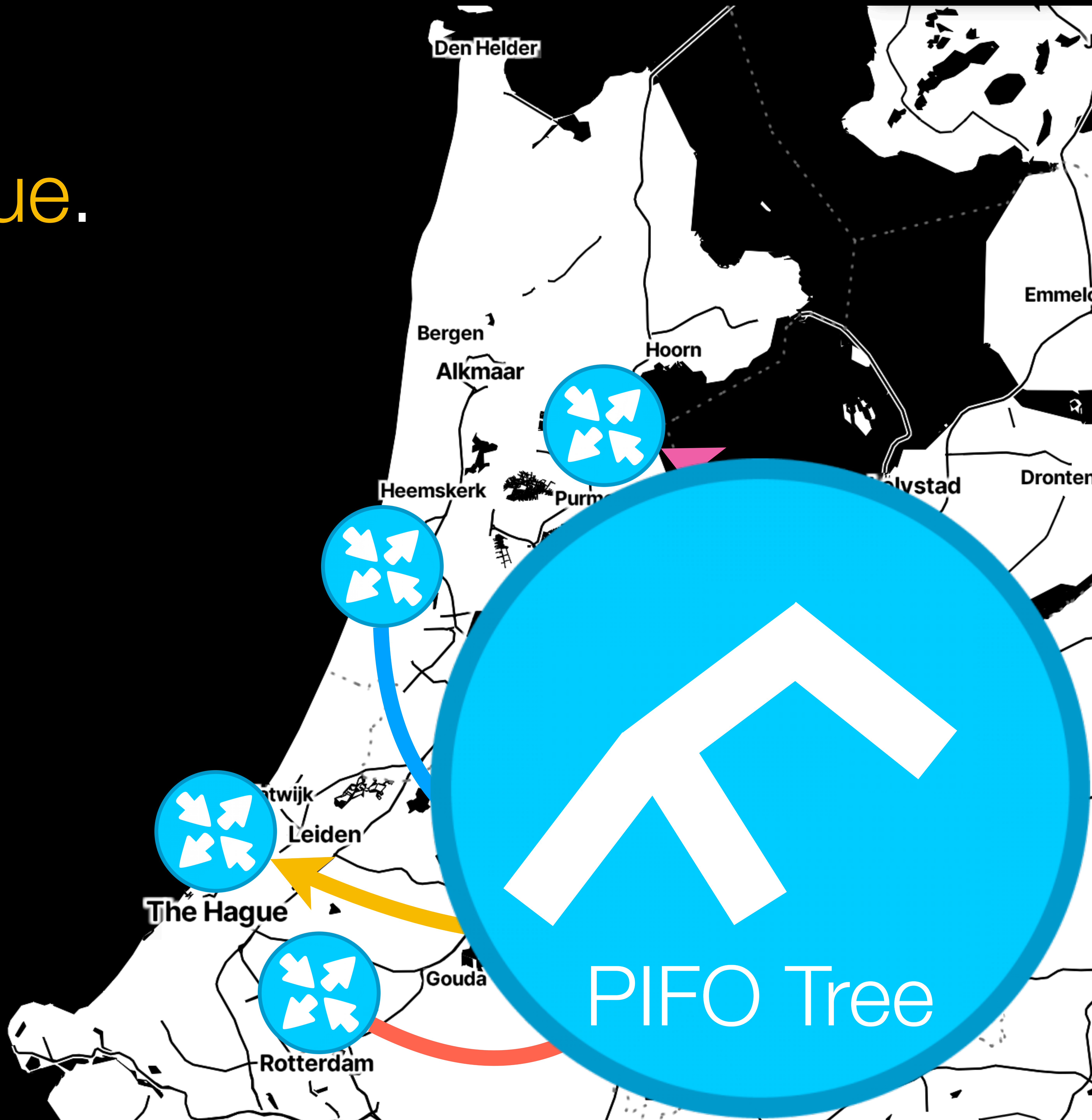


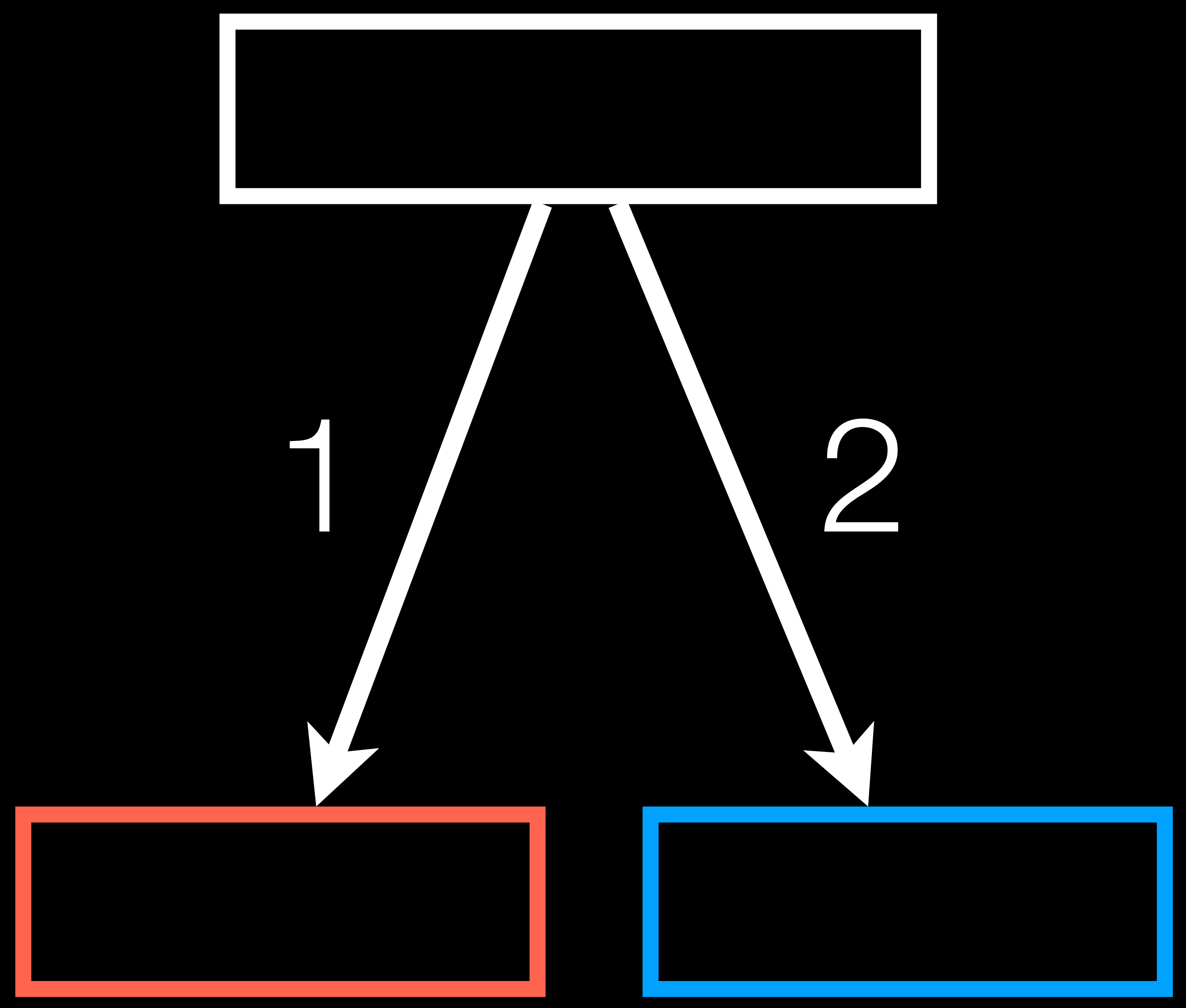


Aside: PIFO Trees

Sivaraman et al. at SIGCOMM ’16

Key Insight

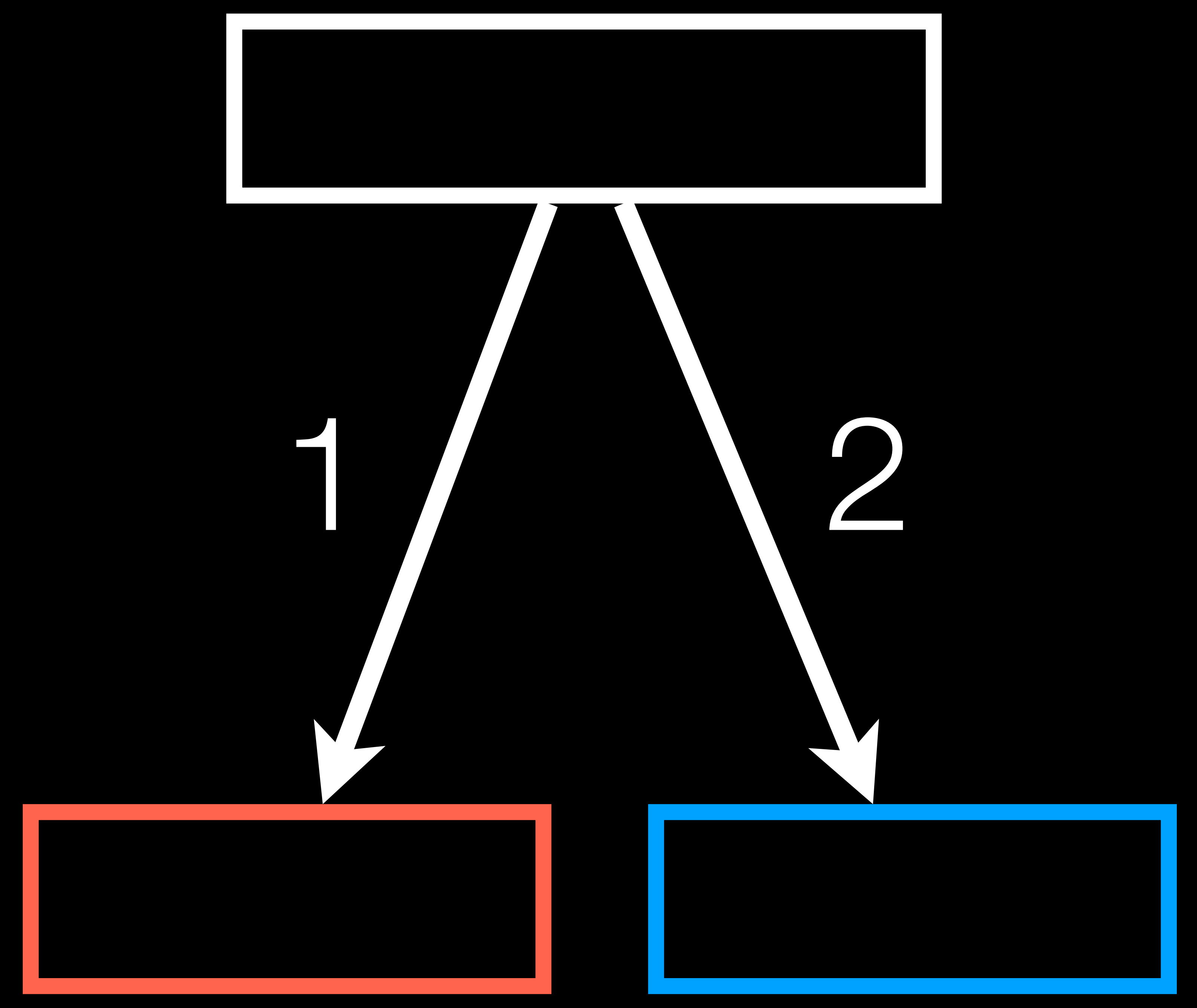
A PIFO tree manifests a *programming language.*



Key Insight

A PIFO tree manifests a *programming language.*

A program is precisely a *scheduling algorithm.*



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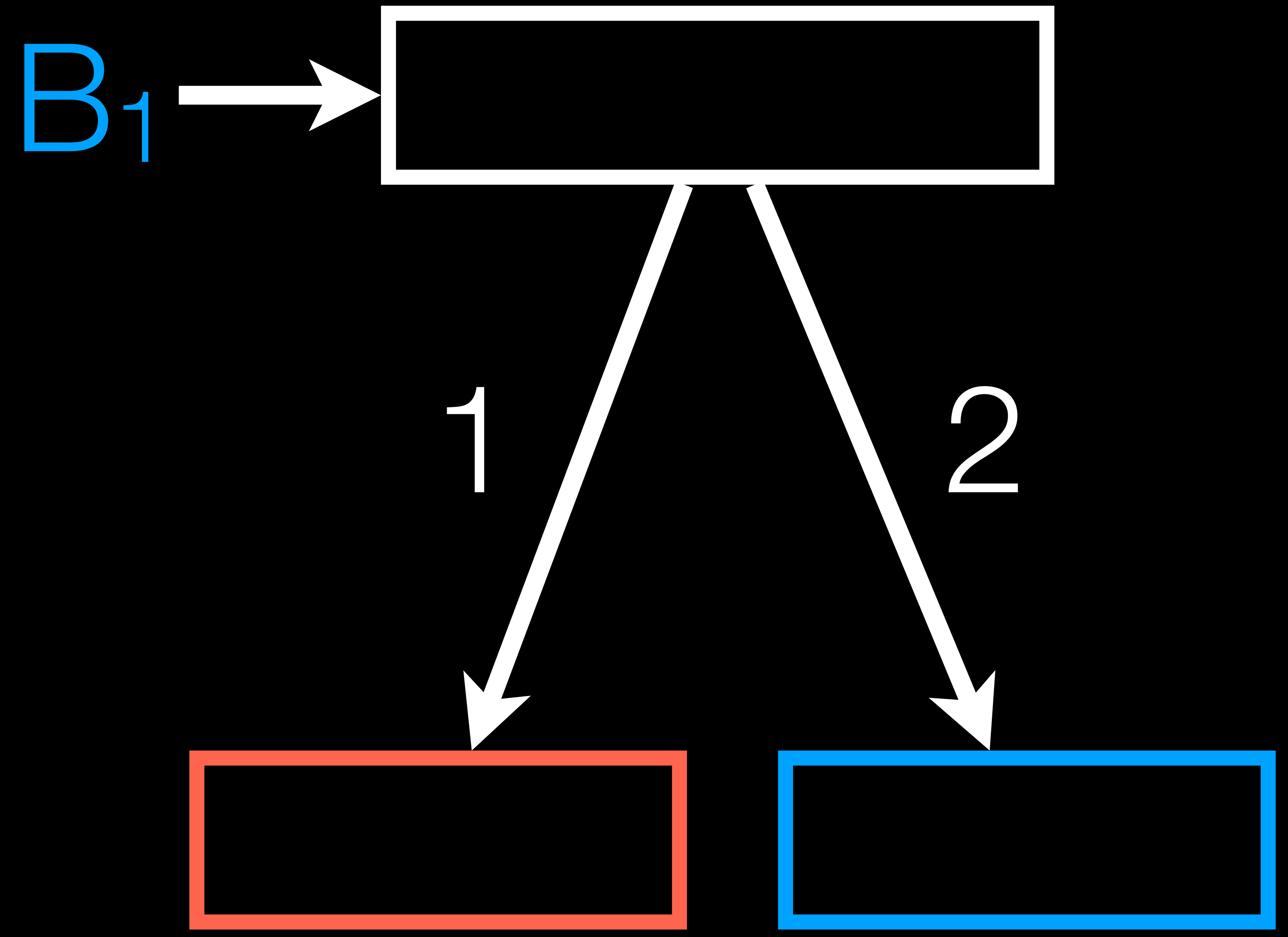
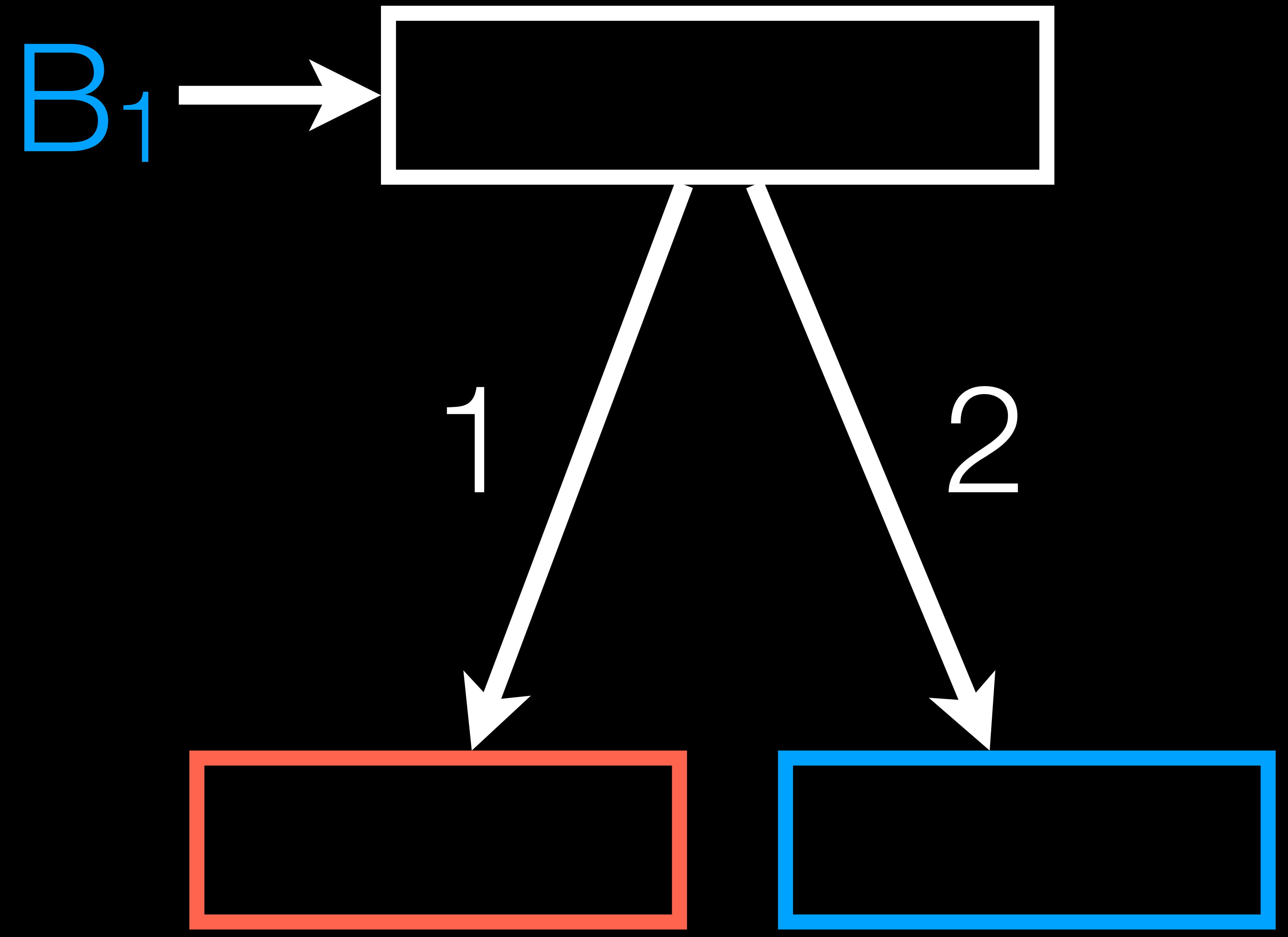


Key Insight

A PIFO tree manifests a *programming language.*

A program is precisely a *scheduling algorithm.*

Path: $[(2,r_1),(B_1,r_2)]$



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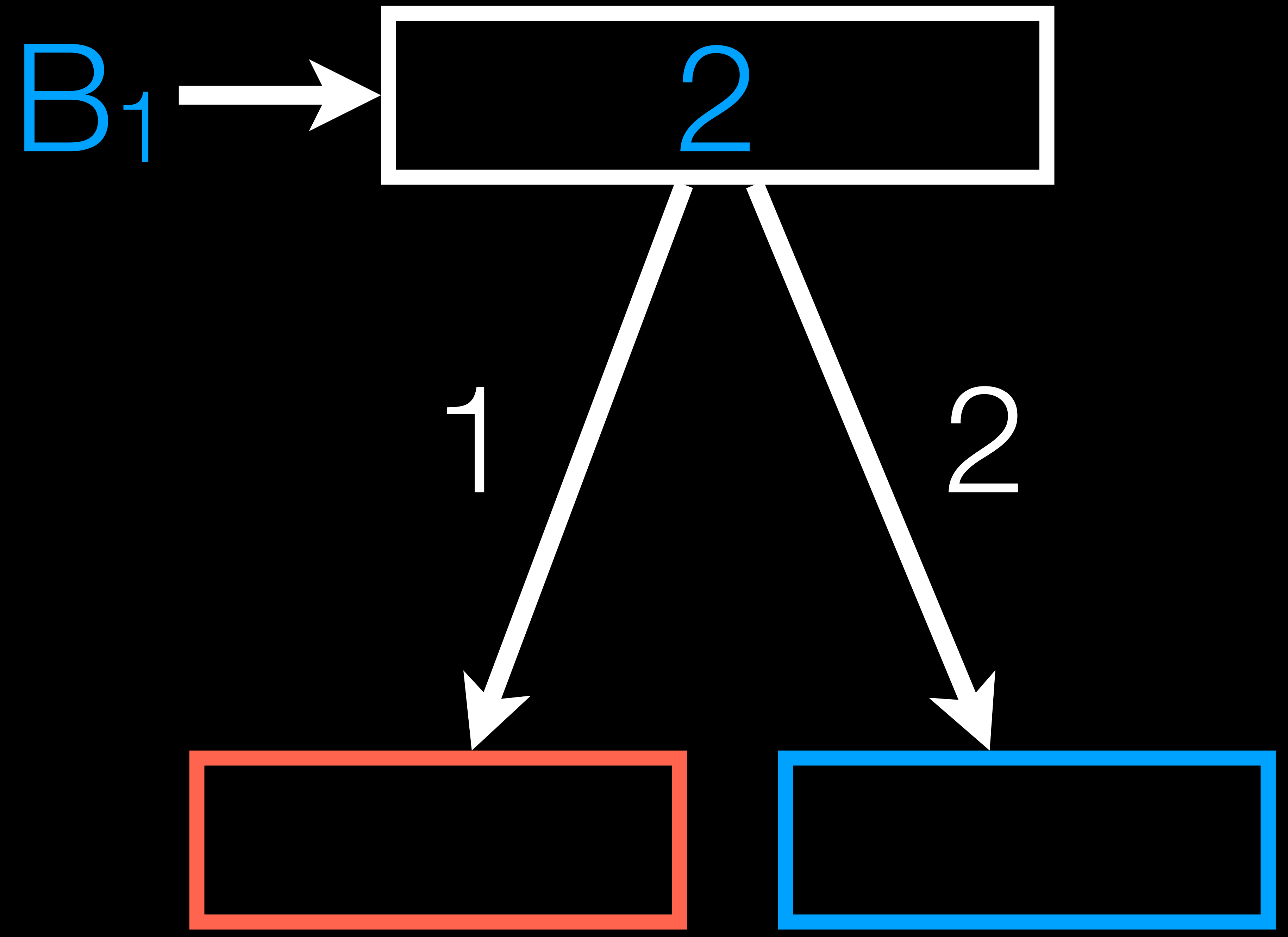
Path: [(2,r_1),(B1,r_2)]



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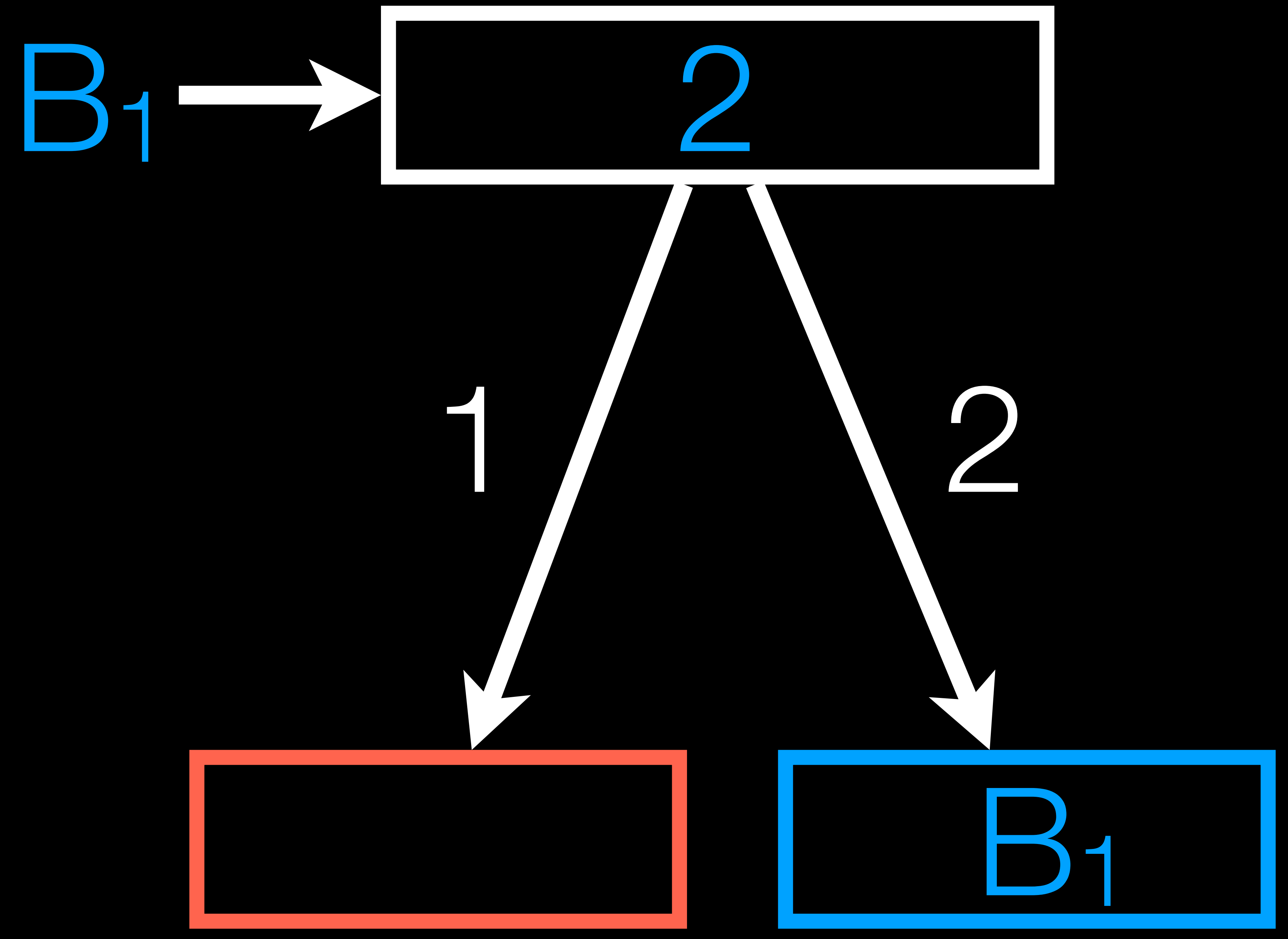
Path: $[(2, r_1), (B_1, r_2)]$



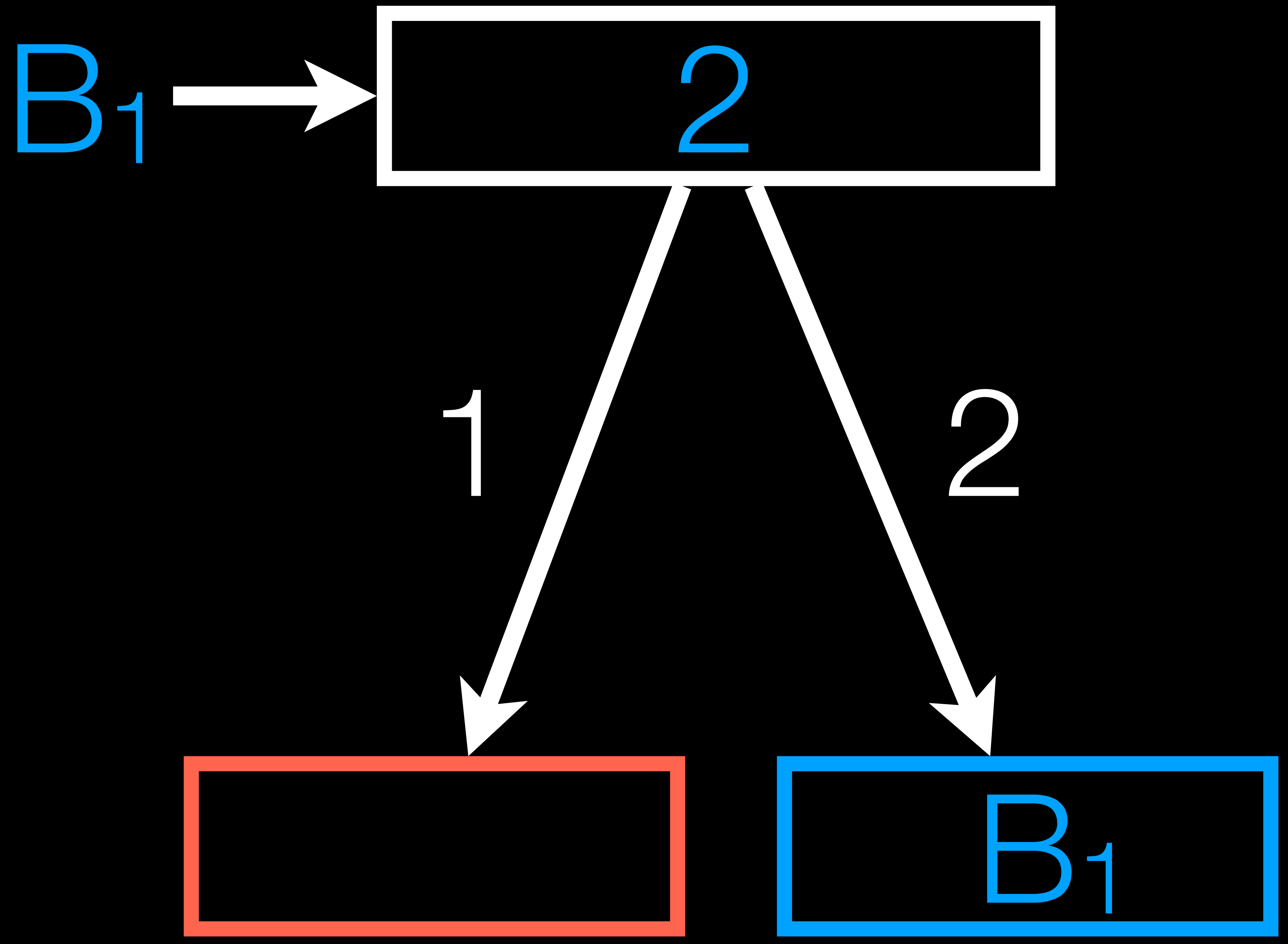
Key Insight

A PIFO tree manifests a *programming language.*

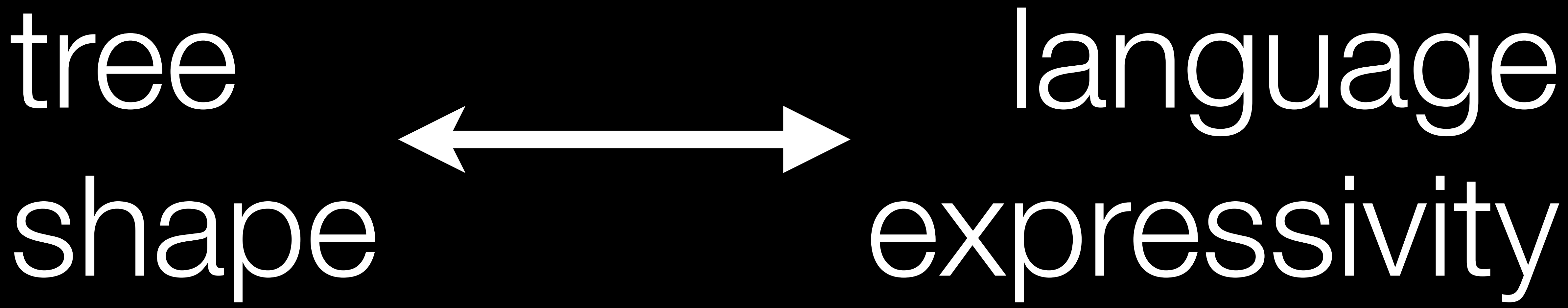
A program is precisely a *scheduling algorithm.*

tree shape ⟷ language expressivity

Path: $[(2, r_1), (B_1, r_2)]$



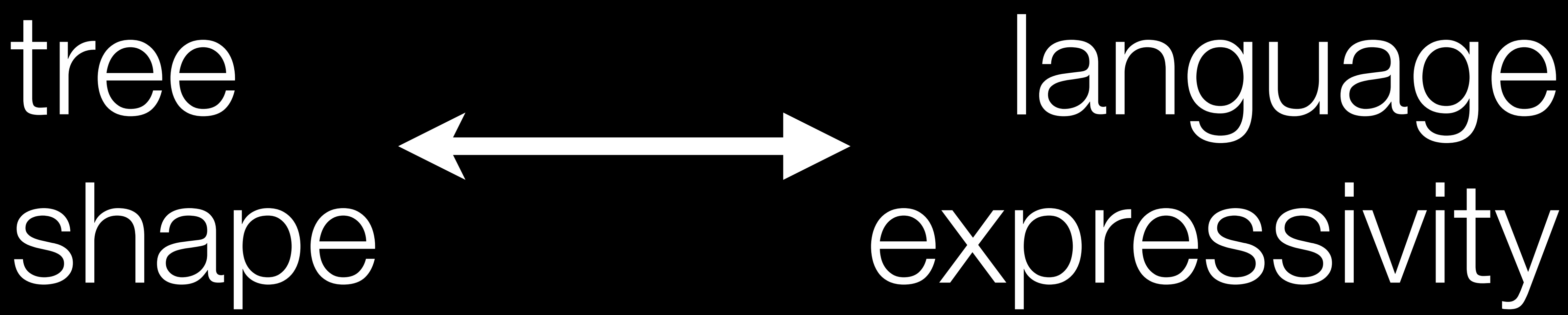
Which leads to some very PL-ey questions:



tree shape

language expressivity

Which leads to some very PL-ey questions:



tree shape

language expressivity

Compare expressivity of languages?

Which leads to some very PL-ey questions:

tree shape ↔ language expressivity

Compare expressivity of languages?
Compare expressivity of *trees*?

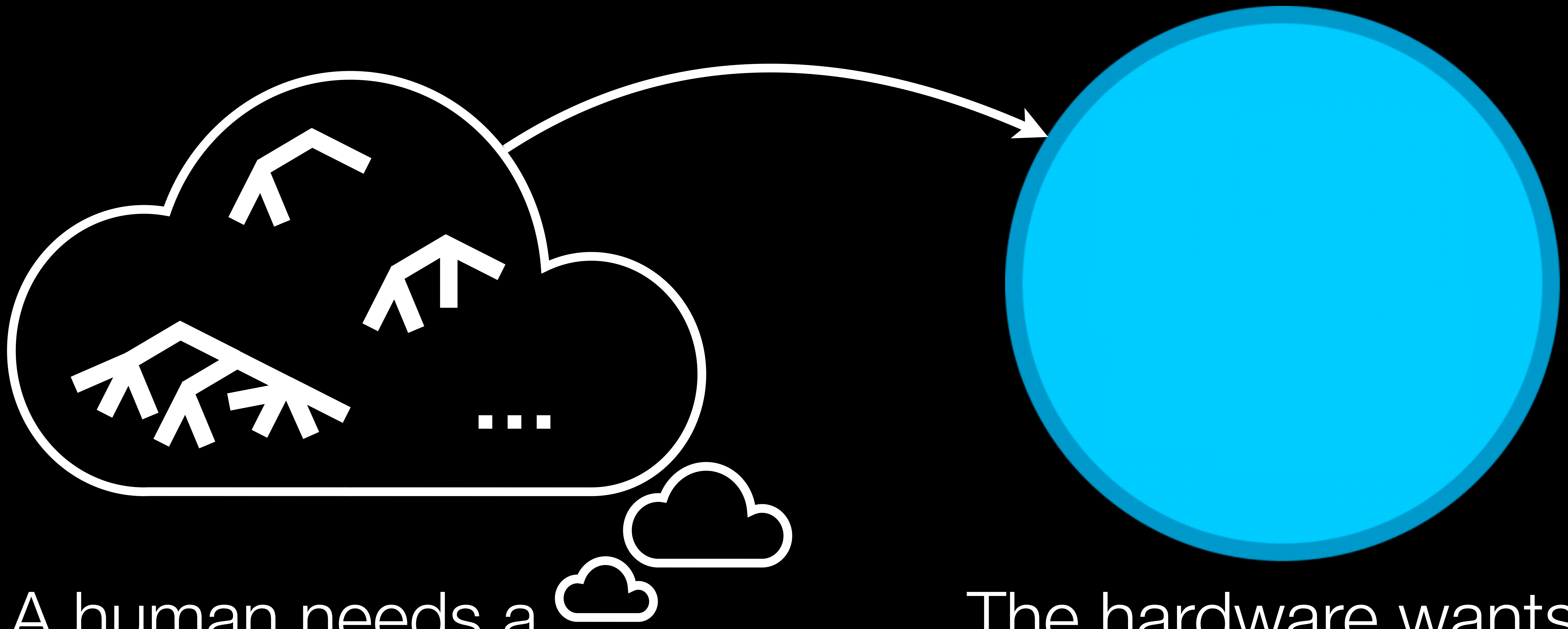
Which leads to some very PL-ey questions:

tree shape ↔ language expressivity

Compare expressivity of languages?
Compare expressivity of *trees*?

Compile a program so it runs against a new tree?

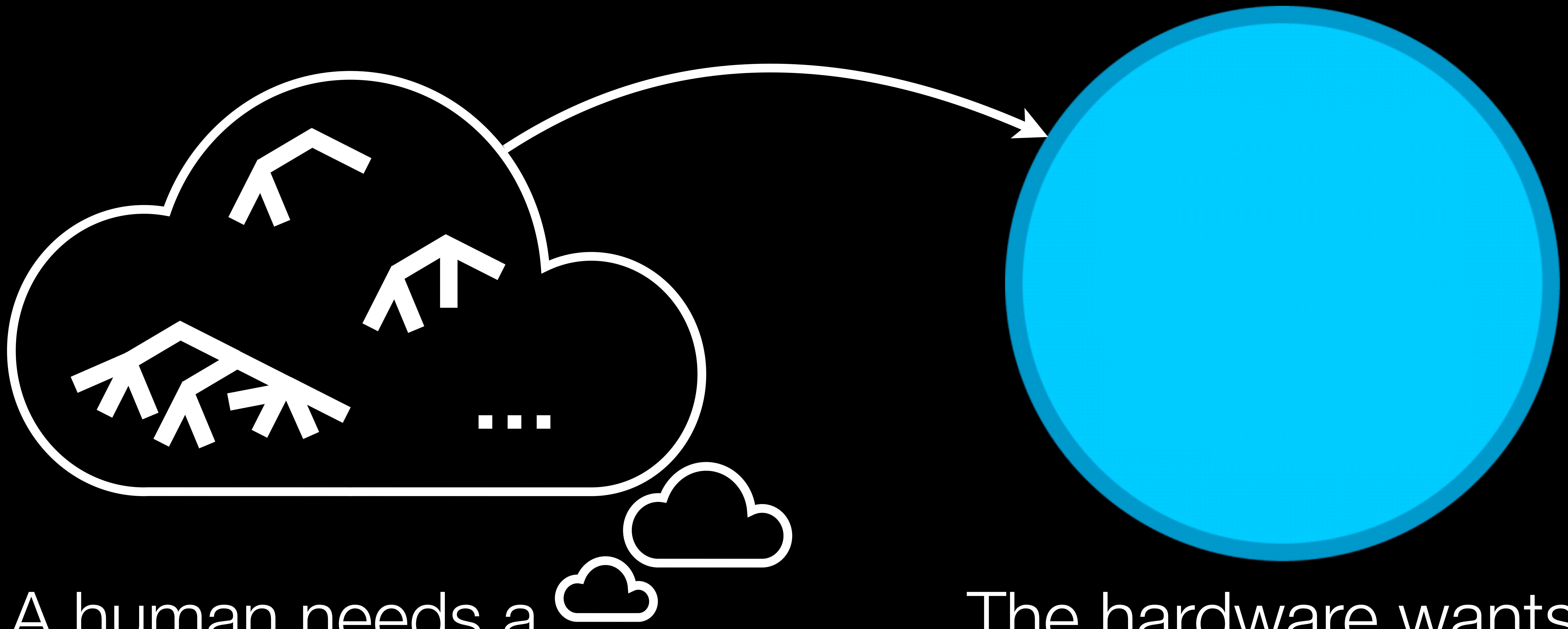
No general way to deploy our gadget.



A human needs a *range* of trees.

The hardware wants to support *one* tree.

~~No general way to deploy our gadget.~~



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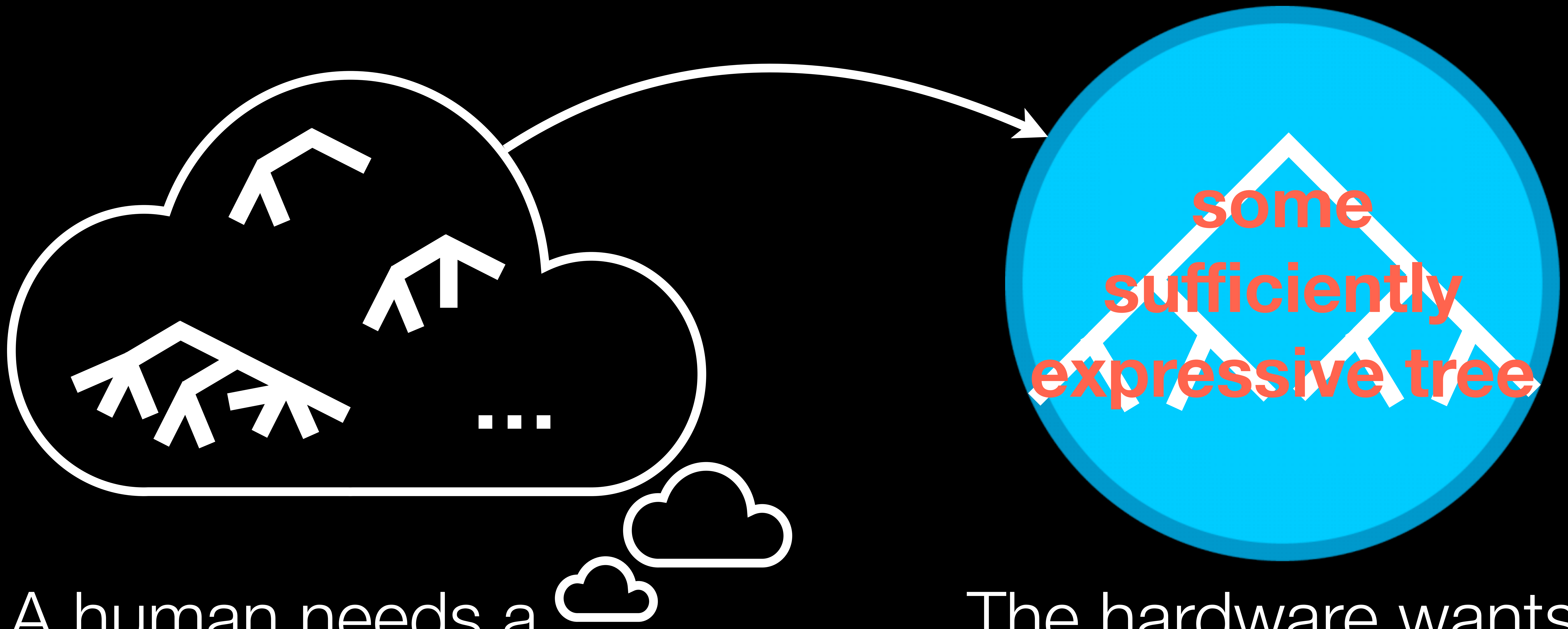

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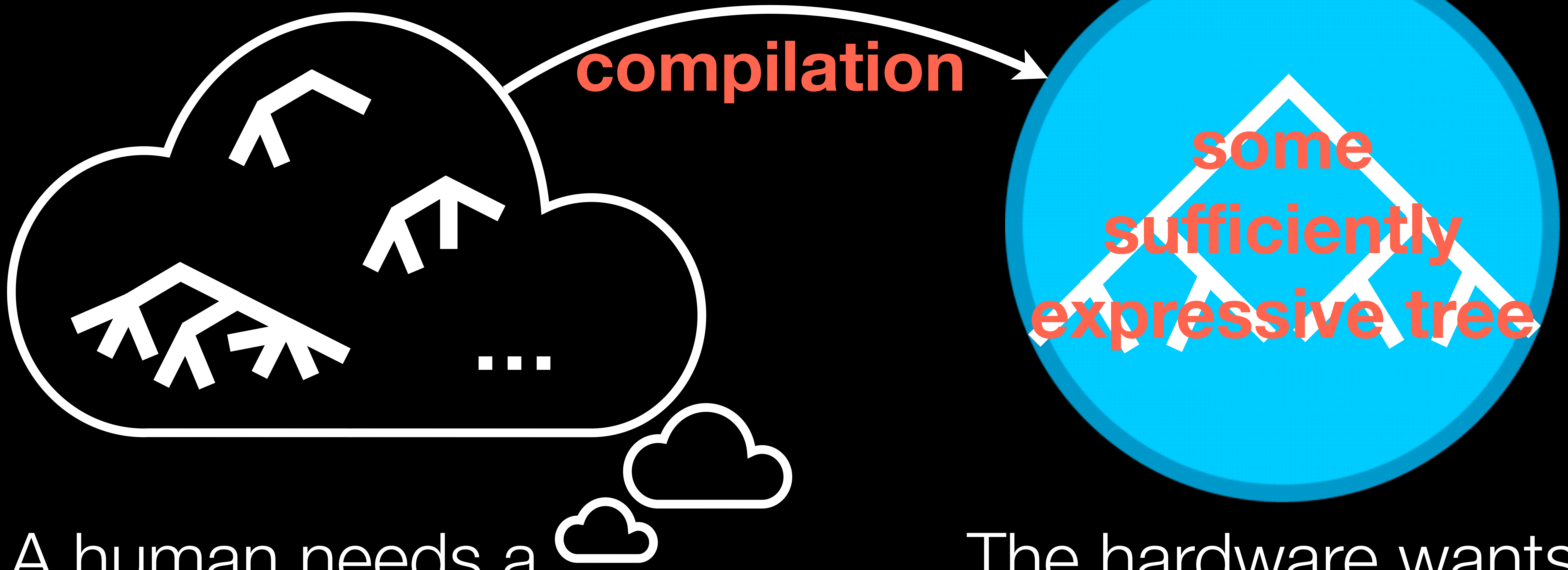


A human needs a *range* of trees.

The hardware wants to support *one* tree.

Contributions

Contributions

Formal model of PIFO trees

Contributions

Formal model of PIFO trees

General theorems of expressiveness w.r.t. tree shape

Contributions

Formal model of PIFO trees

General theorems of expressiveness w.r.t. tree shape

Compiler

Contributions

Formal model of PIFO trees

General theorems of expressiveness w.r.t. tree shape

Compiler

Simulator

Expressivity of trees

Trees with more leaves are more expressive.
Taller trees are more expressive.

Expressivity of trees

Trees with more leaves are more expressive.
Taller trees are more expressive.

Captured elegantly by:

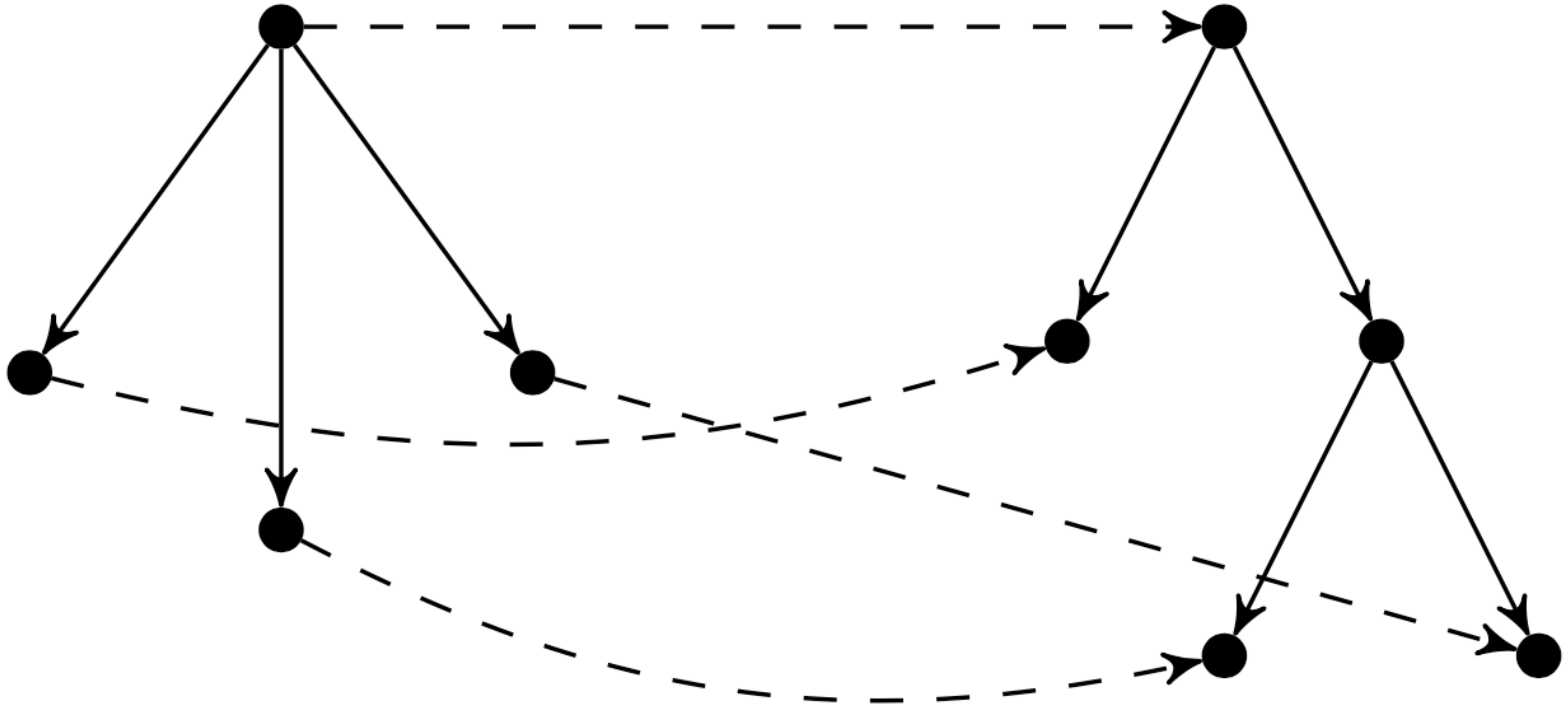
Expressivity of trees

Trees with more leaves are more expressive.
Taller trees are more expressive.

Captured elegantly by:

Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.

Expressivity of trees



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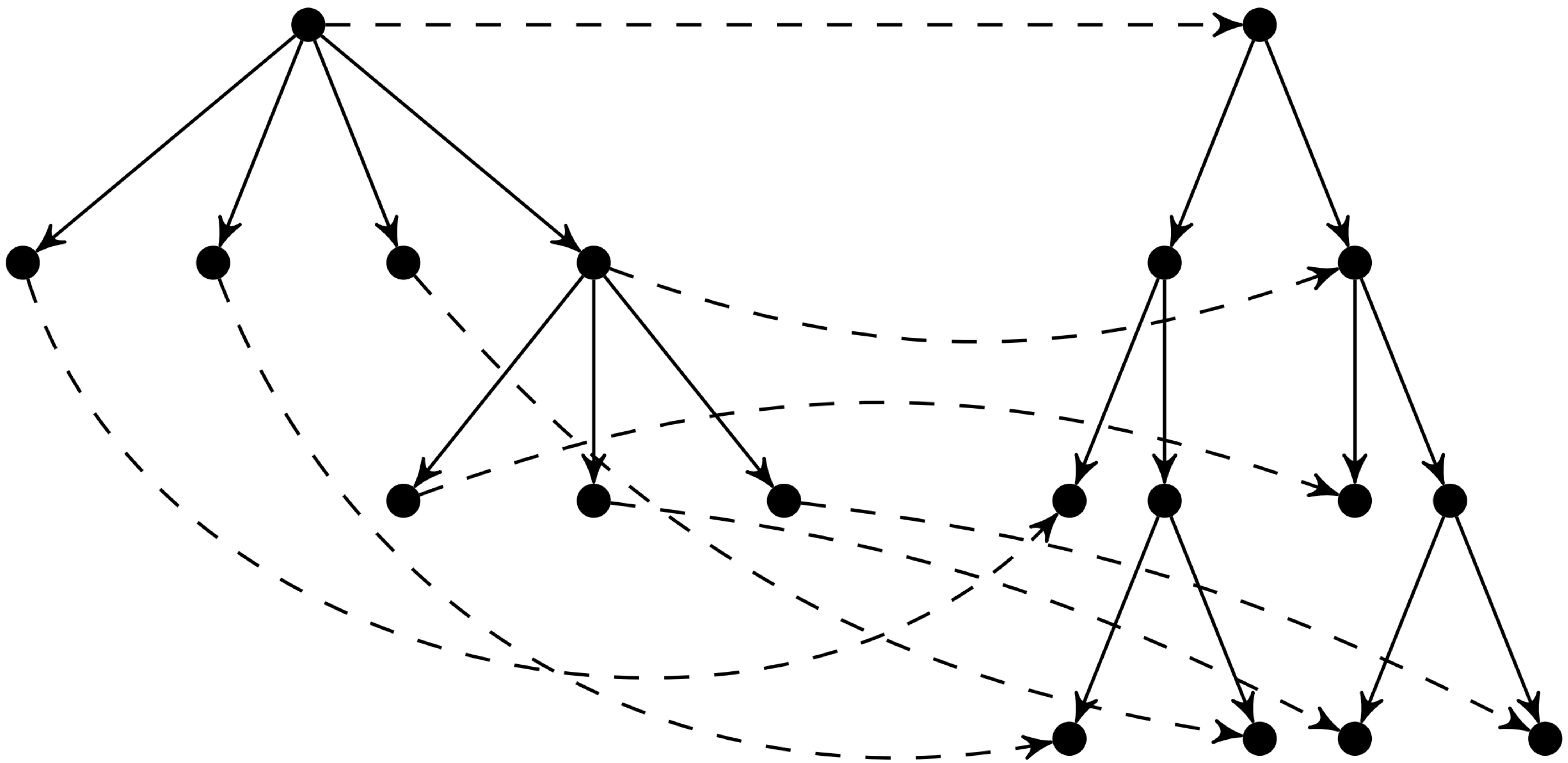
Expressivity of trees



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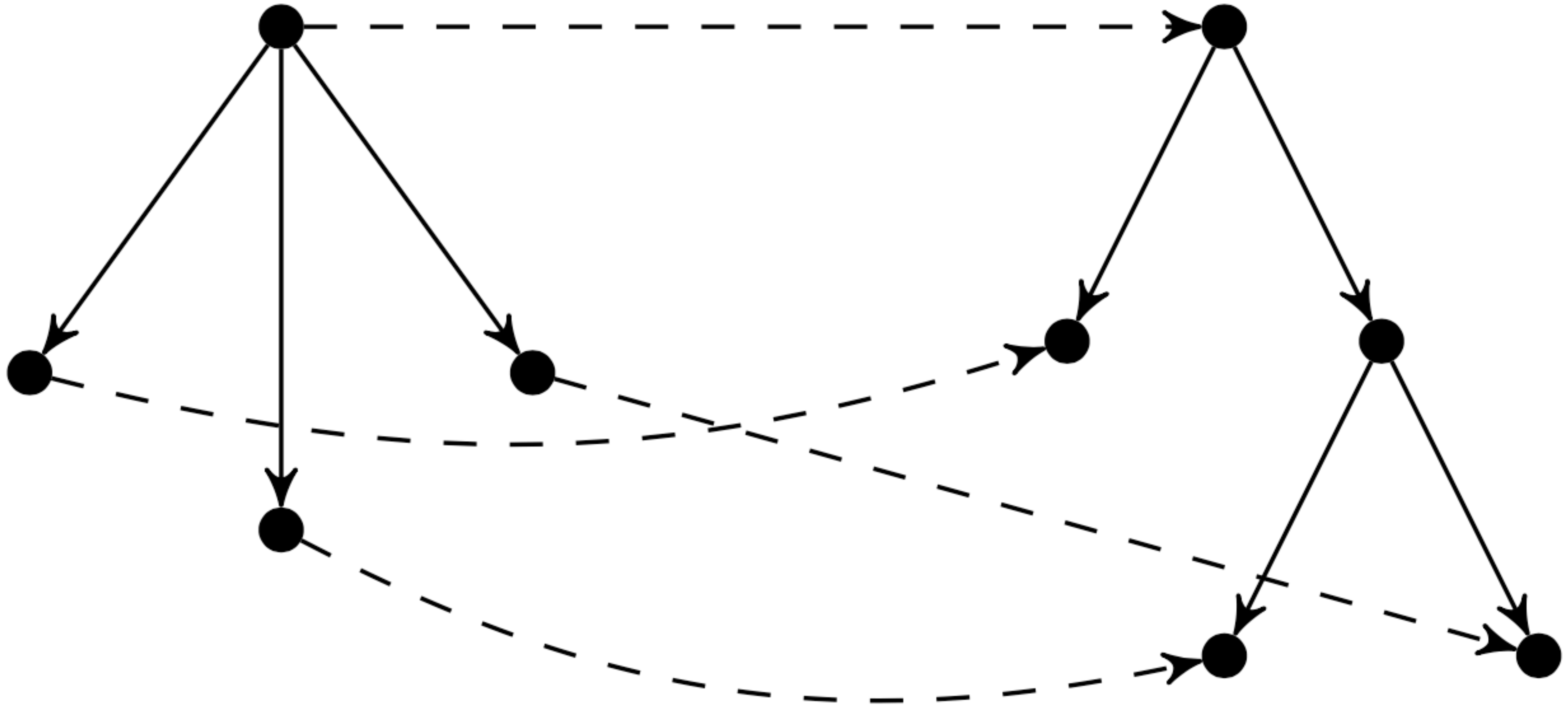
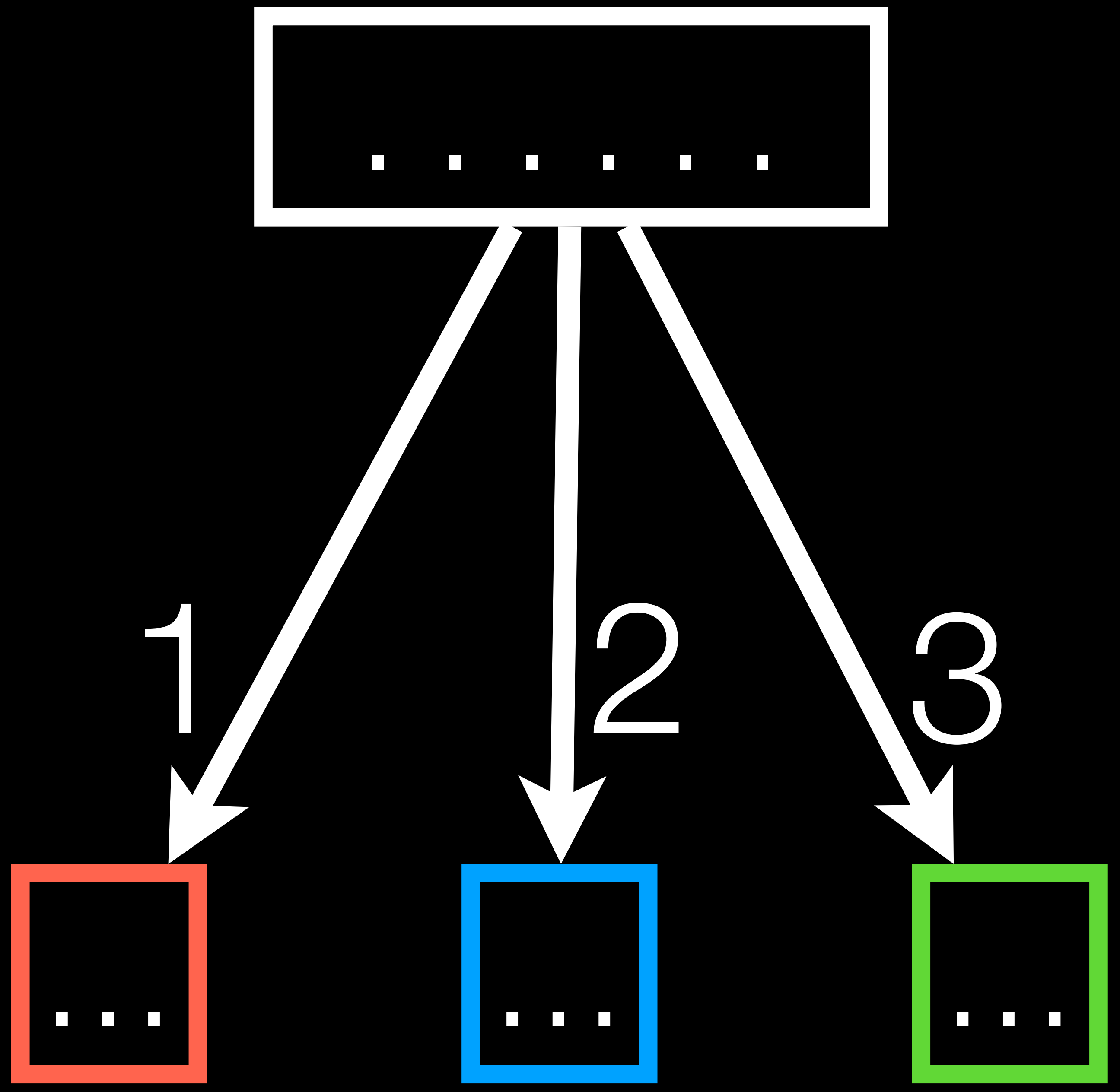
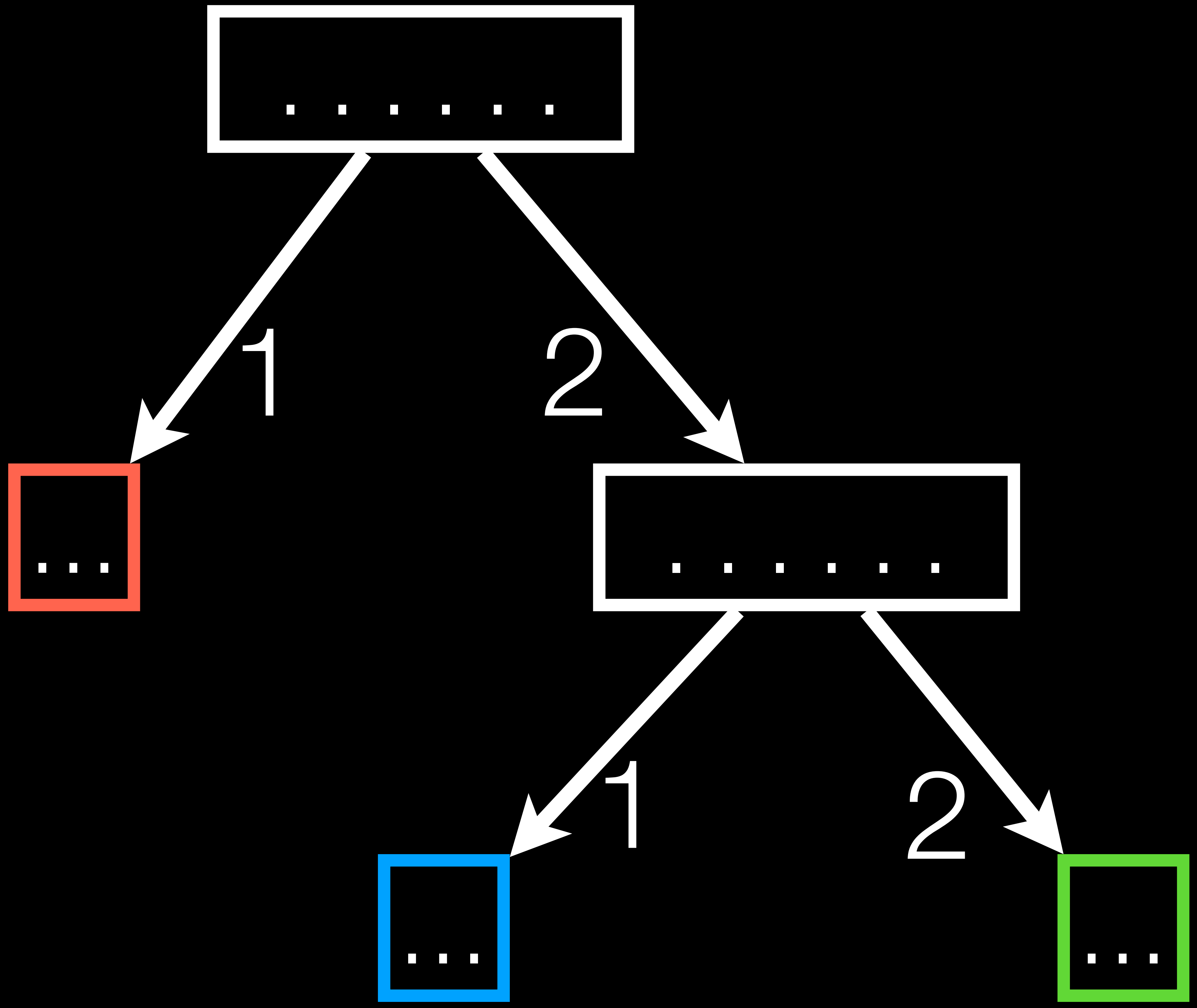


Homomorphic embedding.

Map root to root, leaves to leaves. Respect ancestry.

Compiling programs





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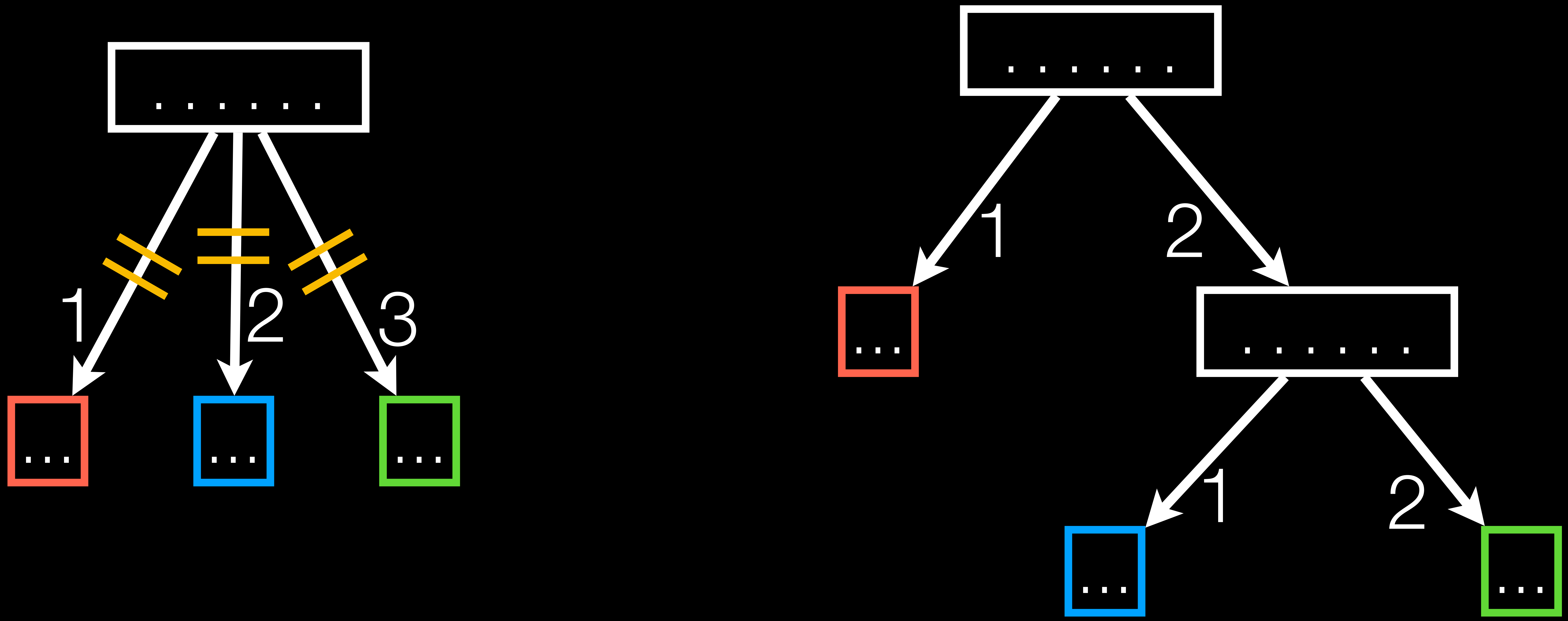

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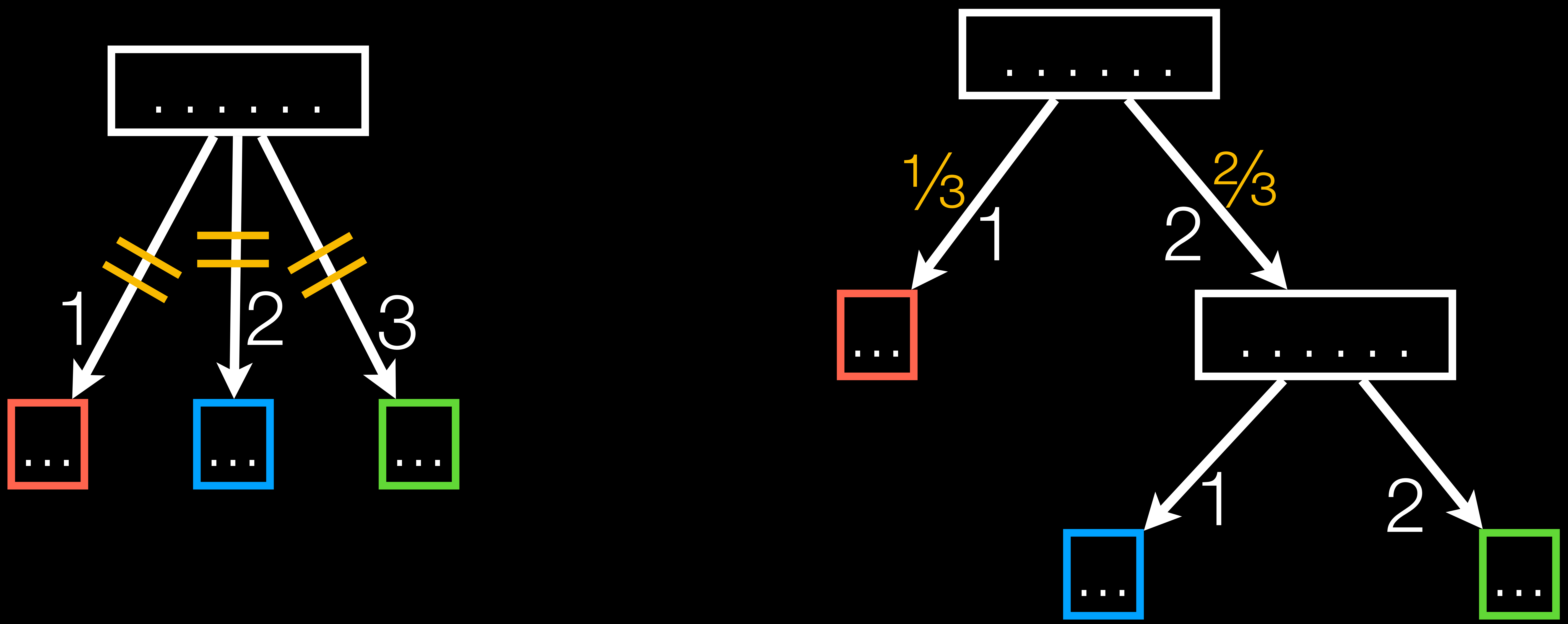

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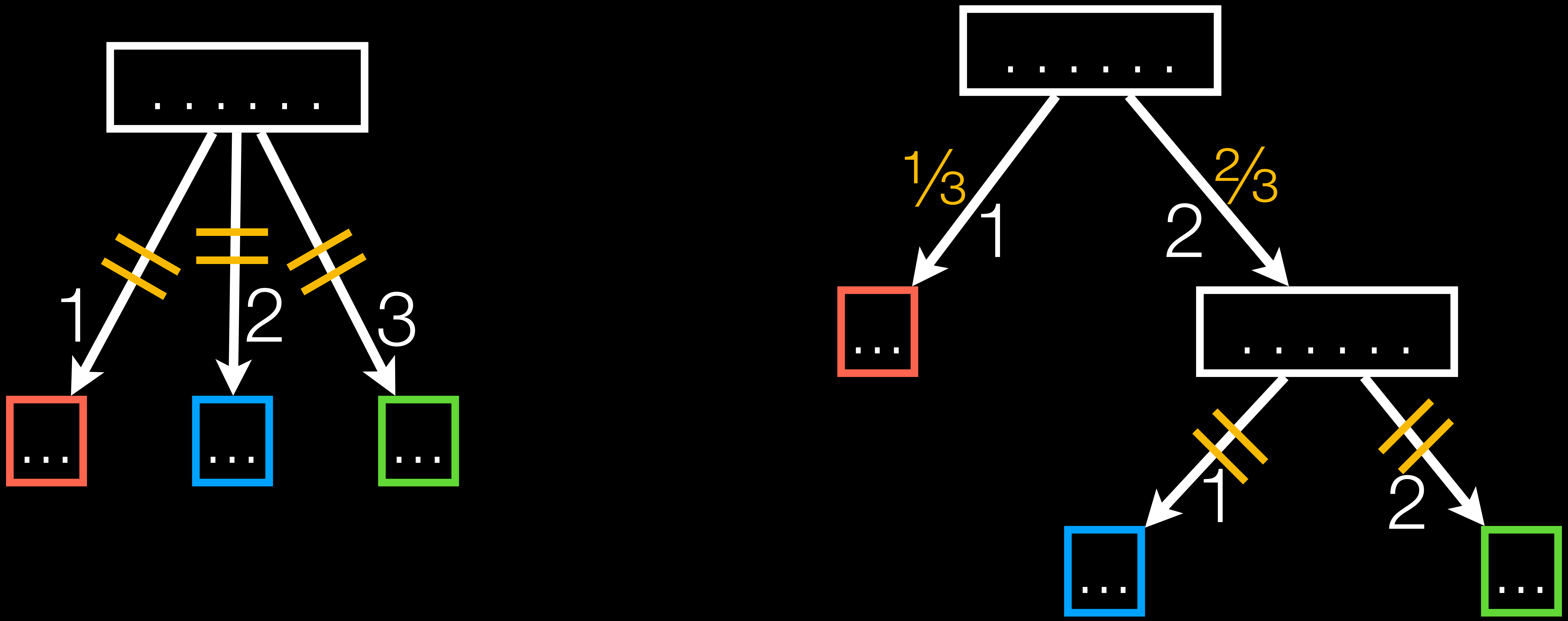

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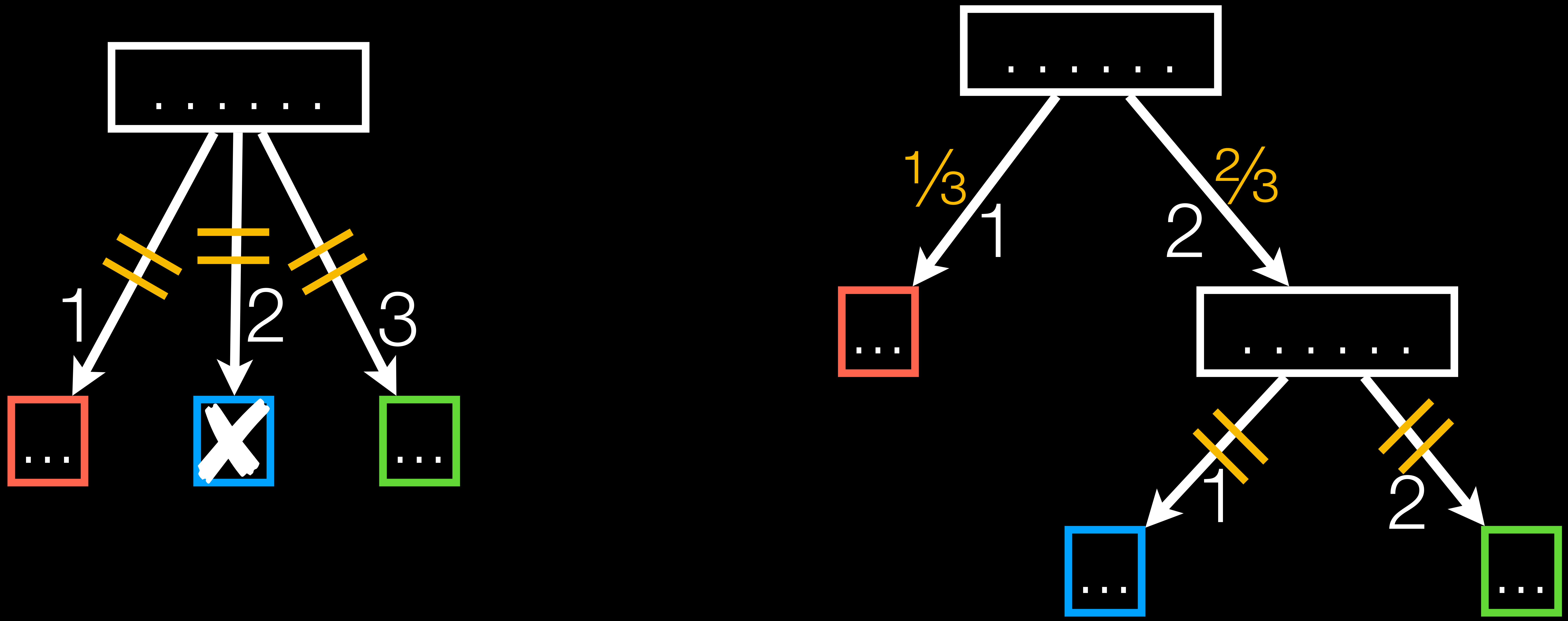

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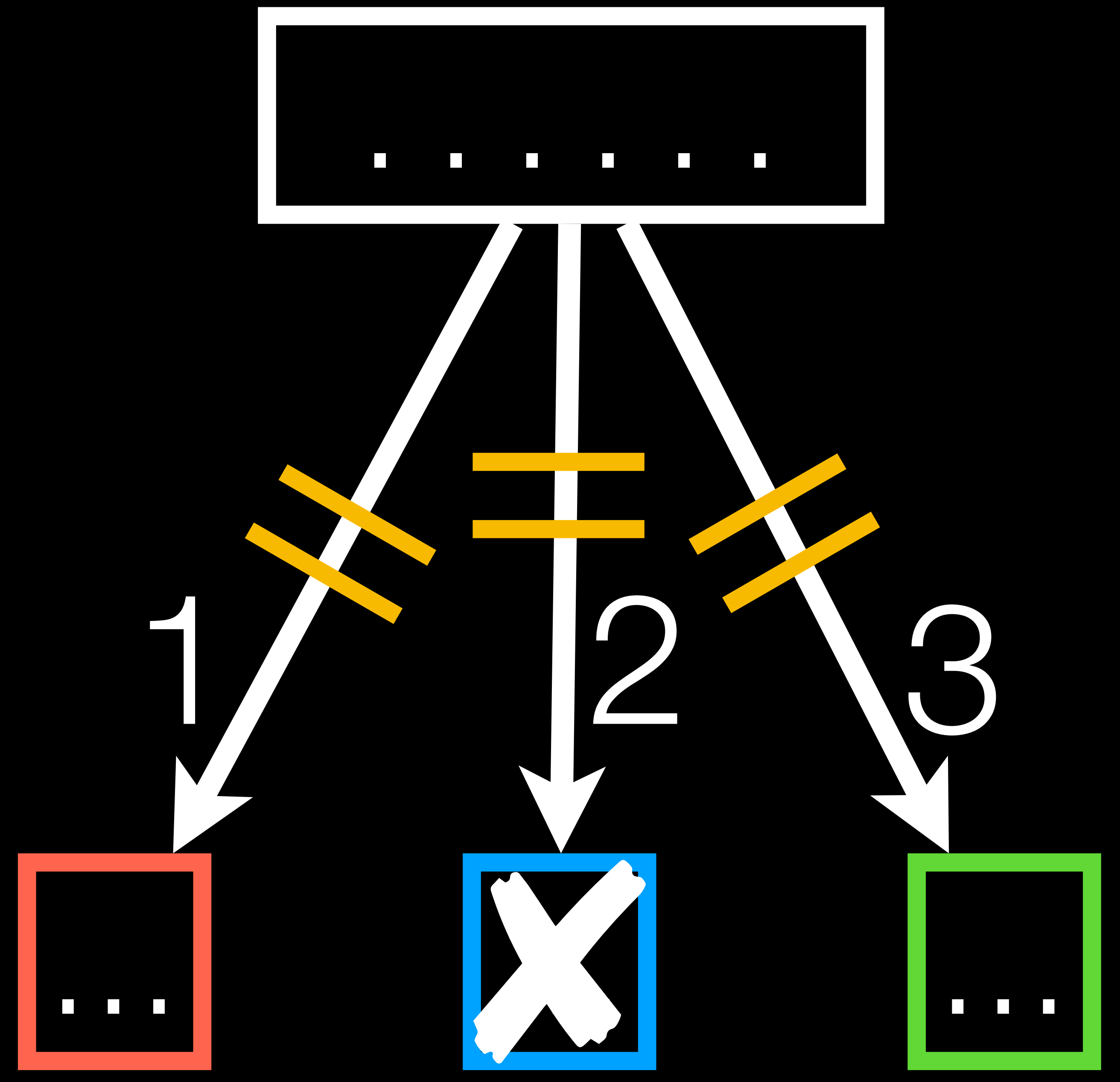

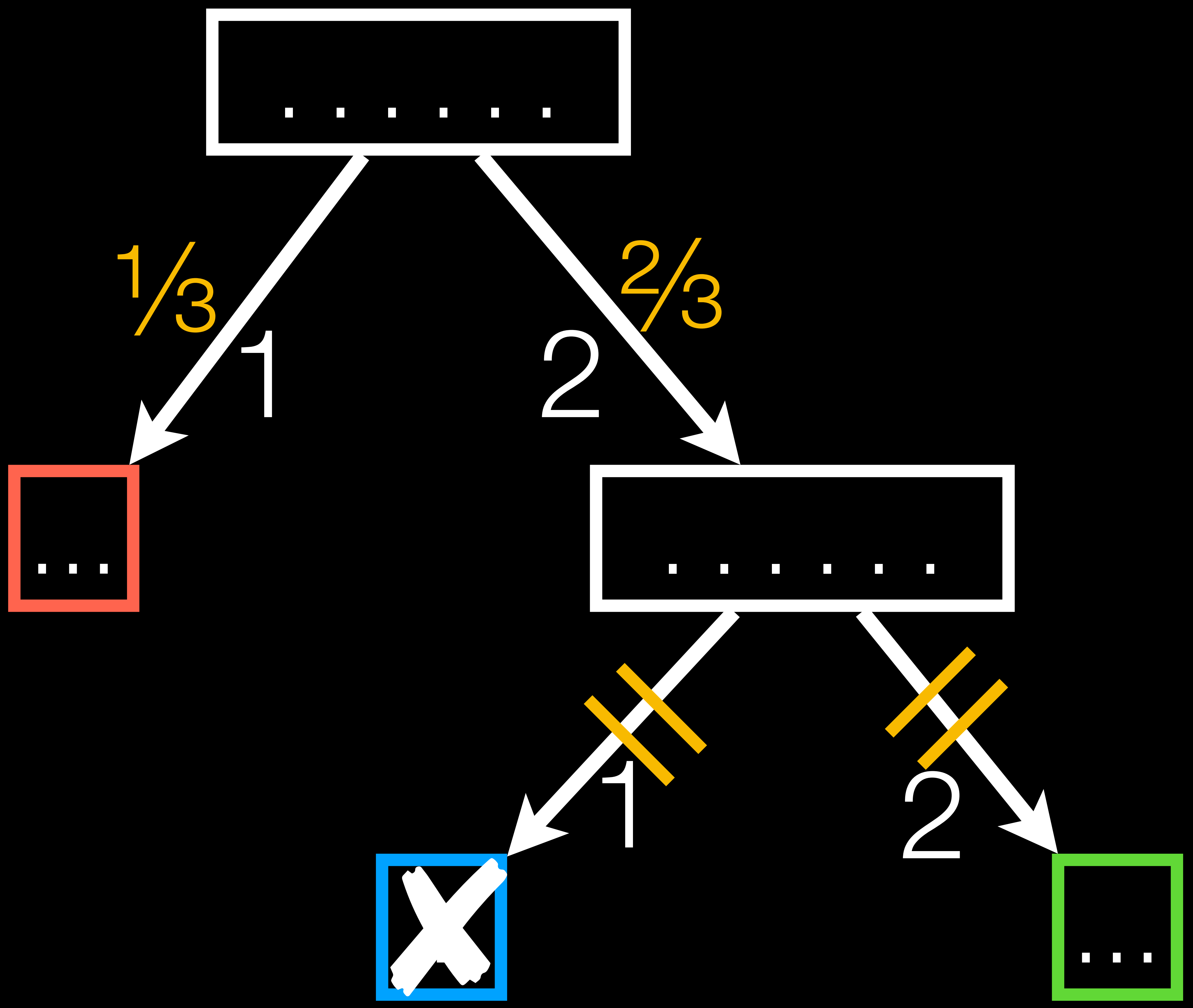

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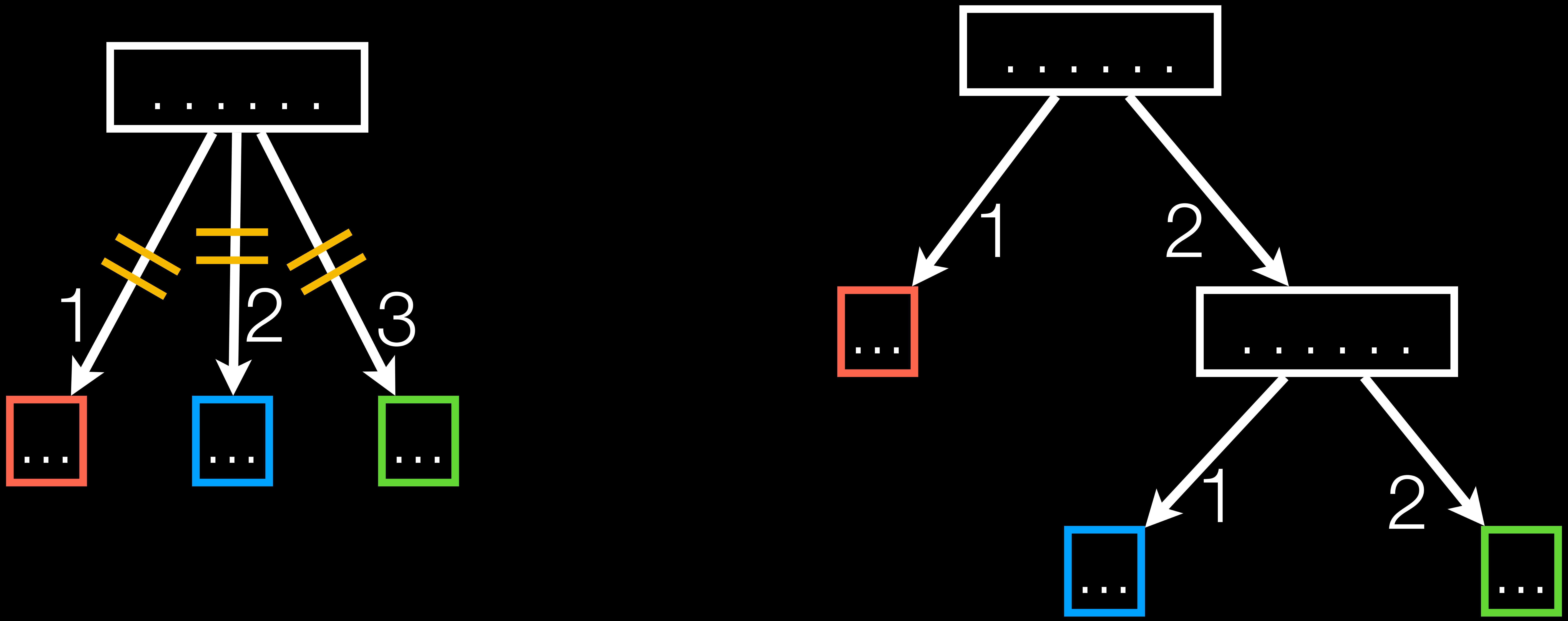

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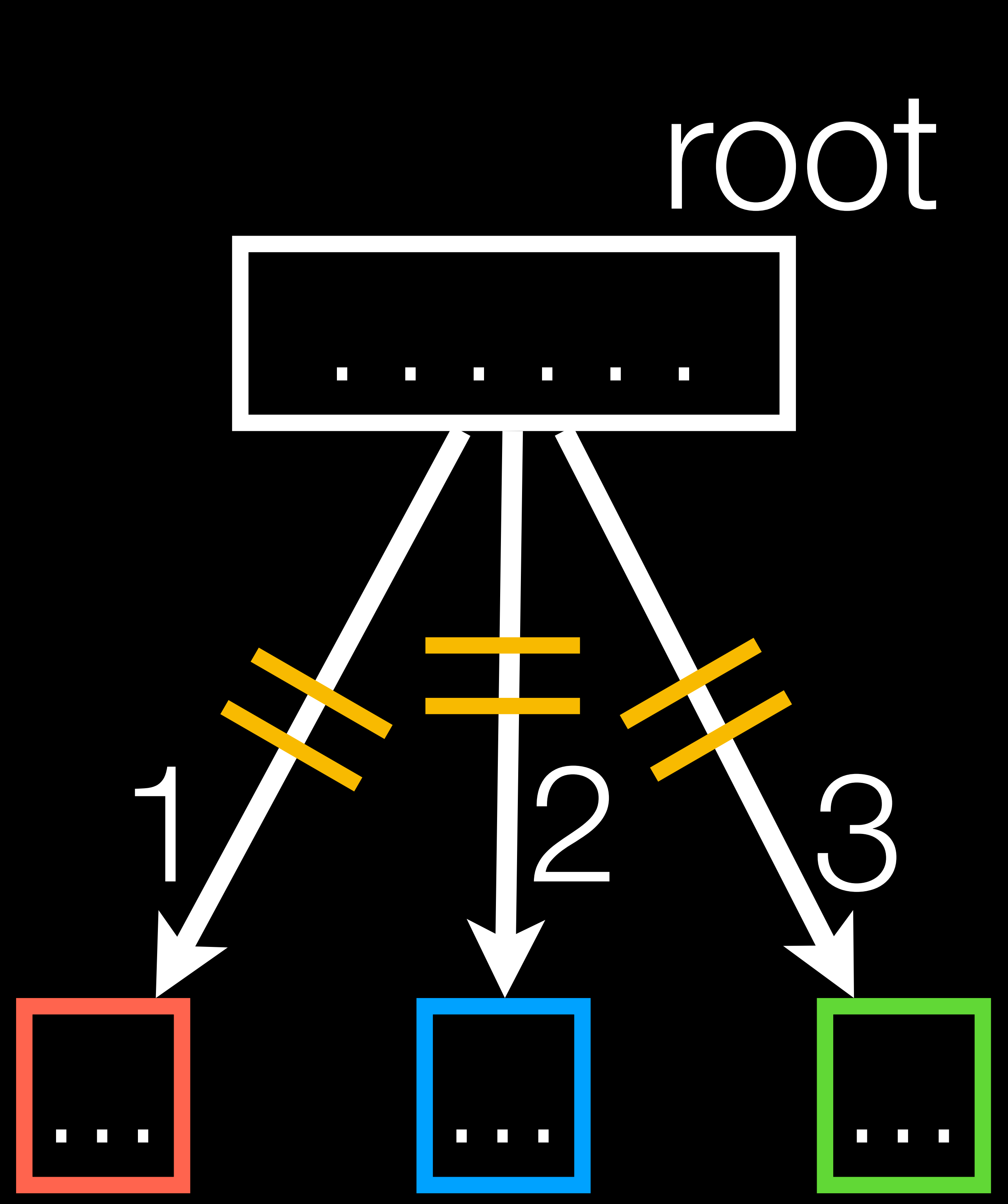

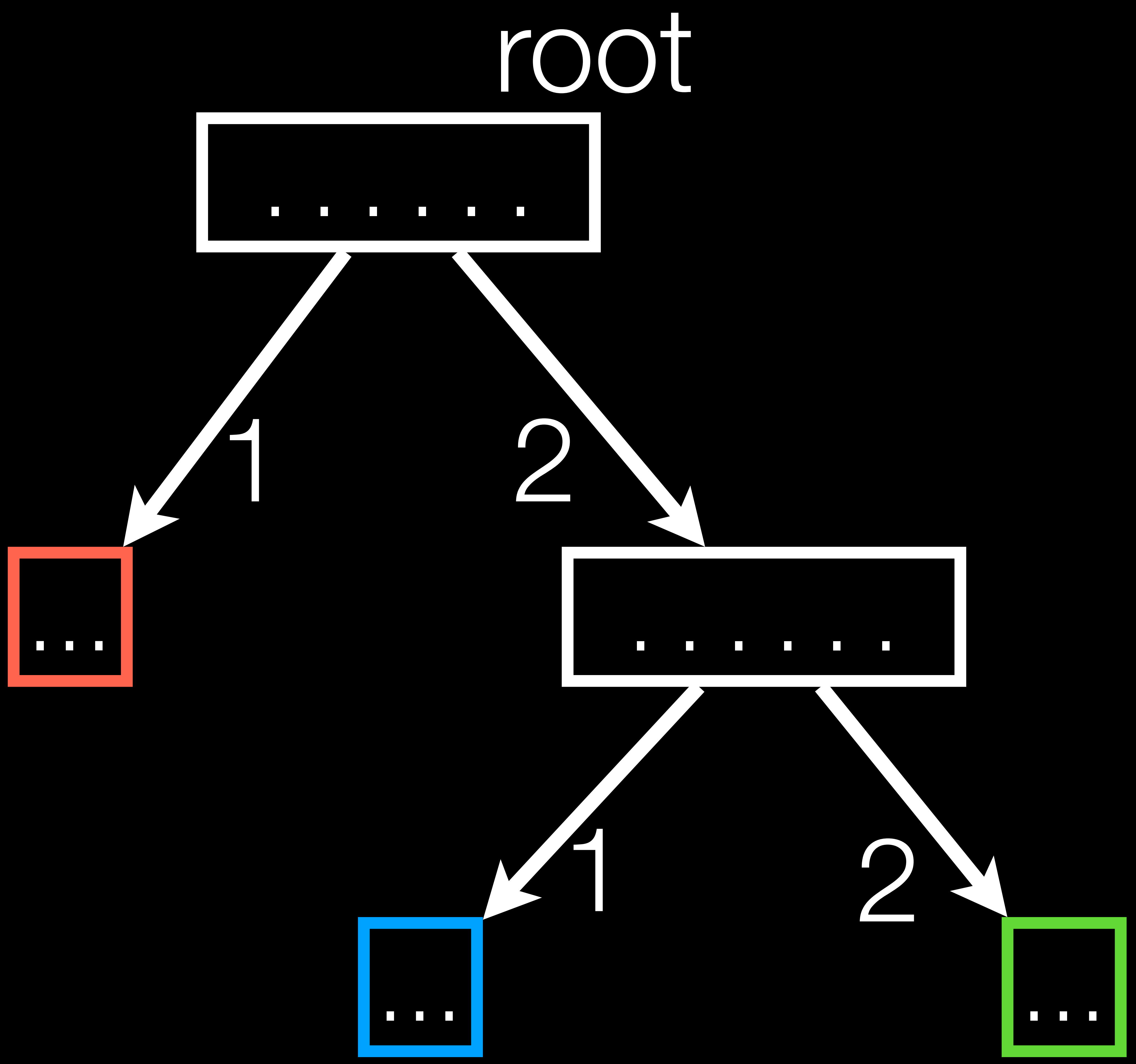

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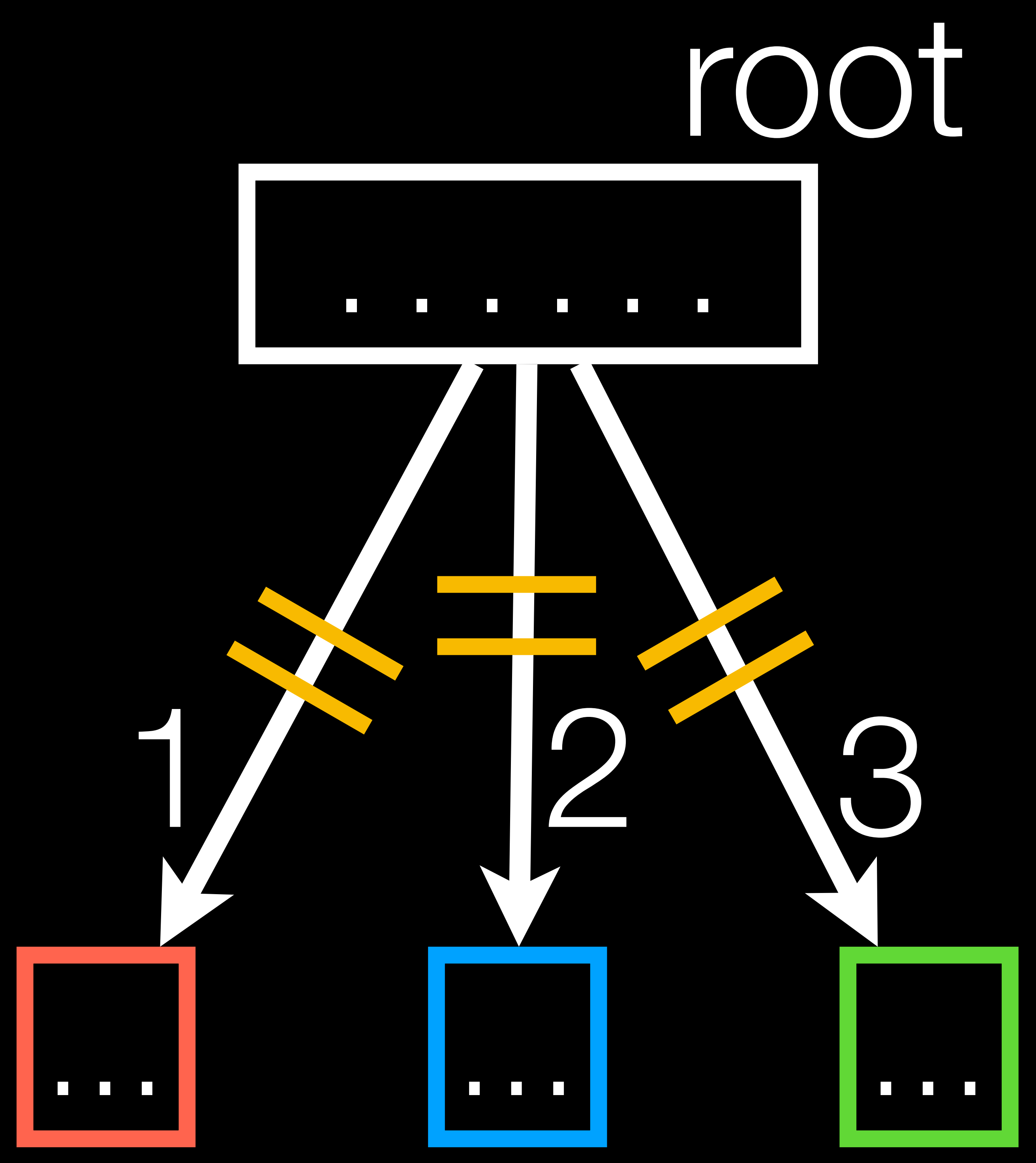

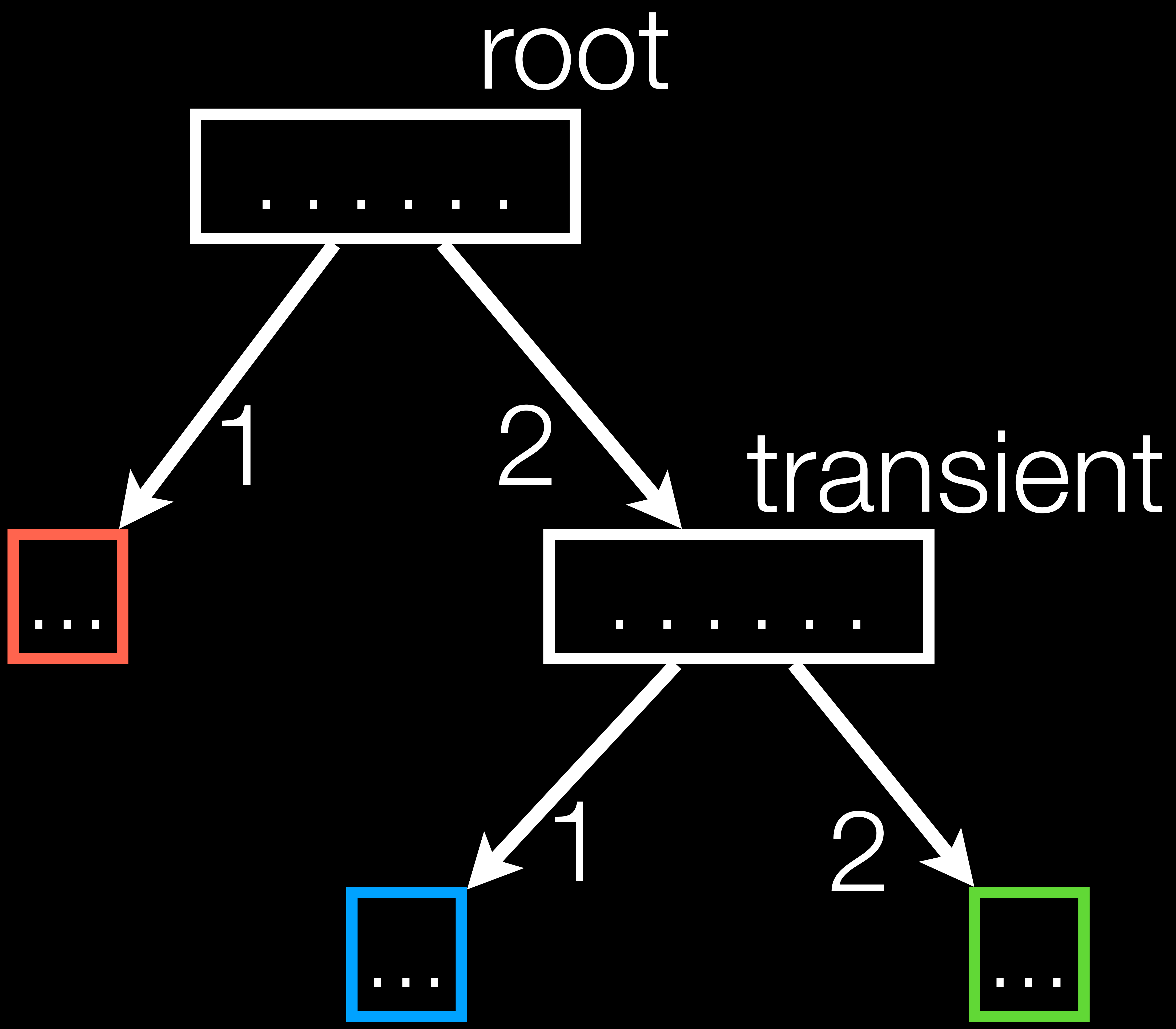

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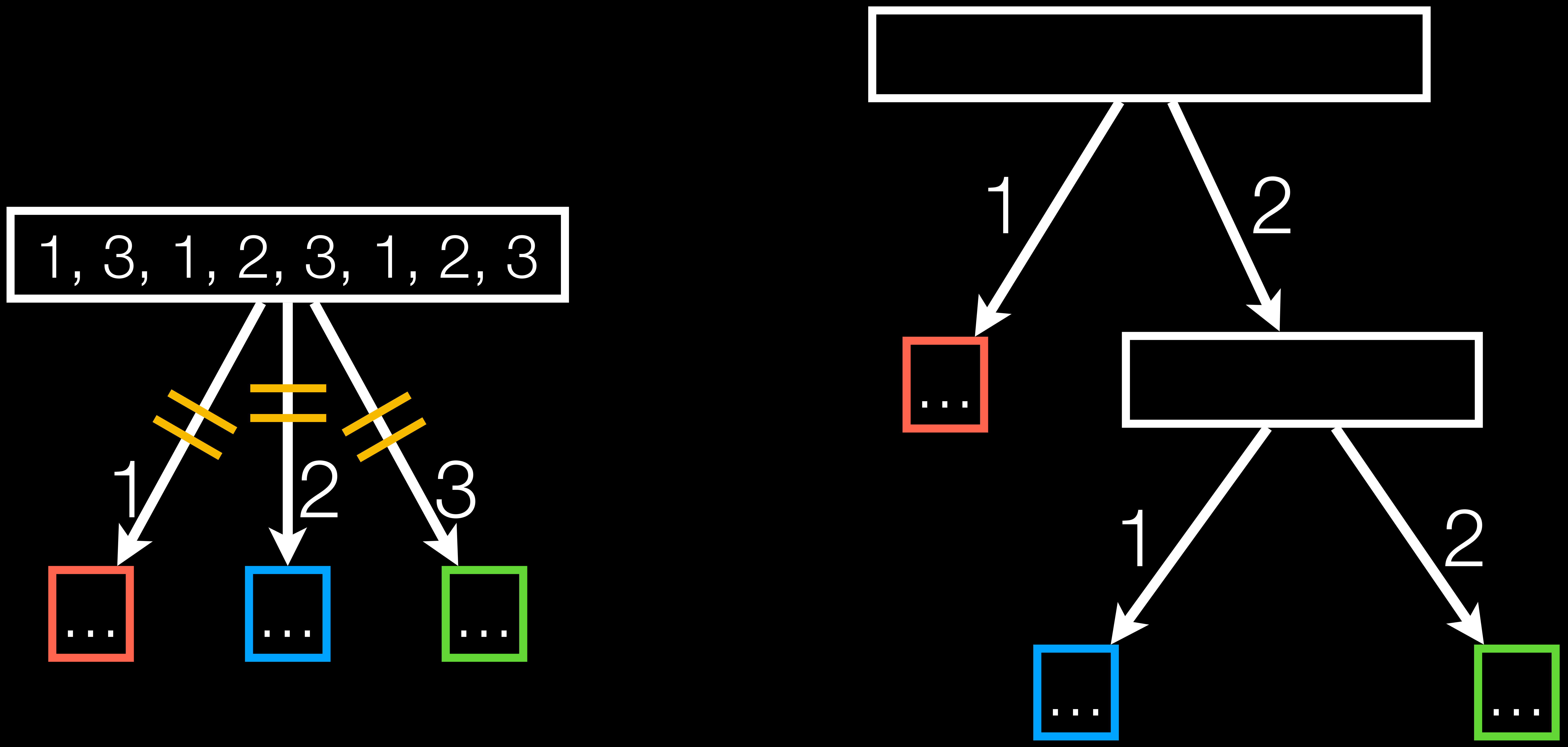

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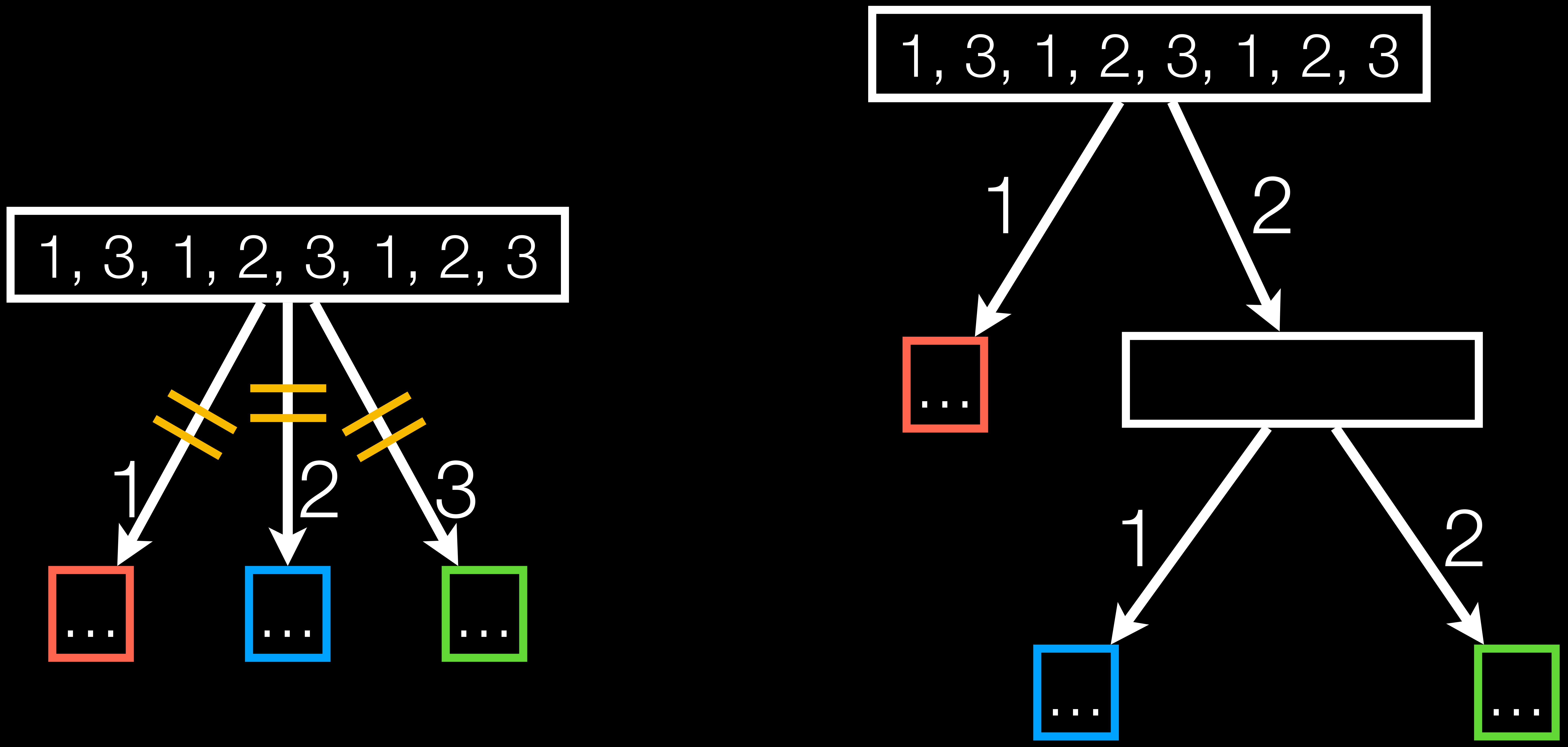

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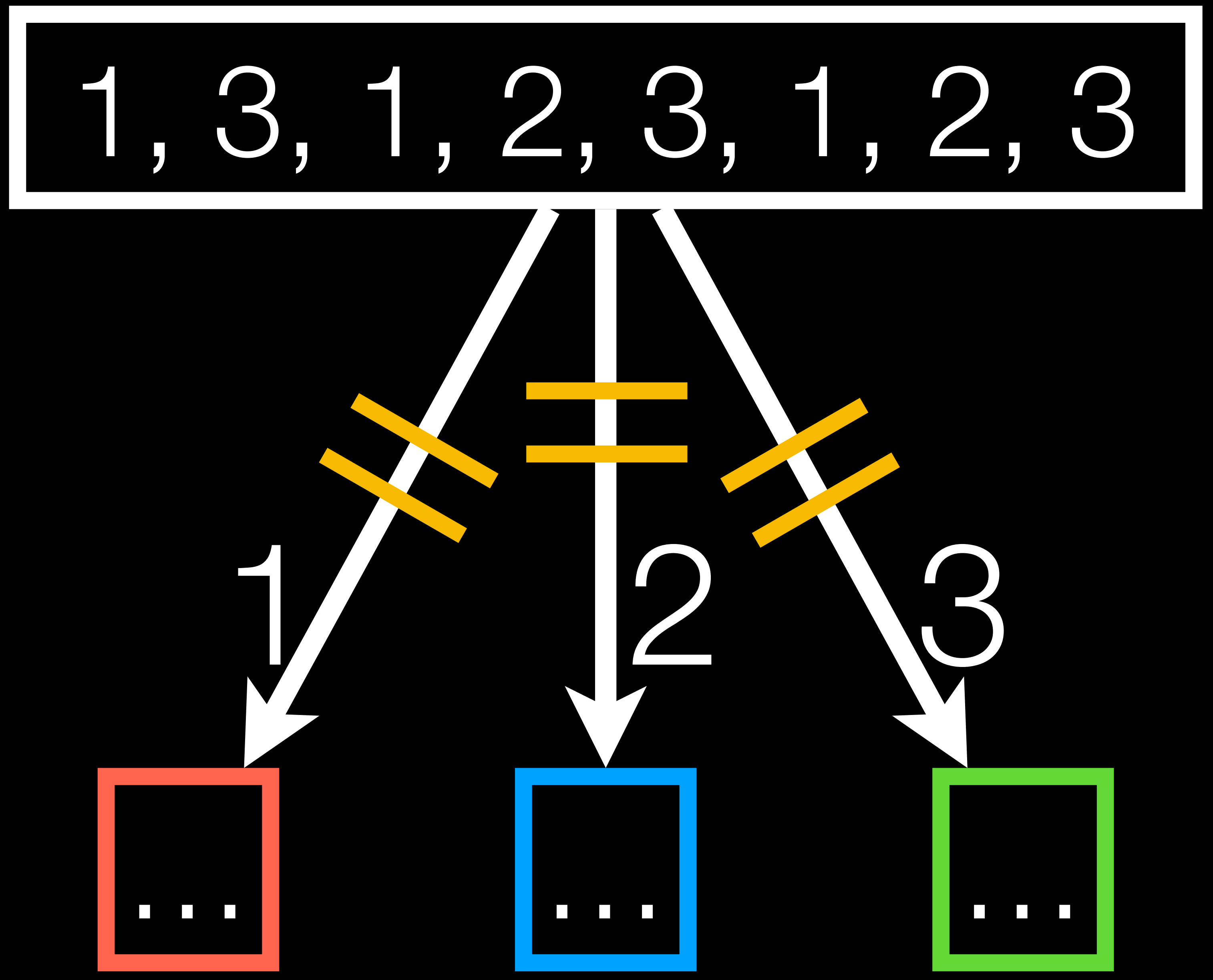

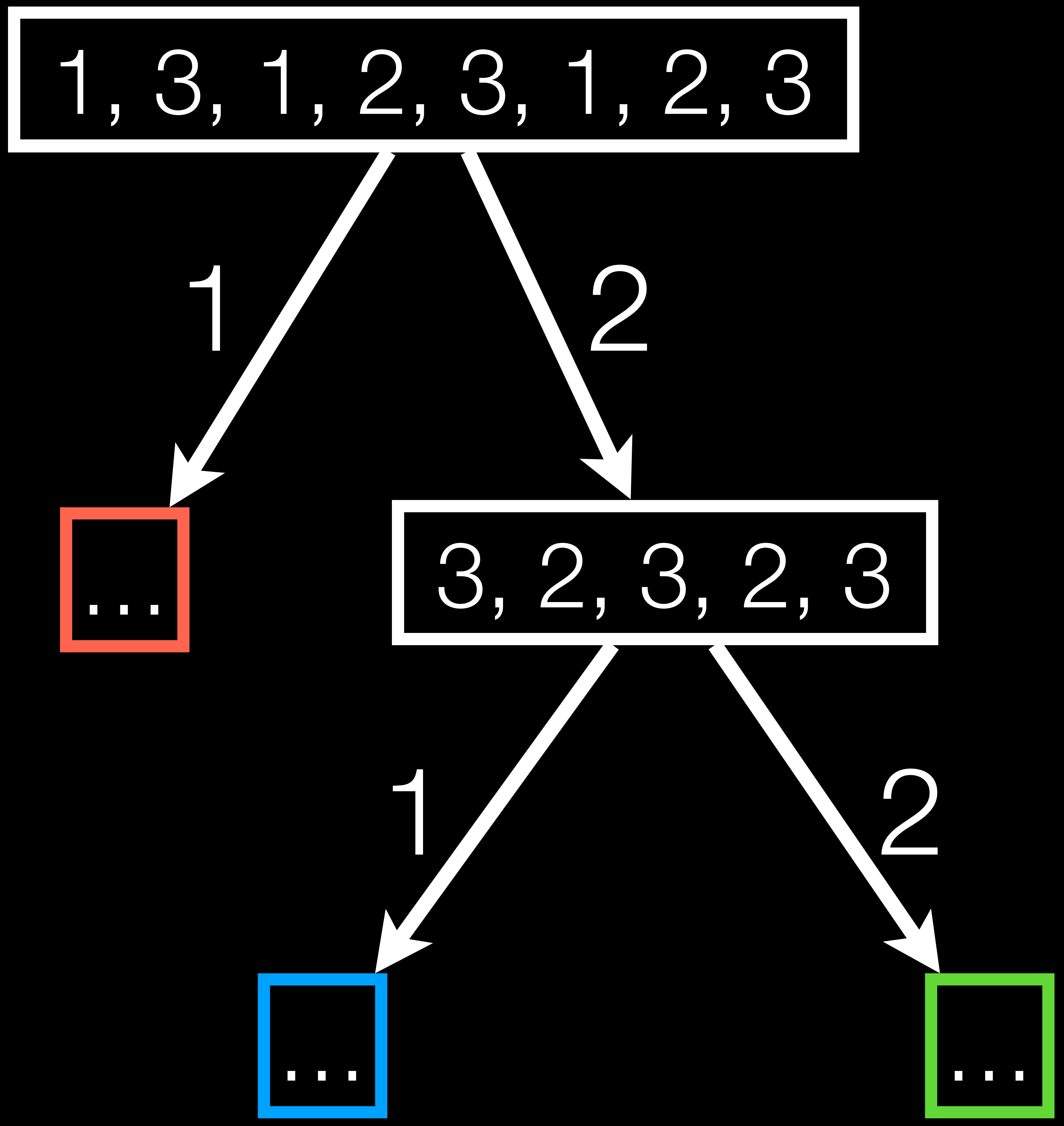

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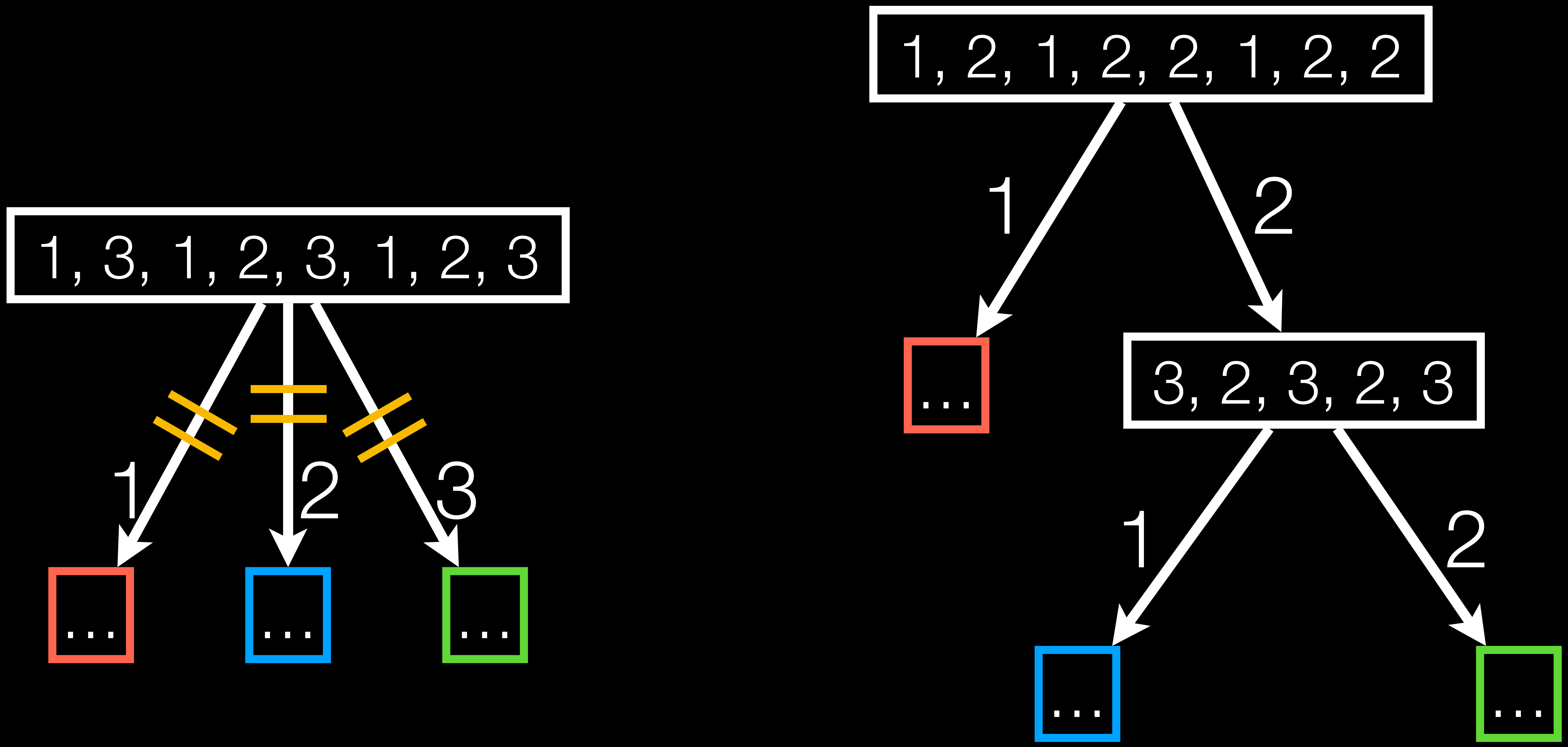

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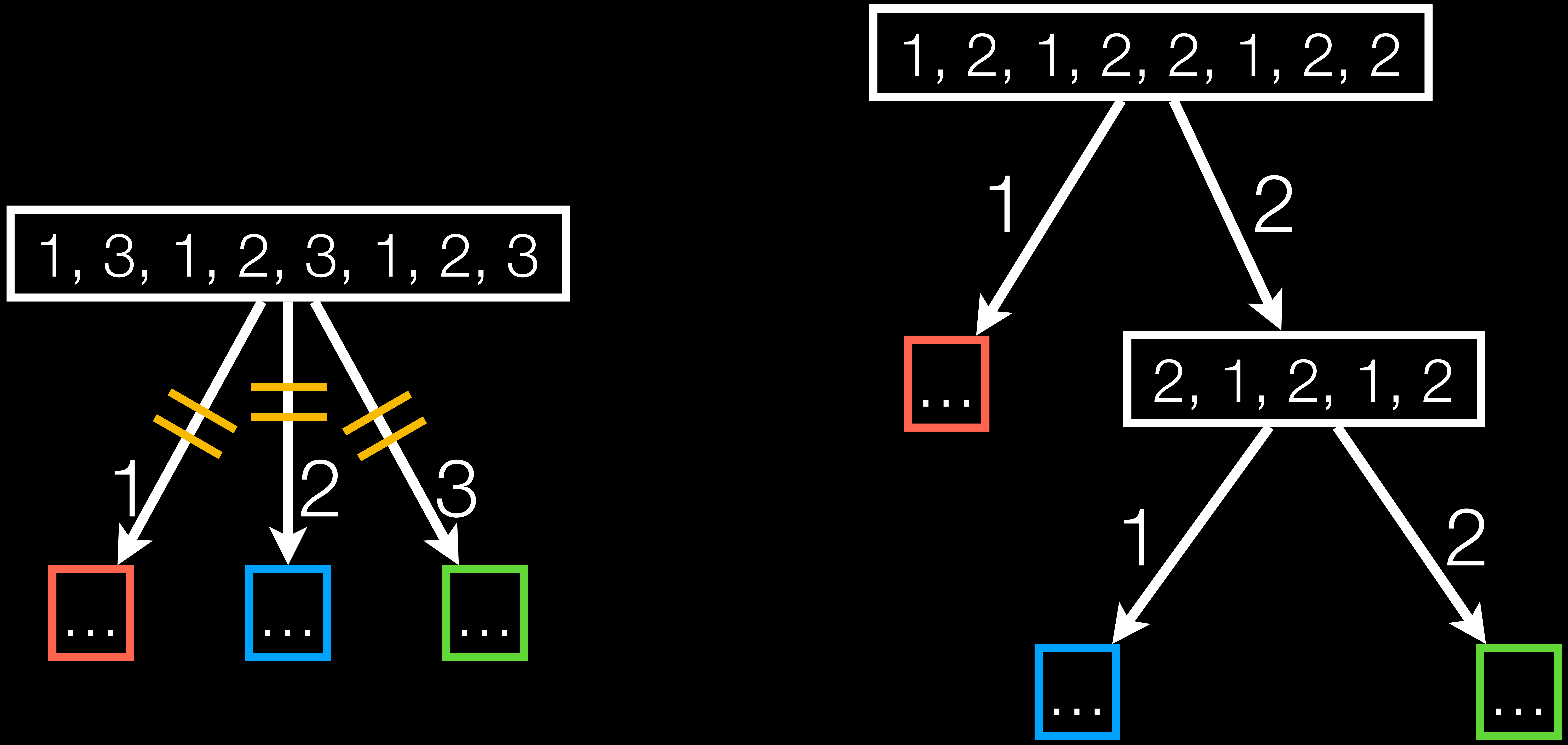

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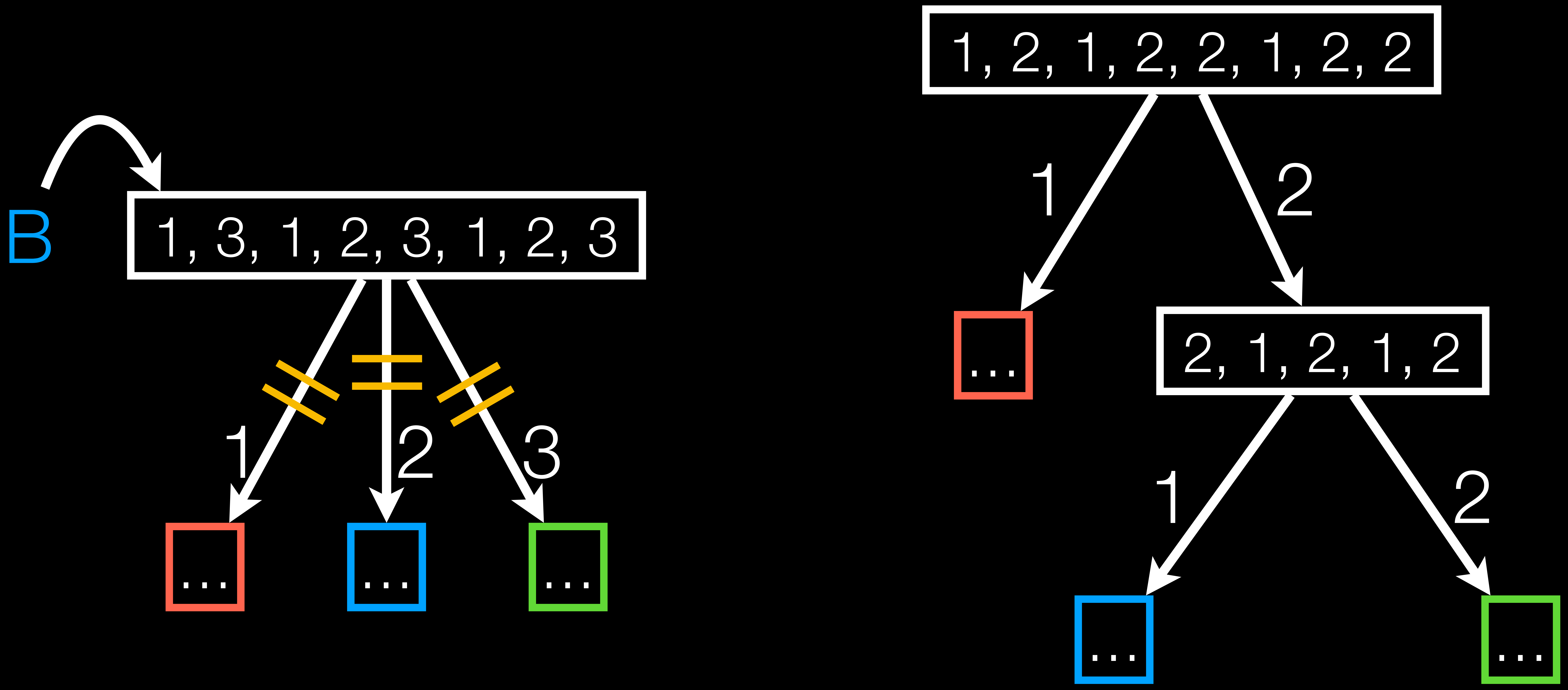

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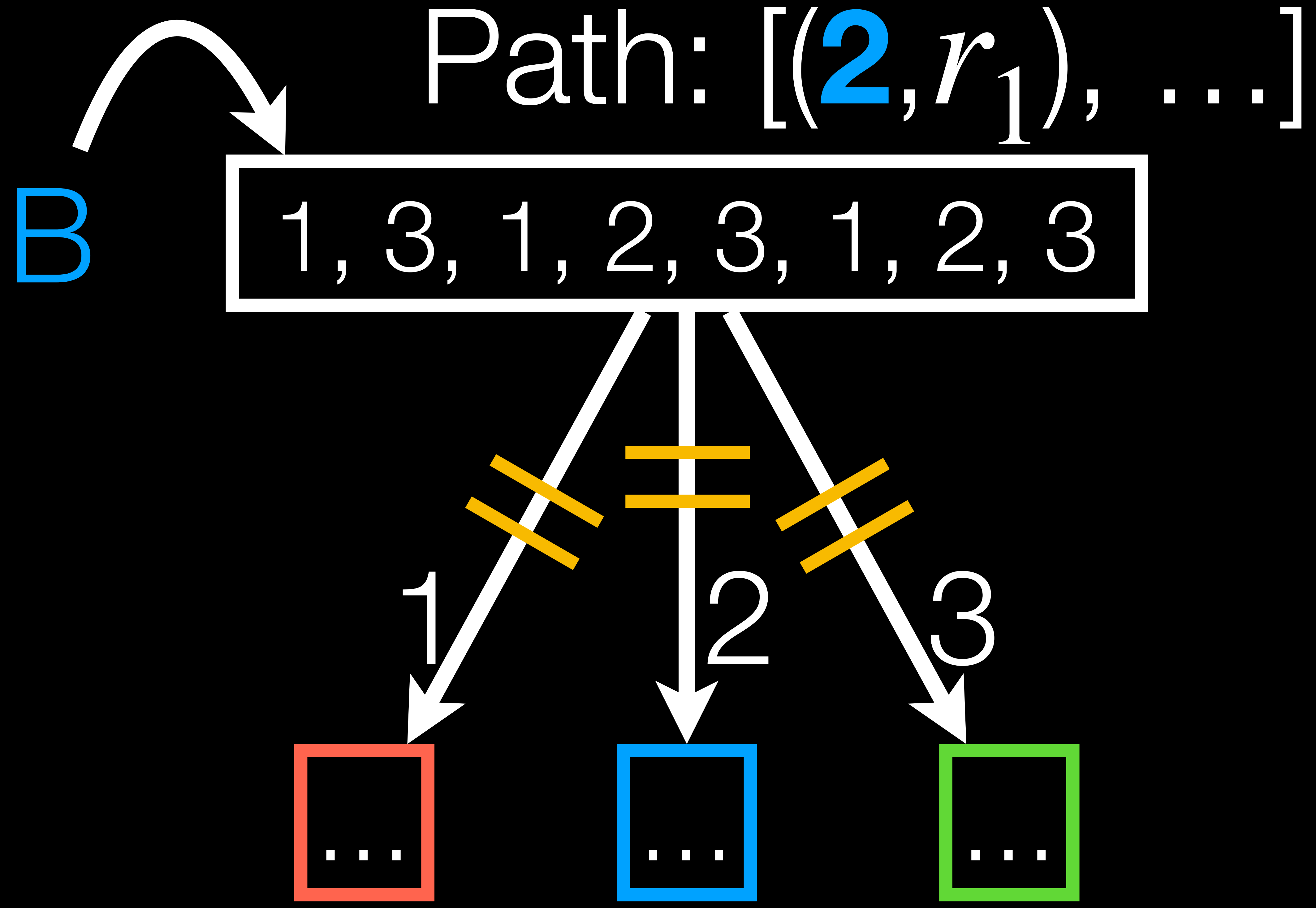

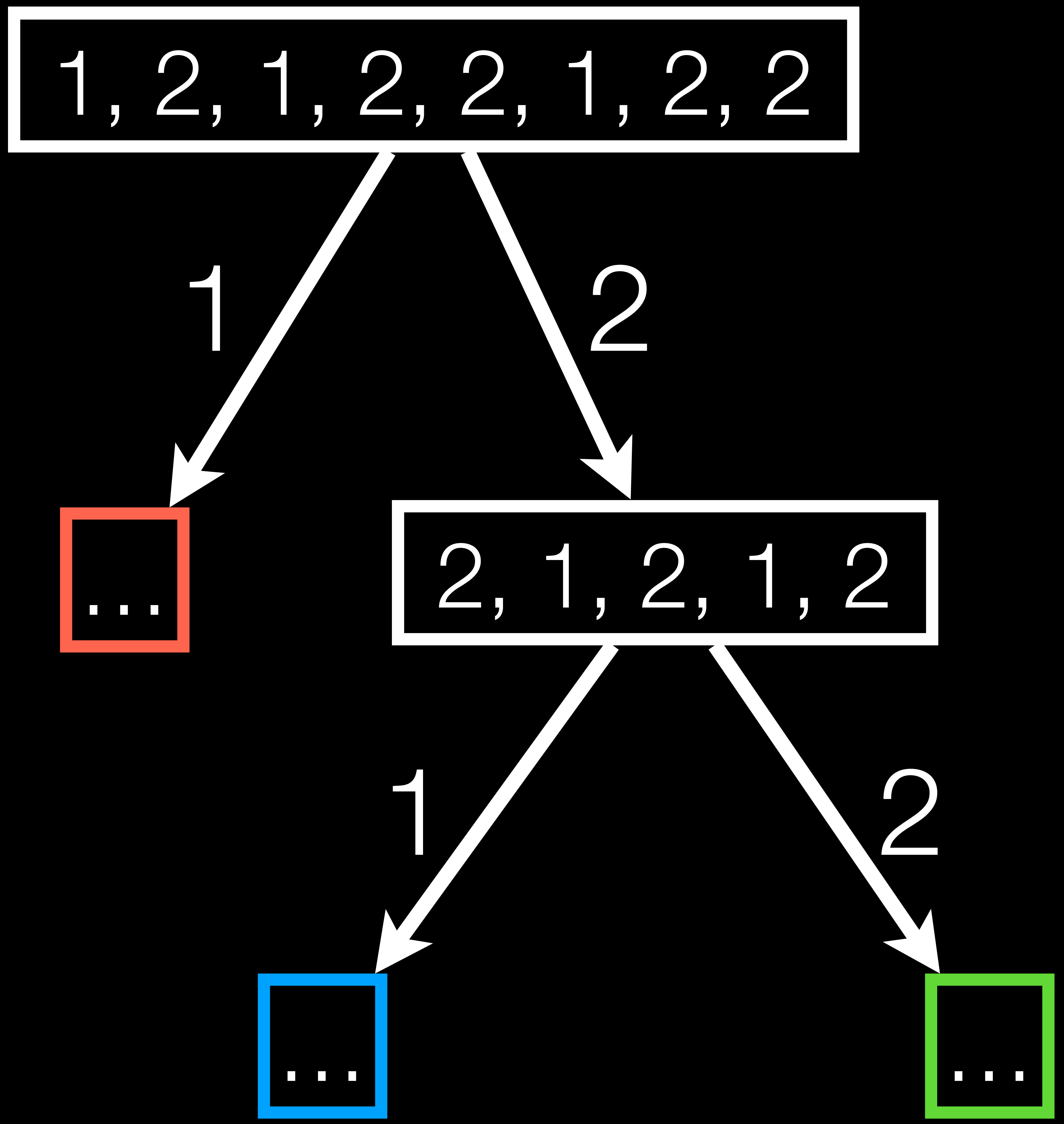

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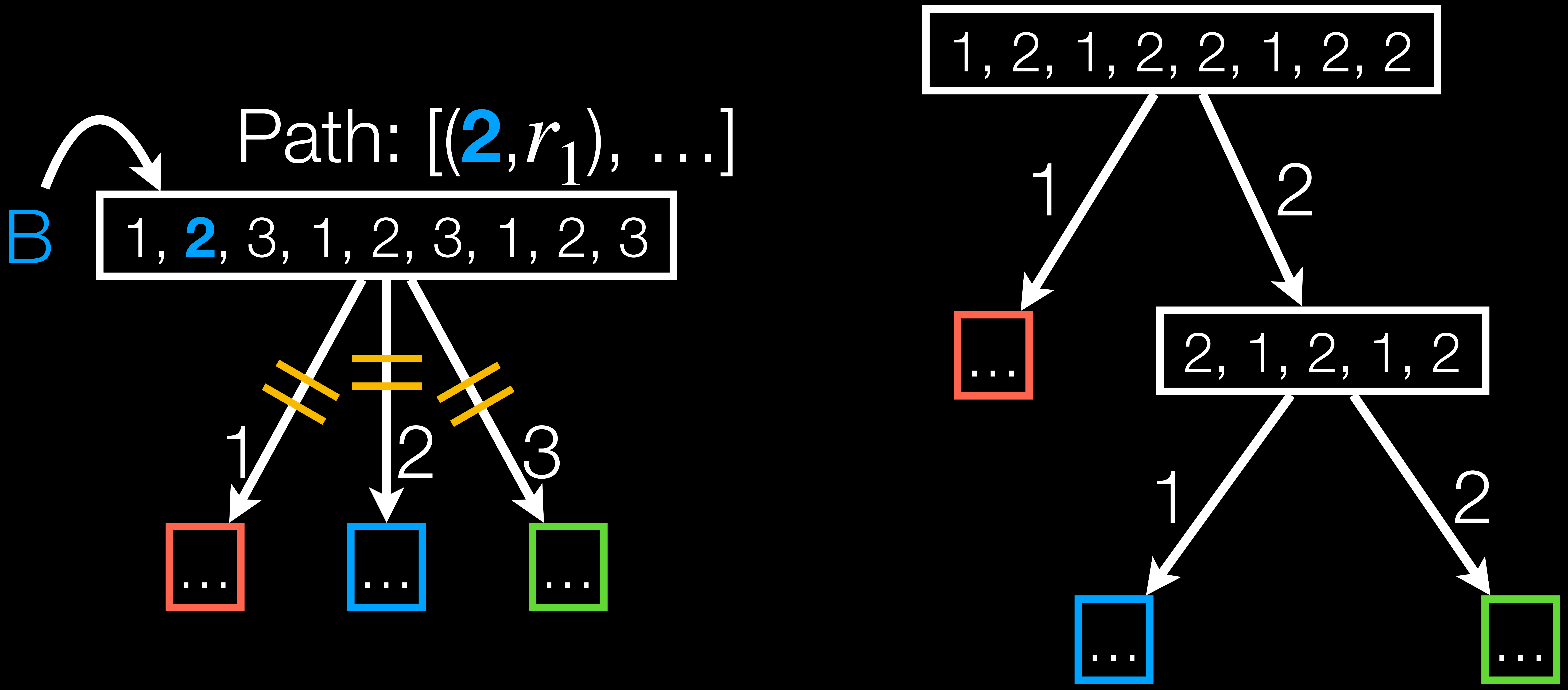

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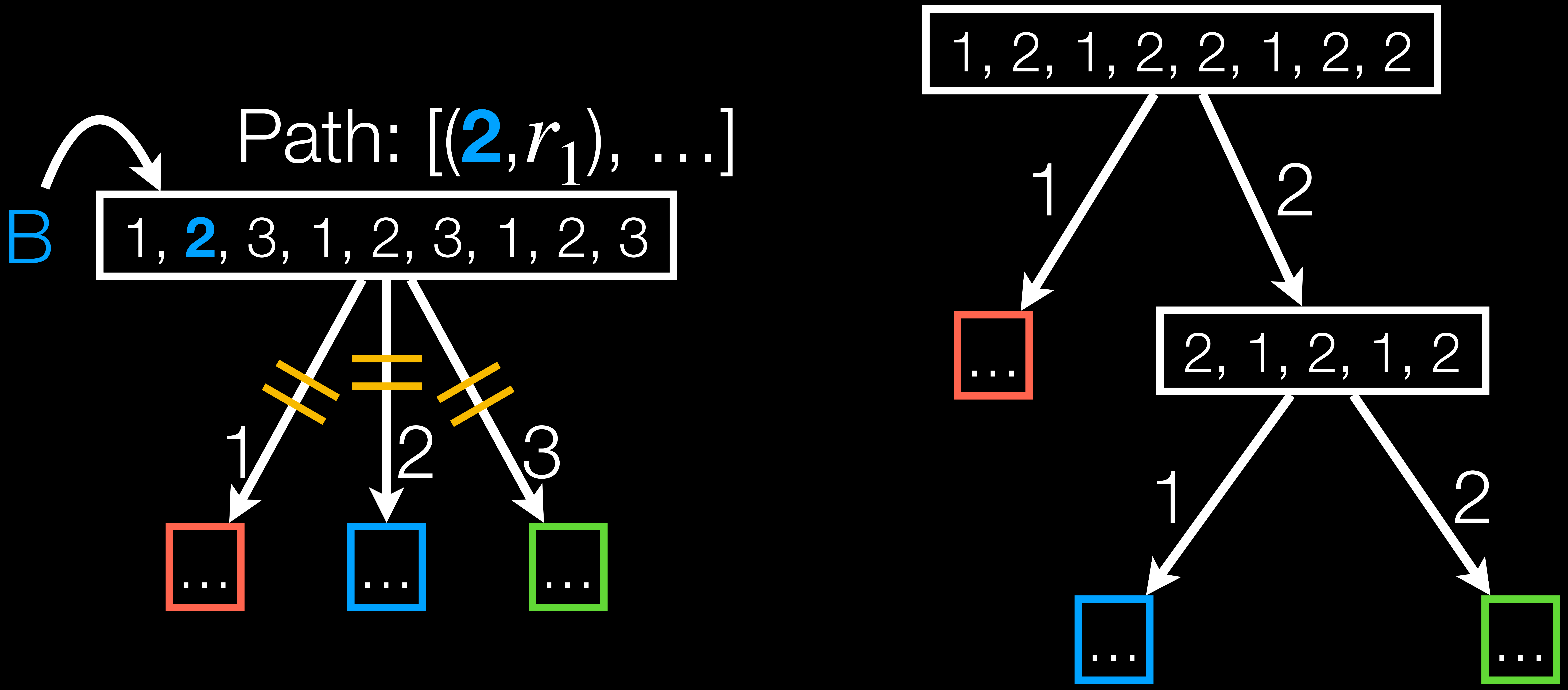
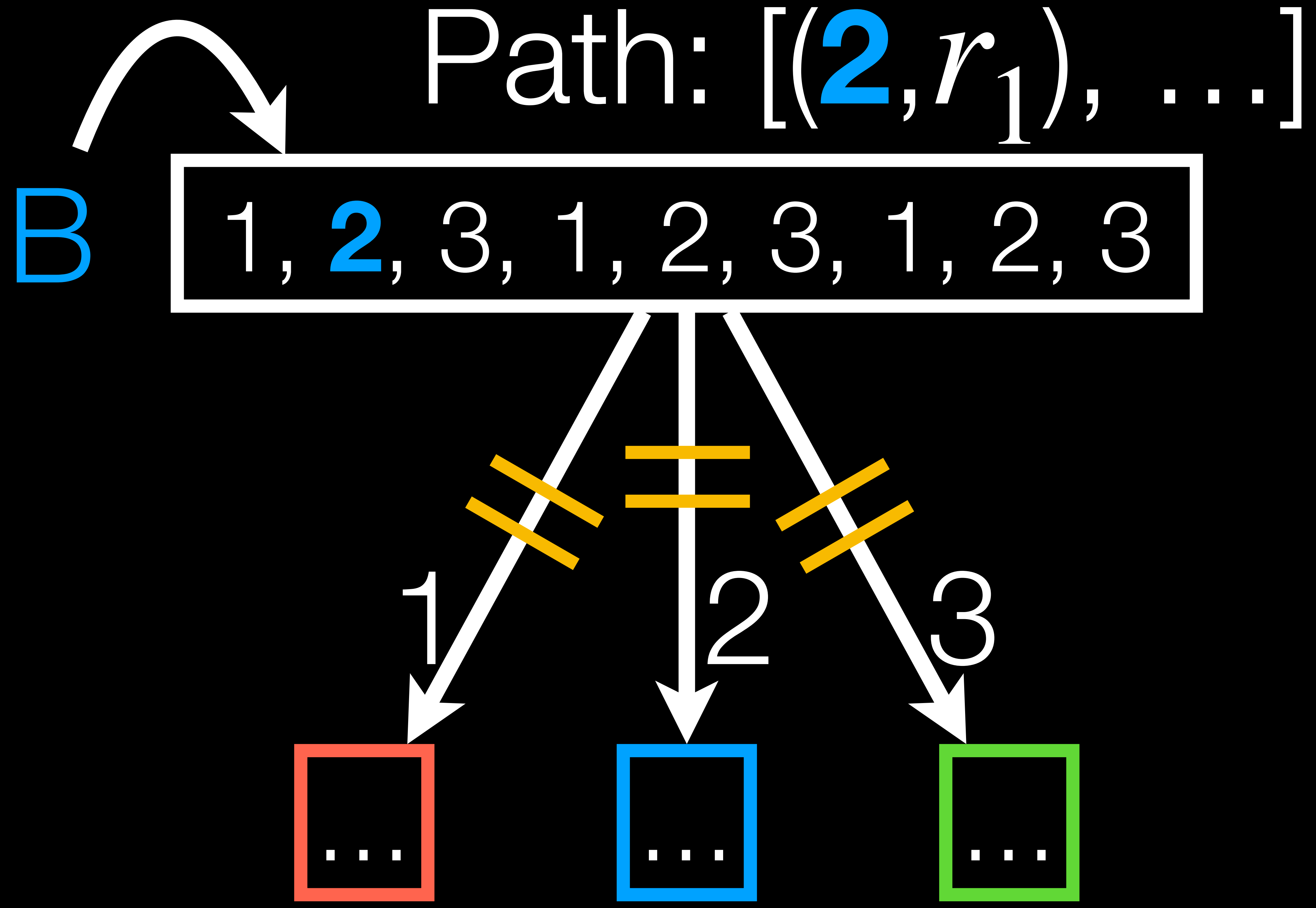


Path: [($\mathbf{2}$, r_1), …]

Compiling programs



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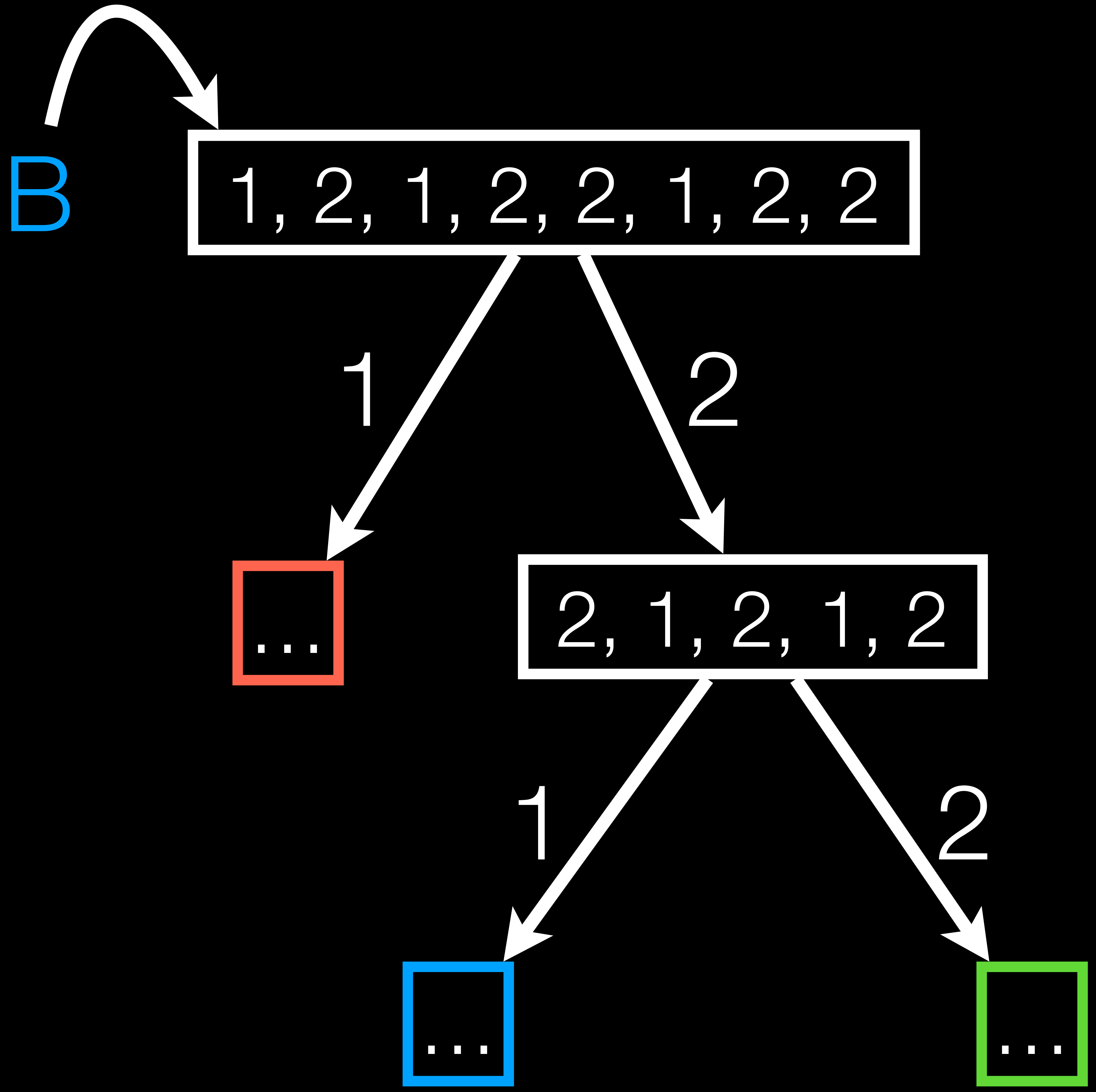

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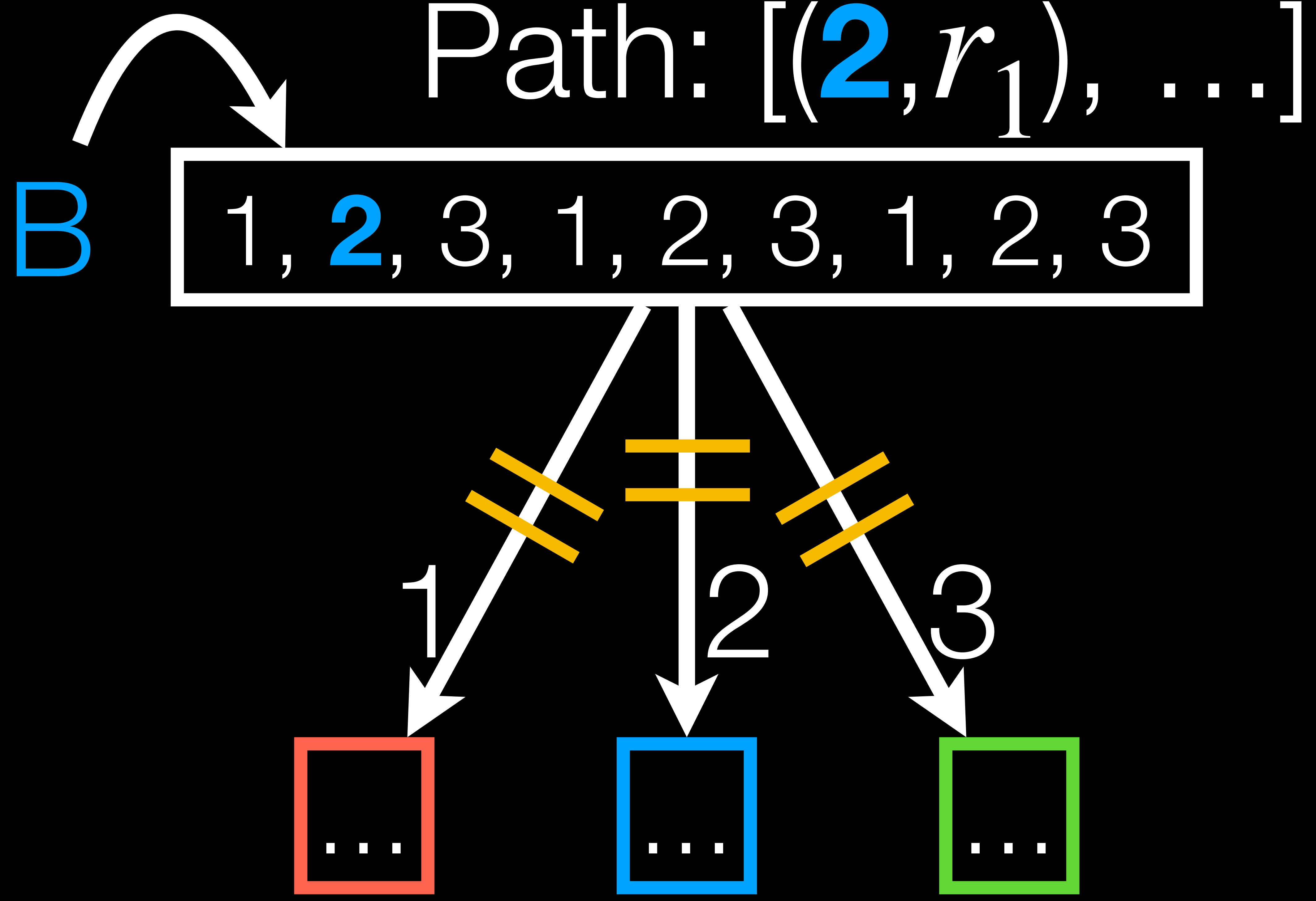


Path: [(**2**,r_1), …]

B

1, **2**, 3, 1, 2, 3, 1, 2, 3

1

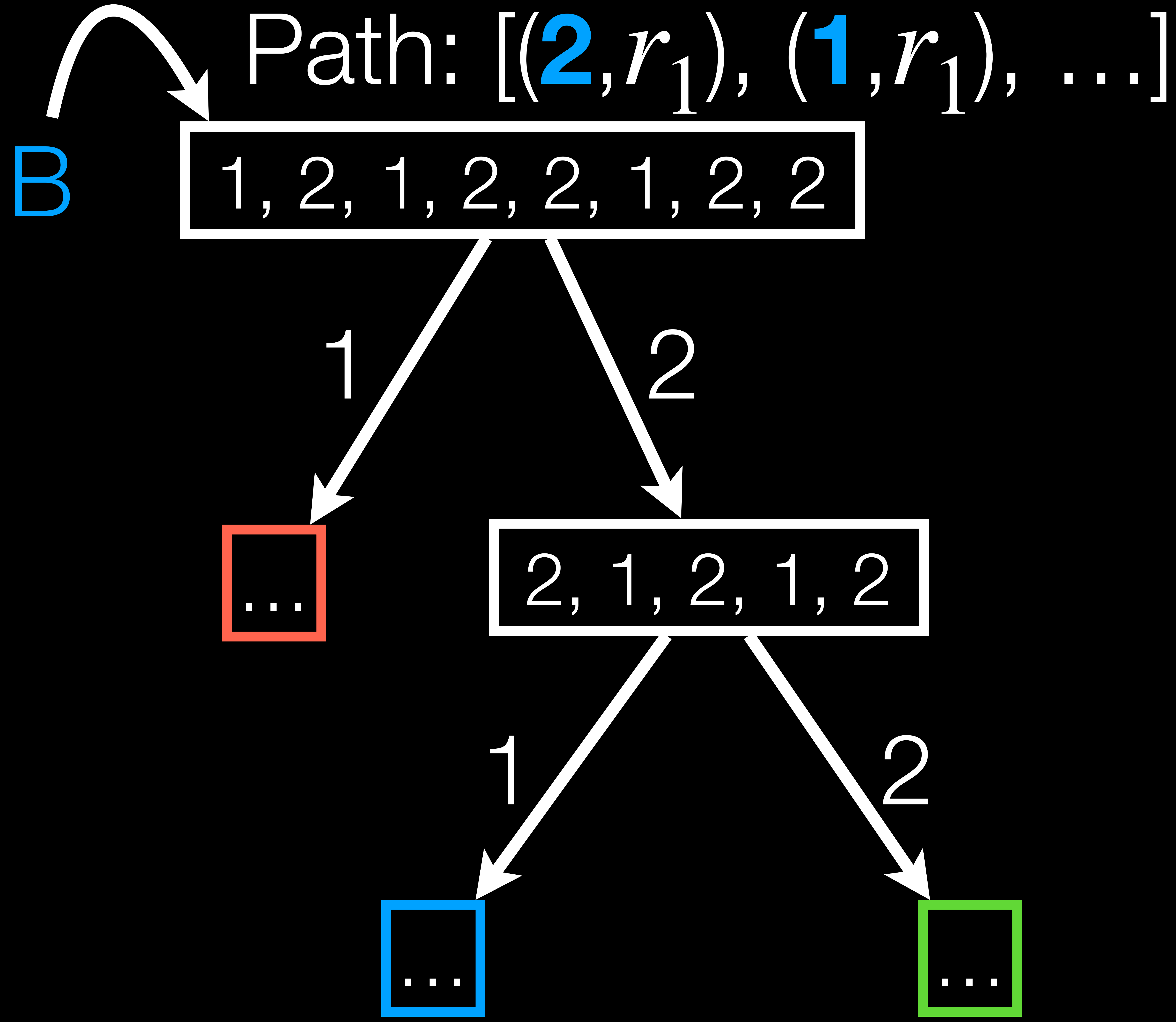
2

3

…

…

…



Path: [(**2**,r_1), (**1**,r_1), …]

B

1, 2, 1, 2, 2, 1, 2, 2

1

2

…

2, 1, 2, 1, 2

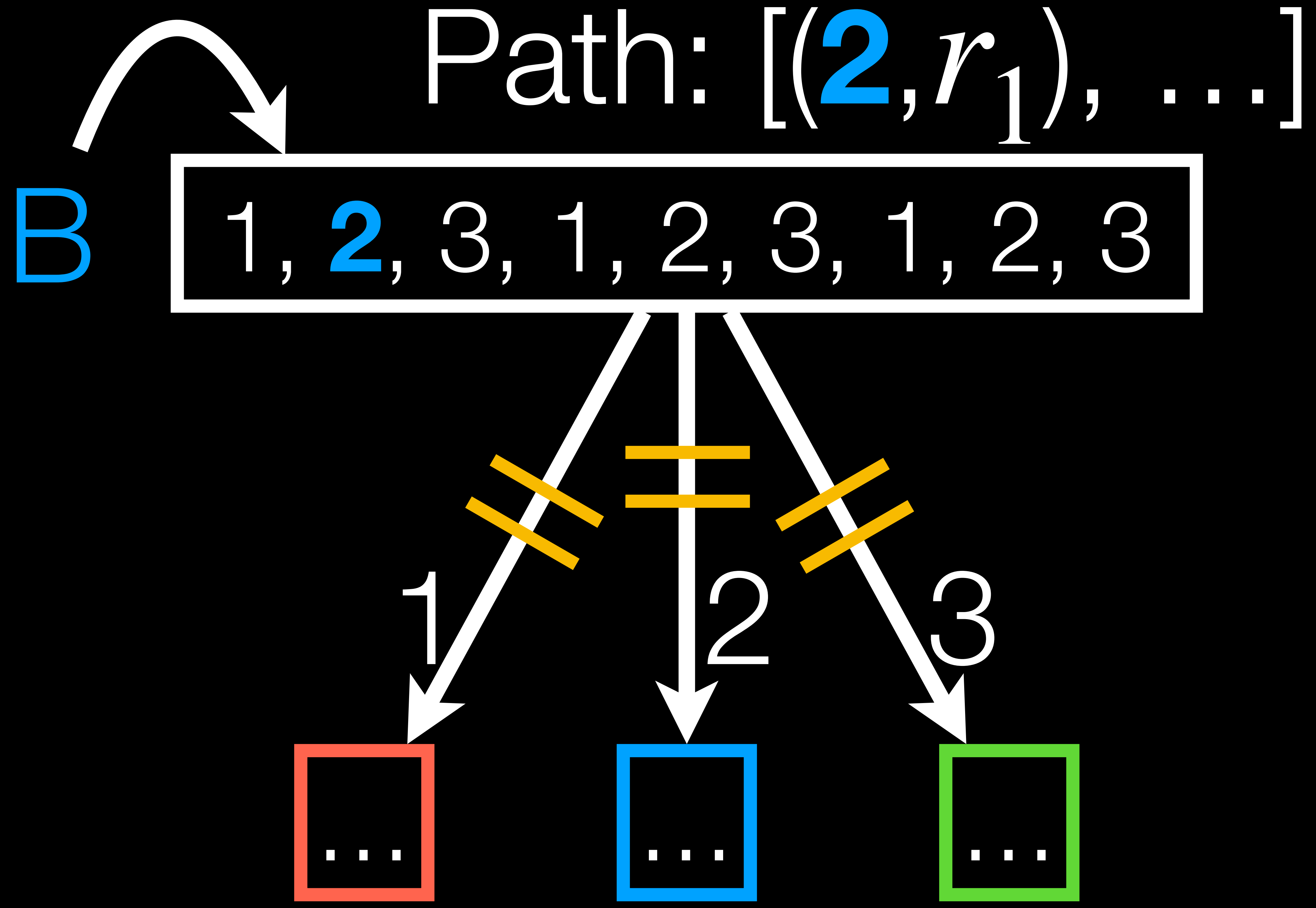
1

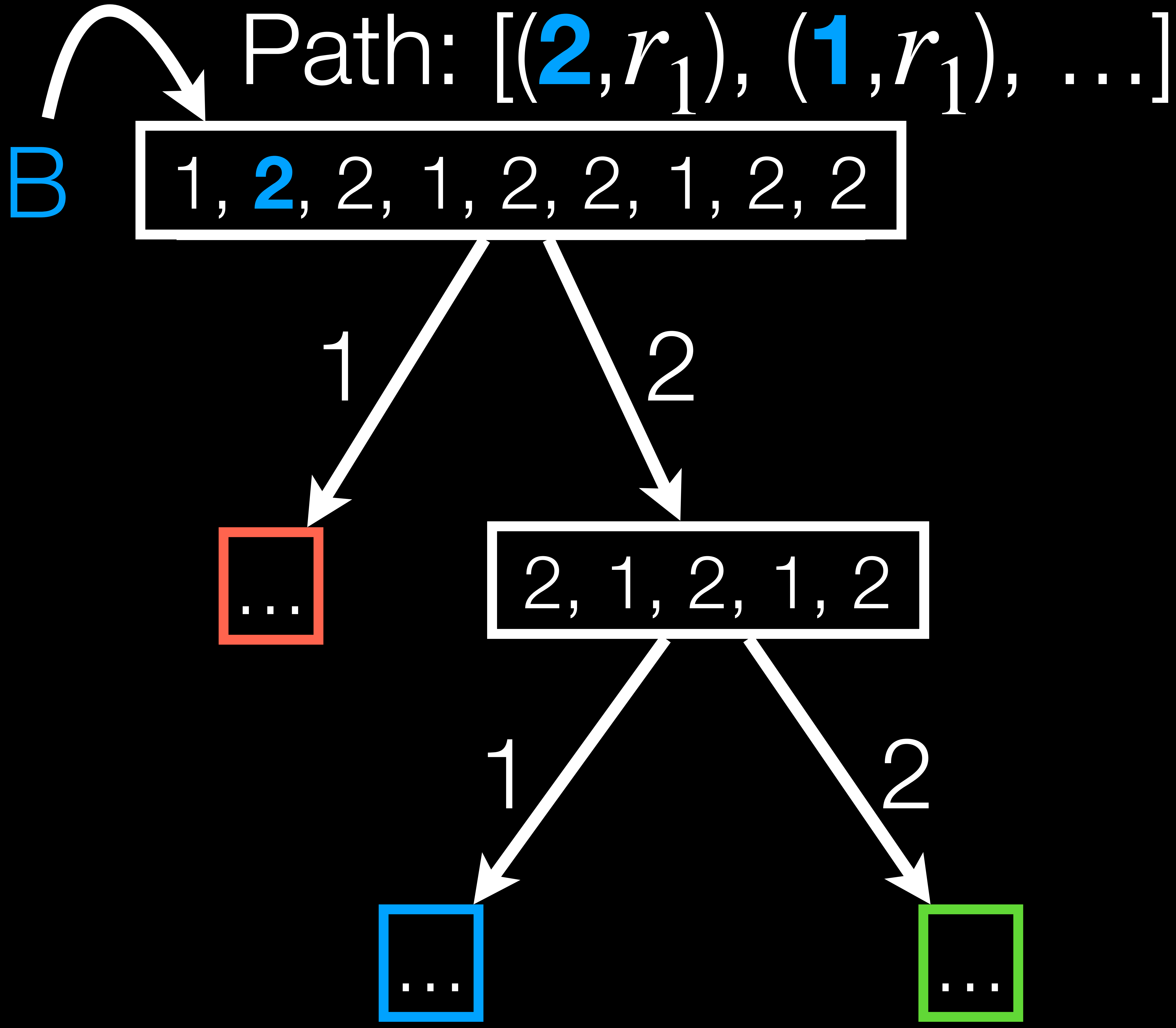
2

…

…

Compiling programs





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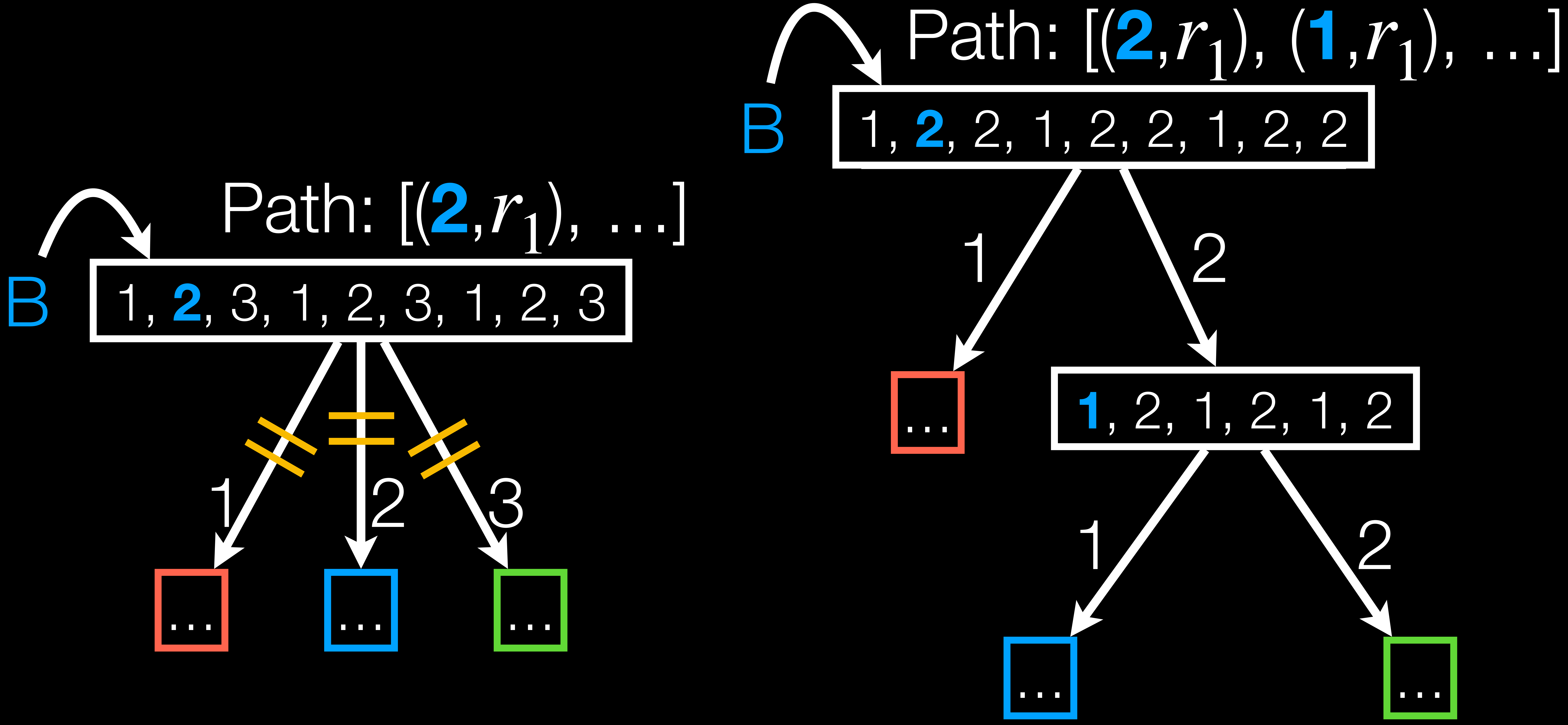

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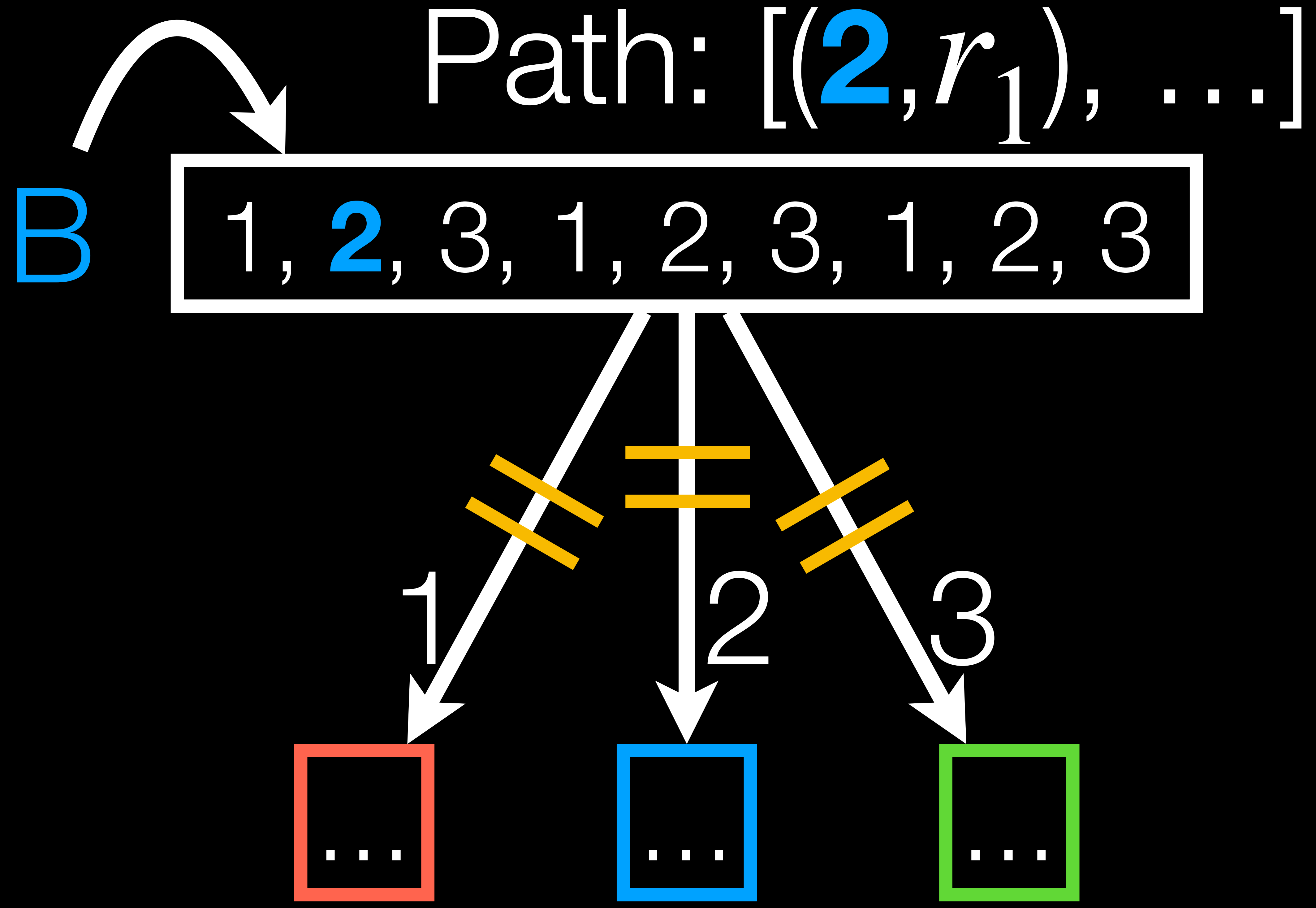
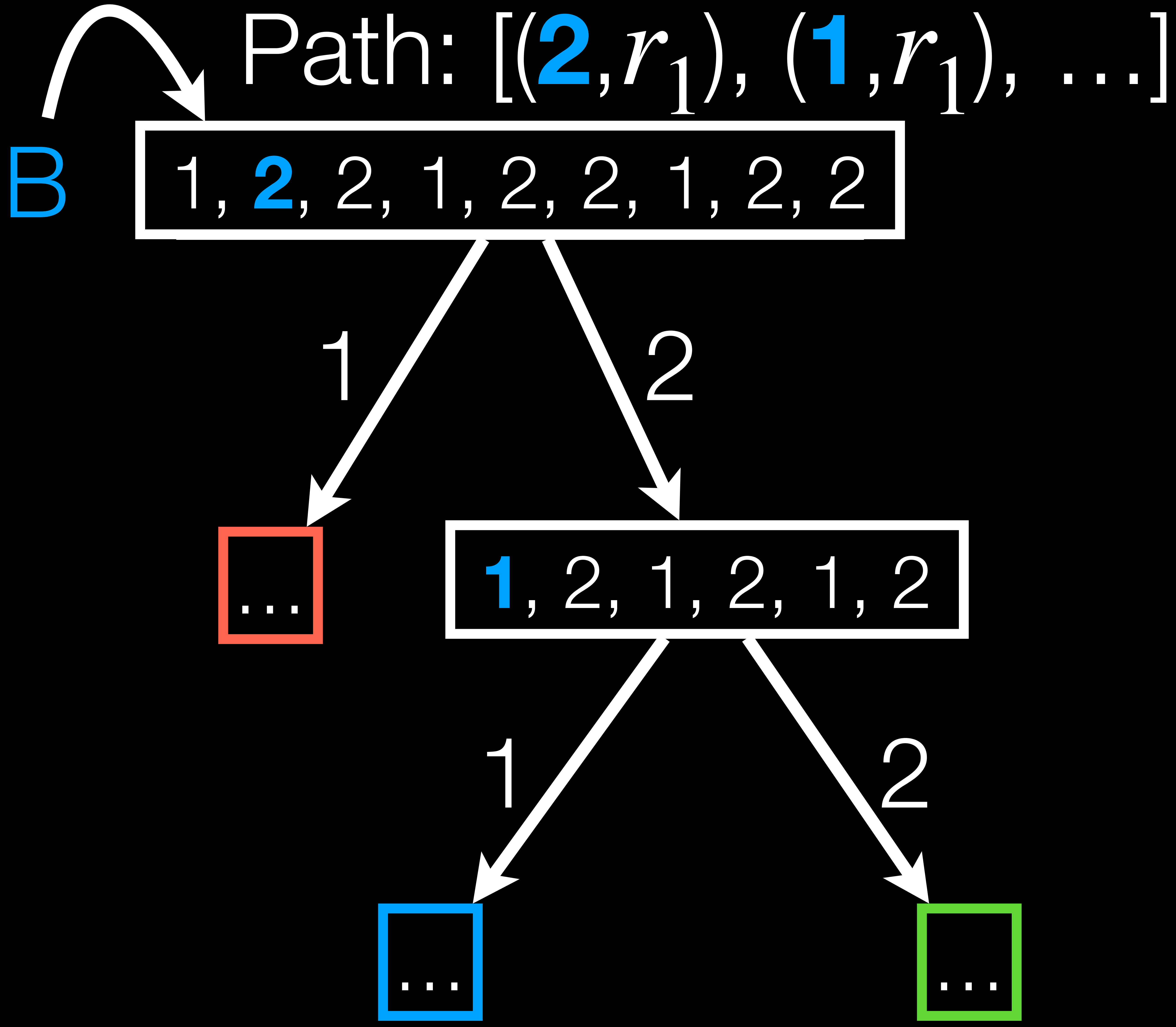
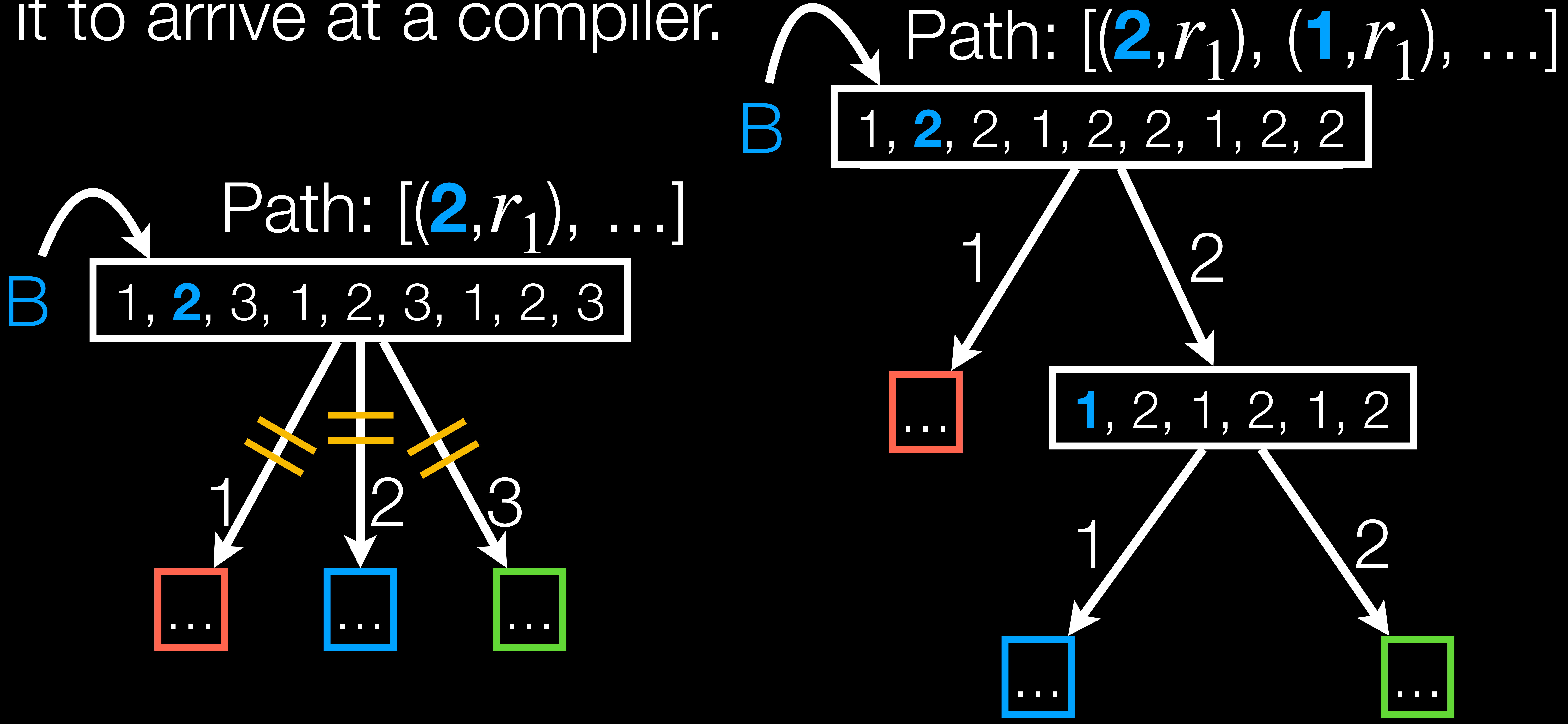




Given an embedding, we *lift* it to arrive at a compiler.

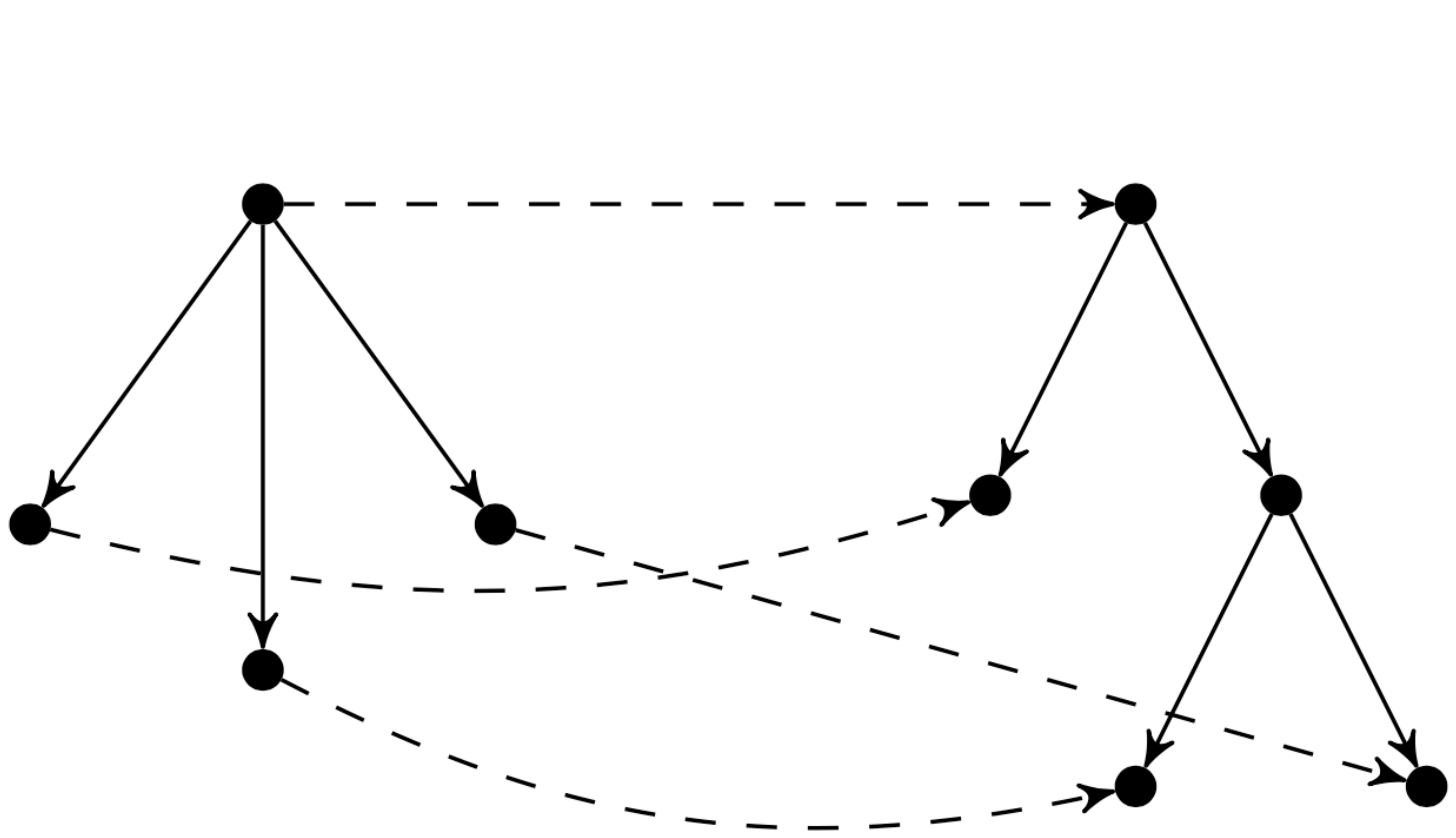


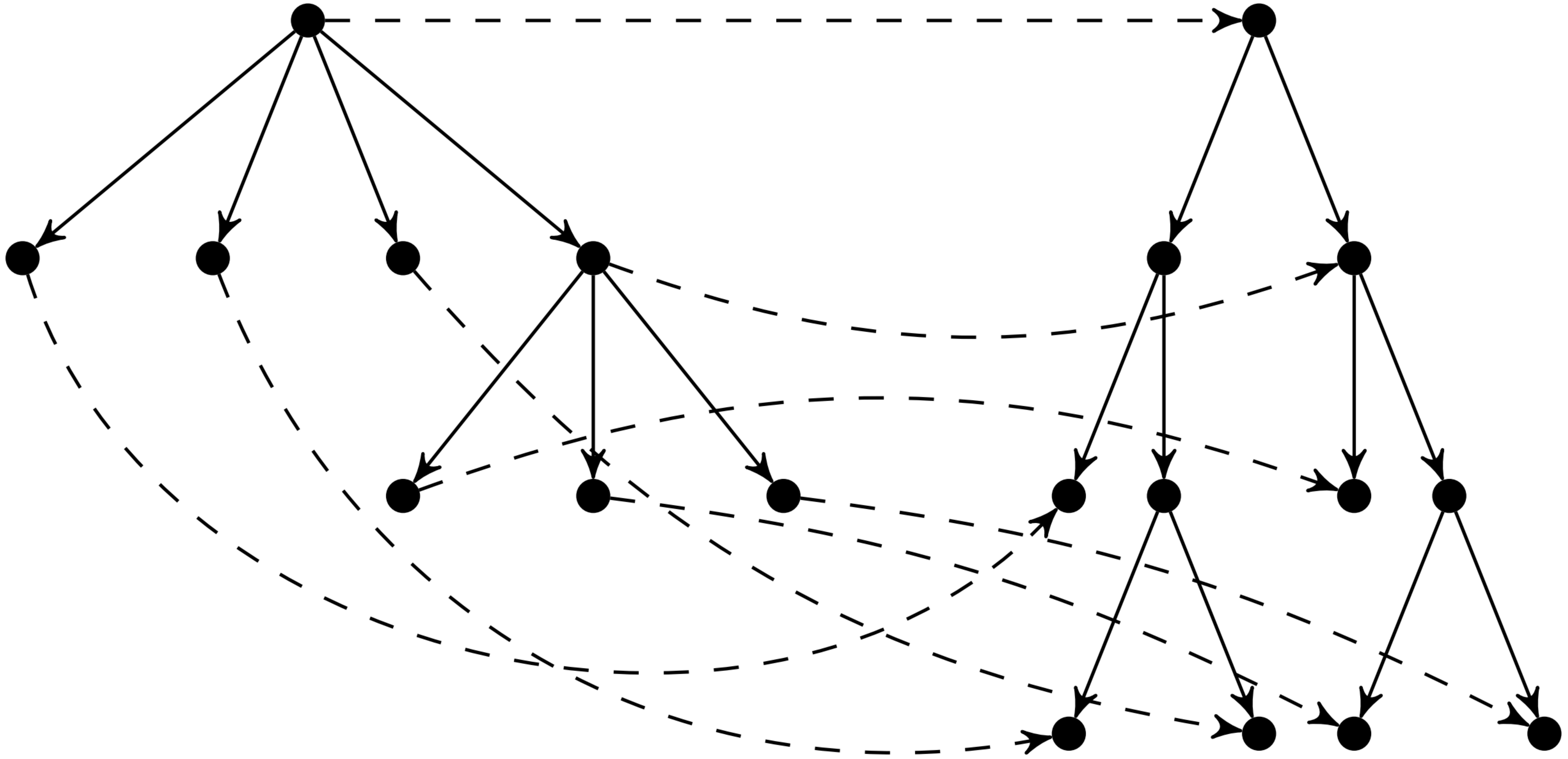
Generating embeddings automatically

Generating embeddings automatically

Homomorphic embedding.

Map root to root, leaves to leaves. Respect ancestry.





Generating embeddings automatically

Homomorphic embedding.
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Two new algorithms,
both starting with heterogeneous source trees.

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1. If target tree is regular *d*-ary for some *d.*

Generating embeddings automatically

Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.

Two new algorithms,
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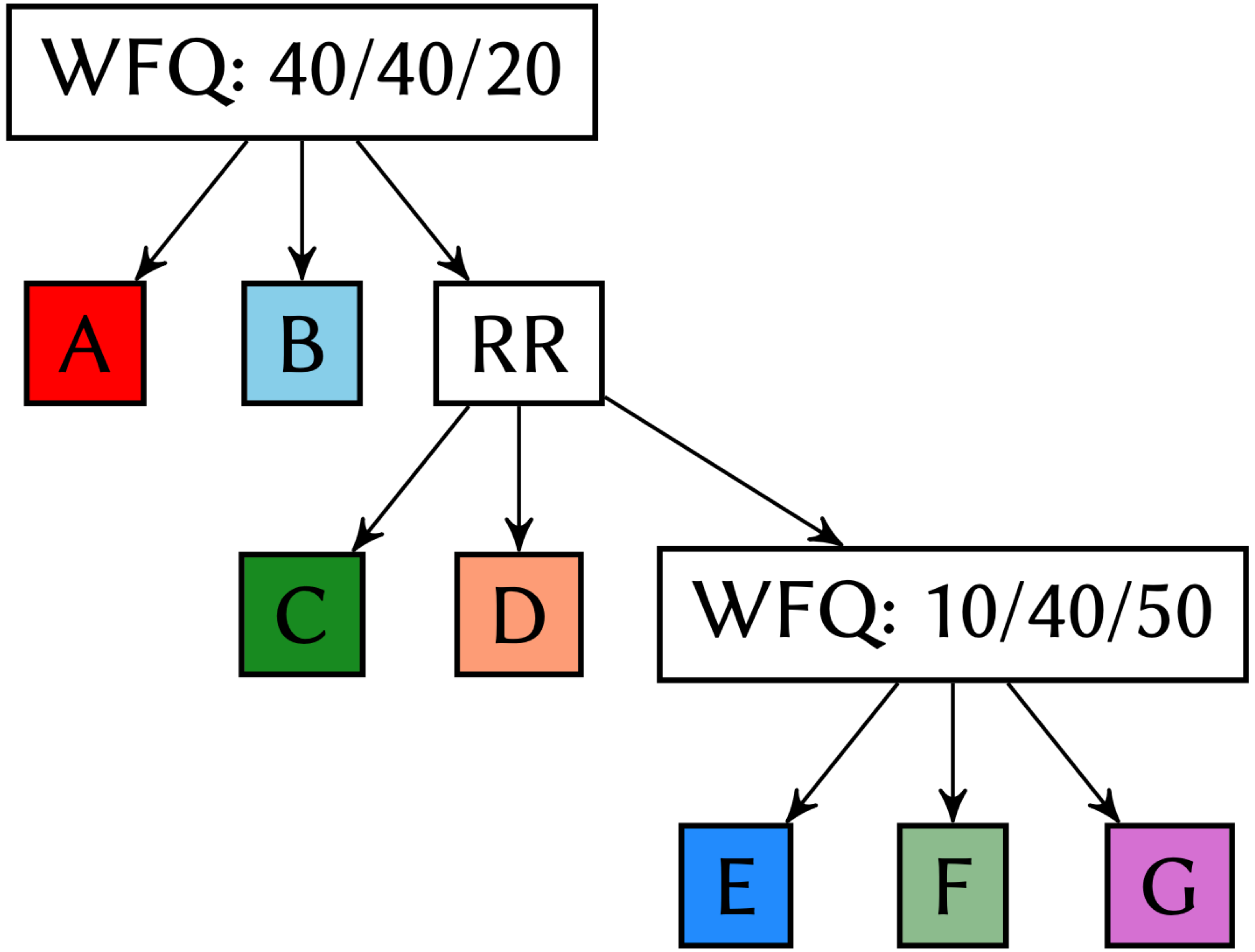
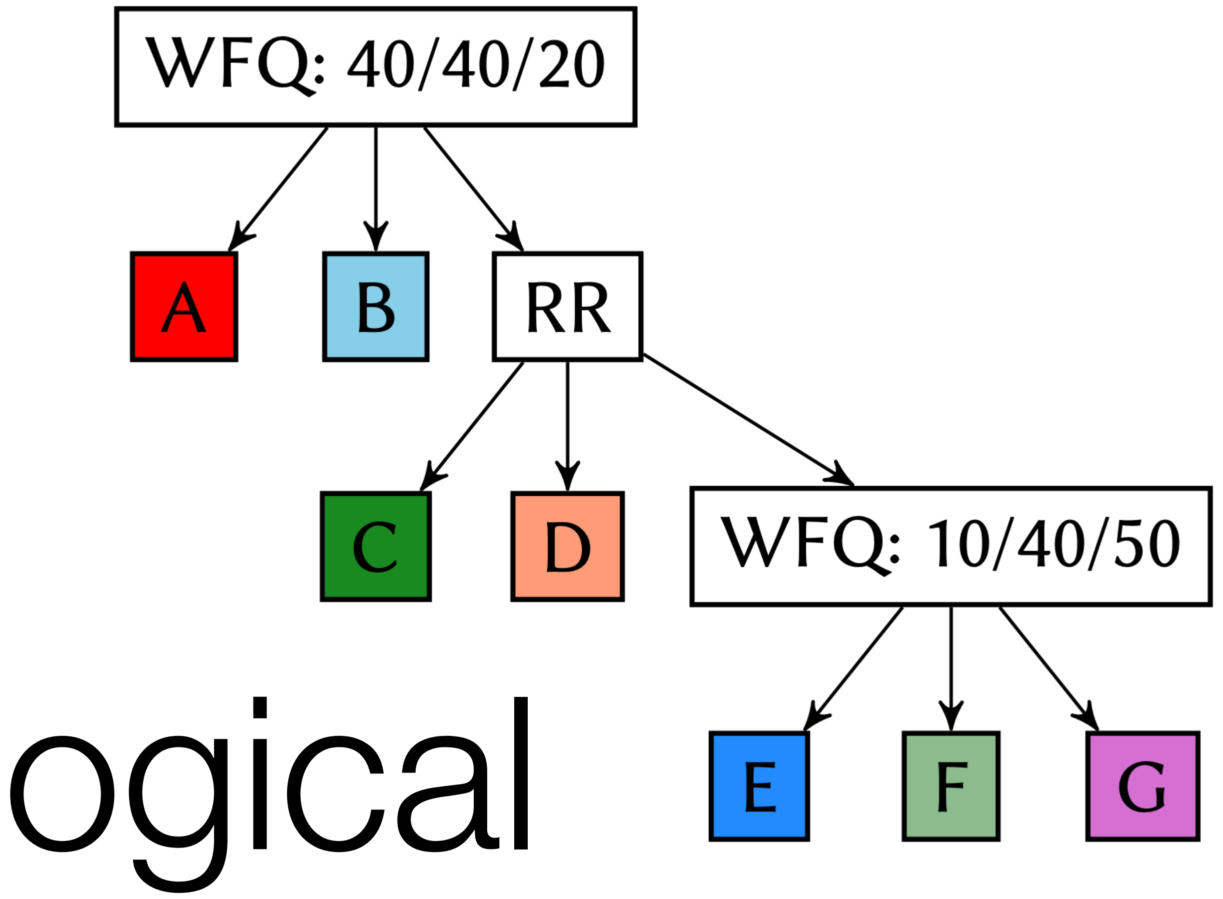
1. If target tree is regular *d*-ary for some *d.*
2. If target tree is itself heterogeneous.

Workflow

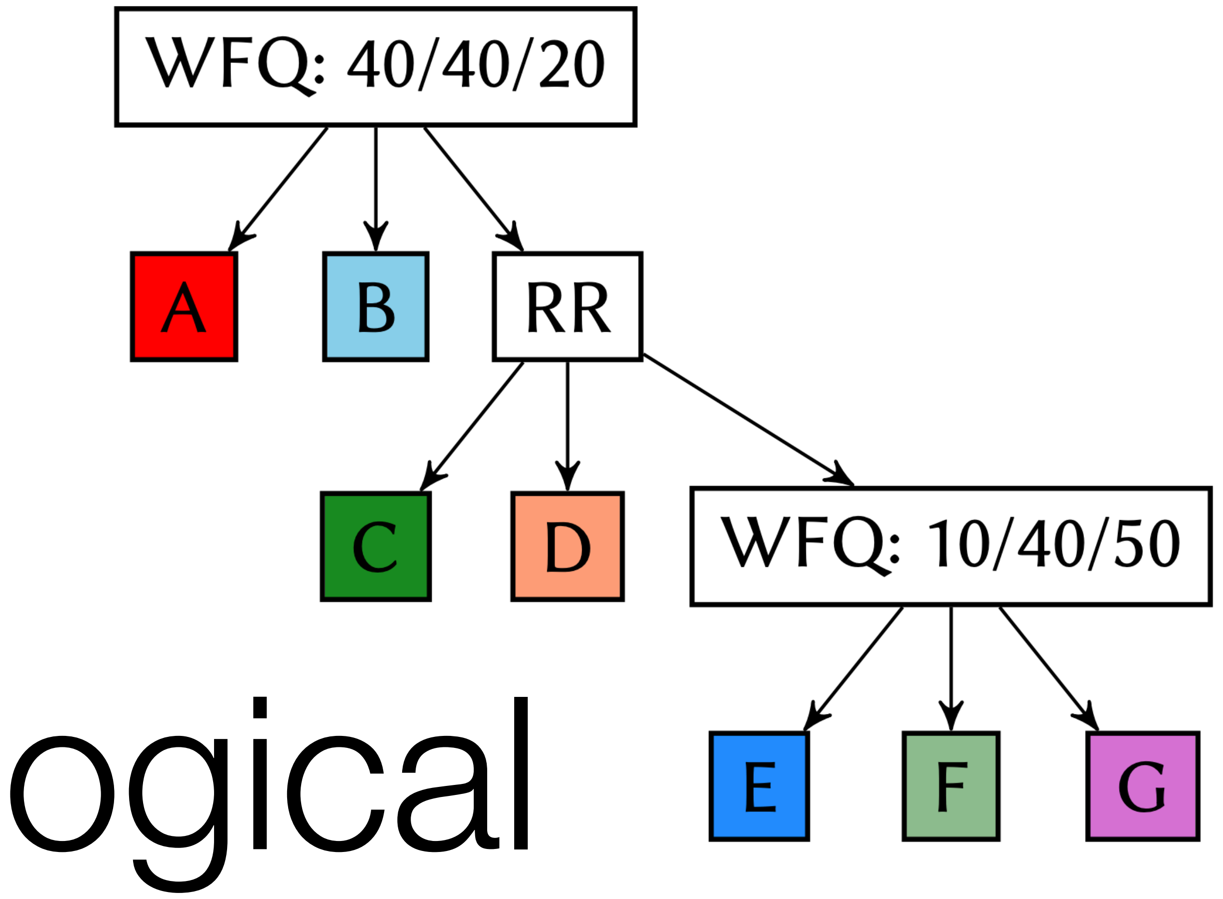


Workflow



logical

Workflow



logical

But the hardware supports
a regular-branching binary tree.

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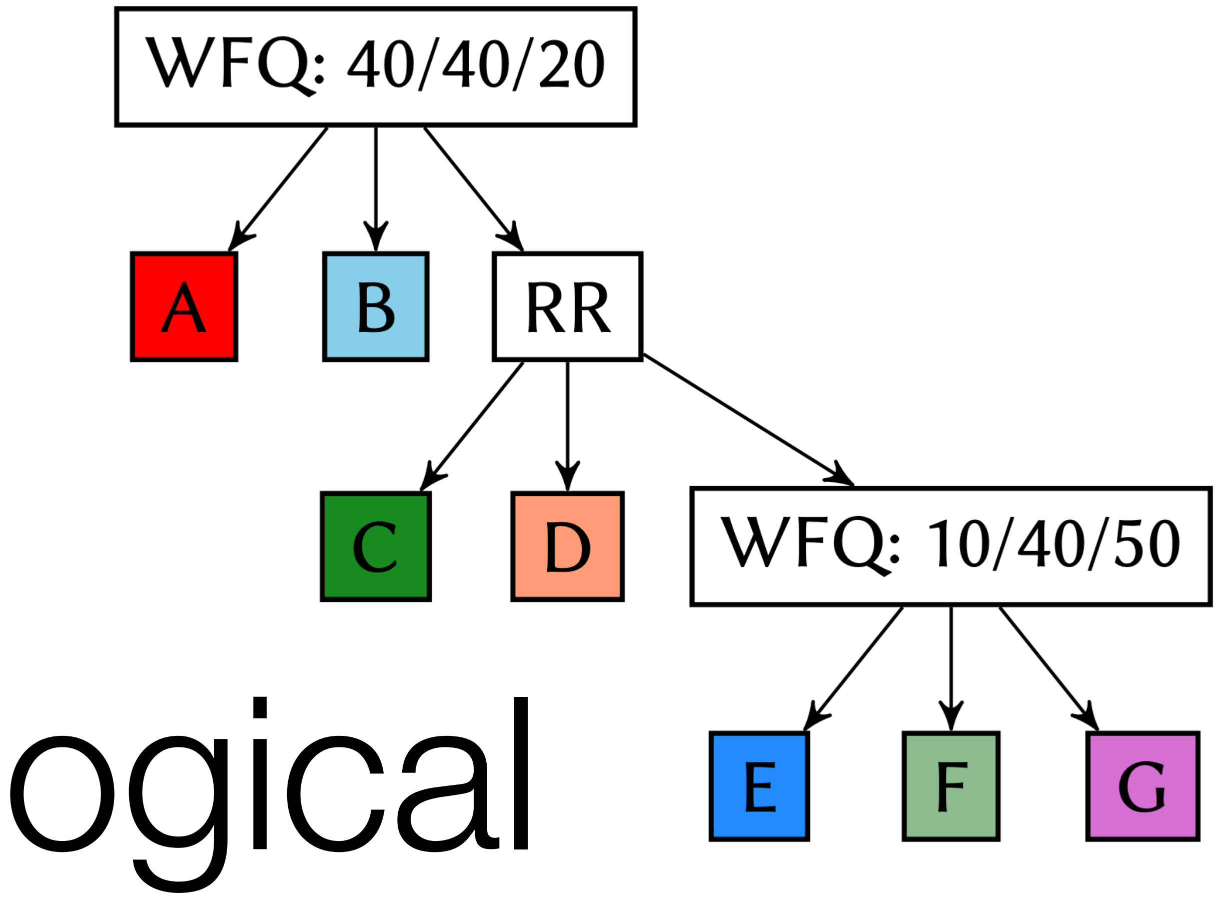

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No problem.
Here's how I'll use that tree.

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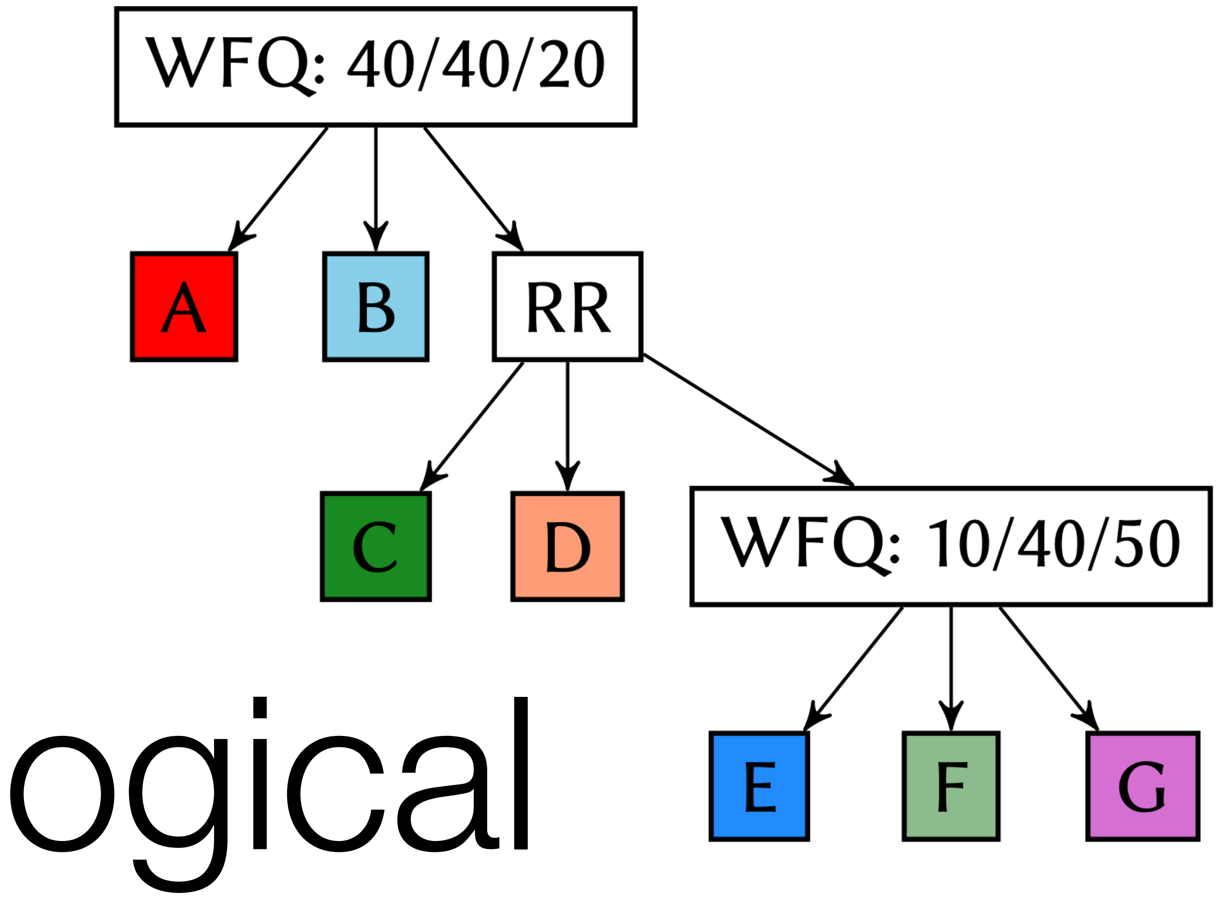

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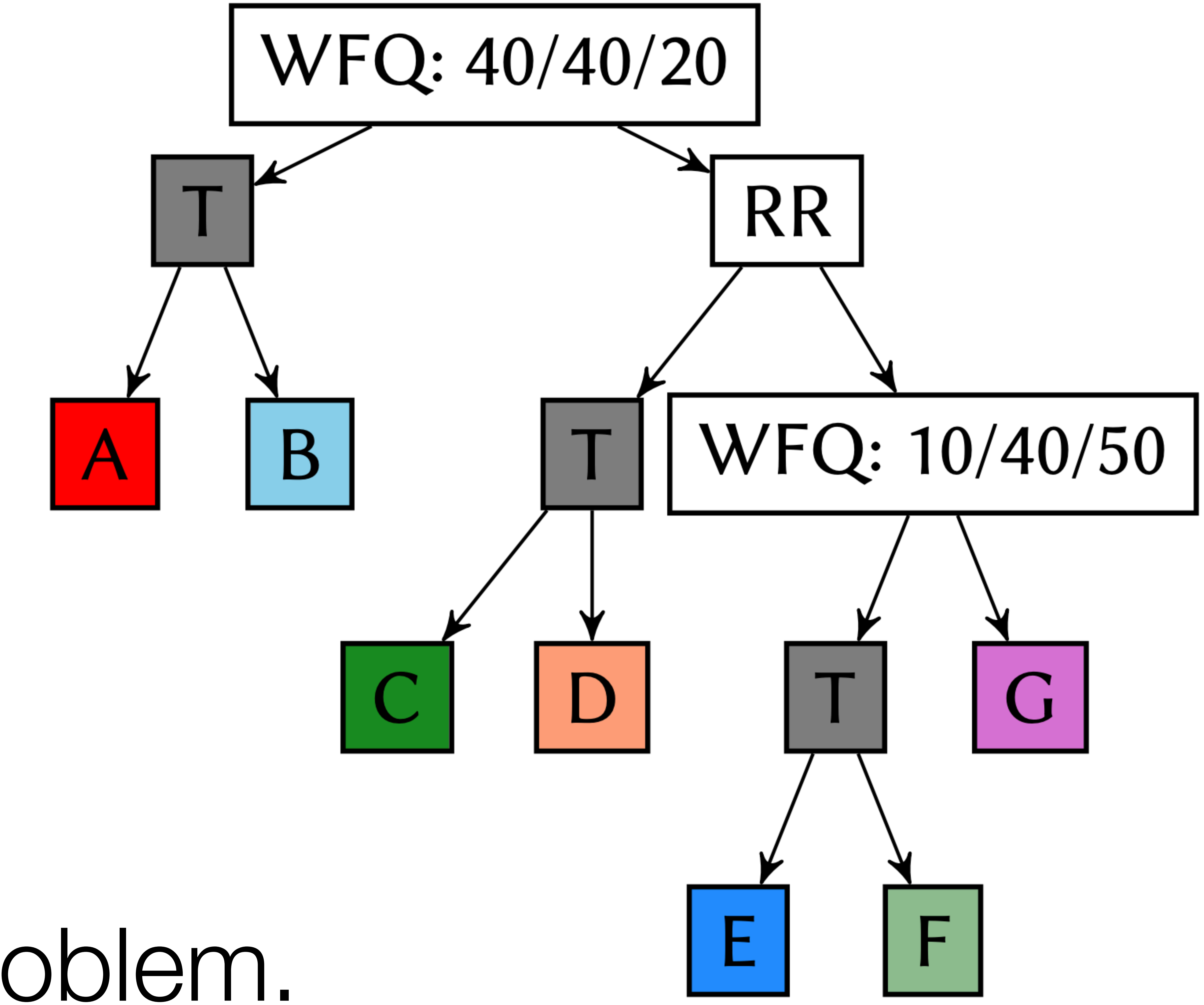

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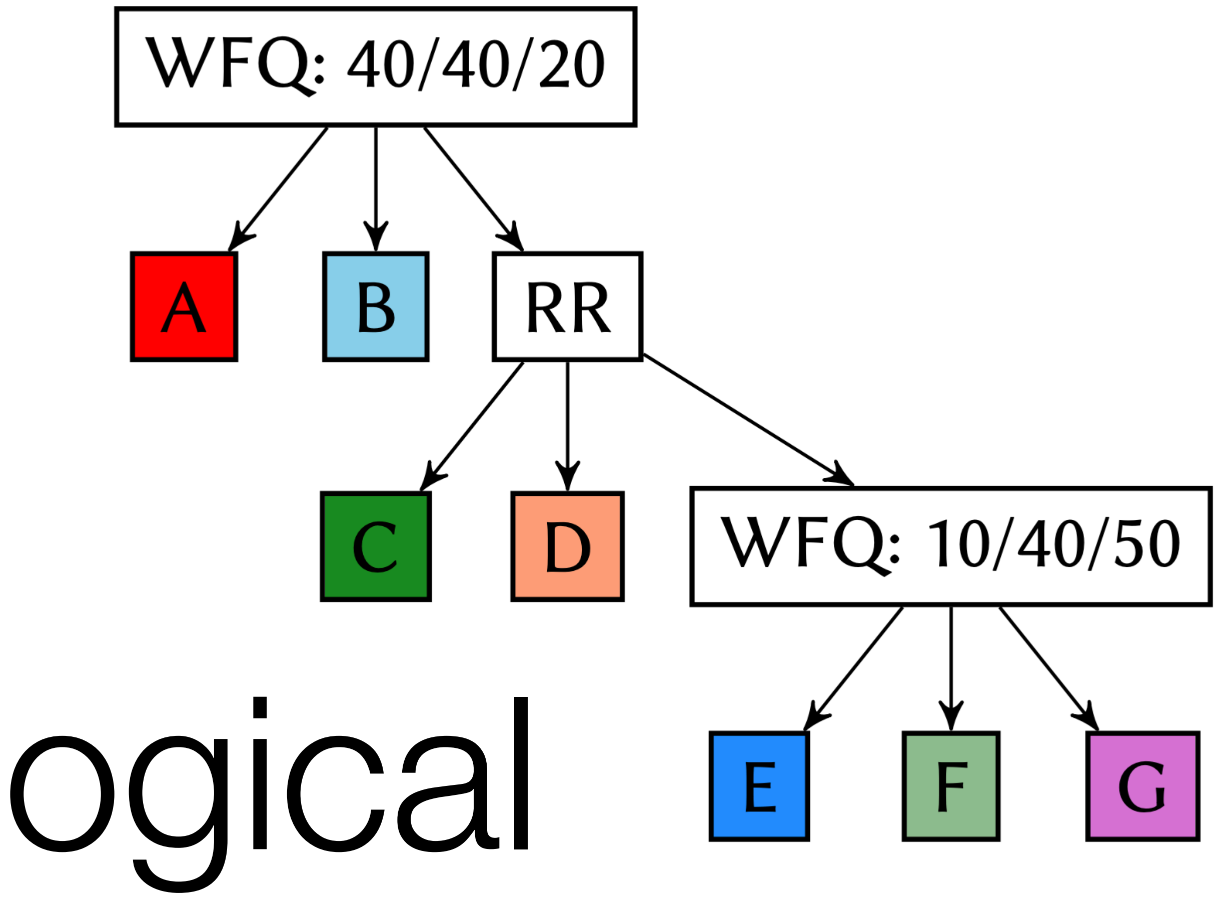
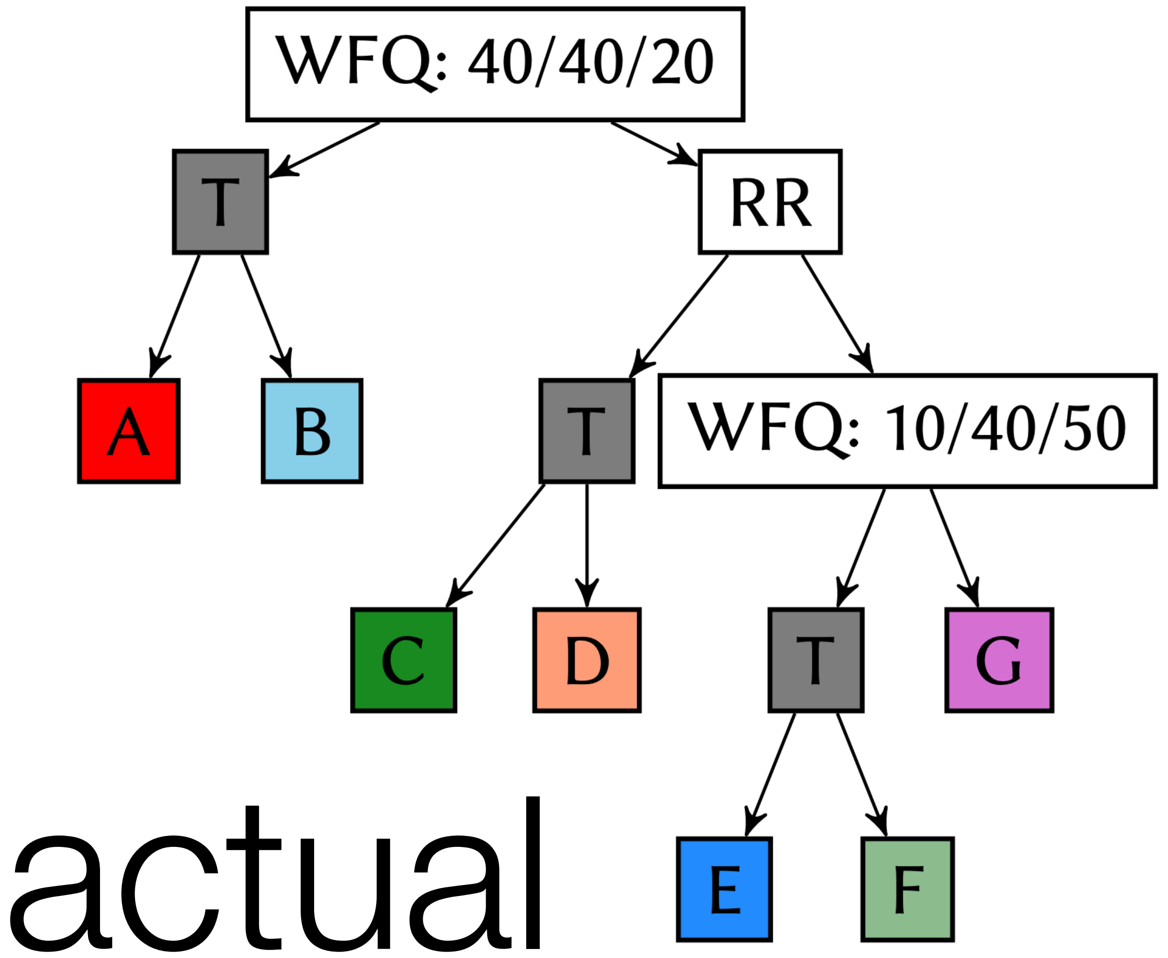
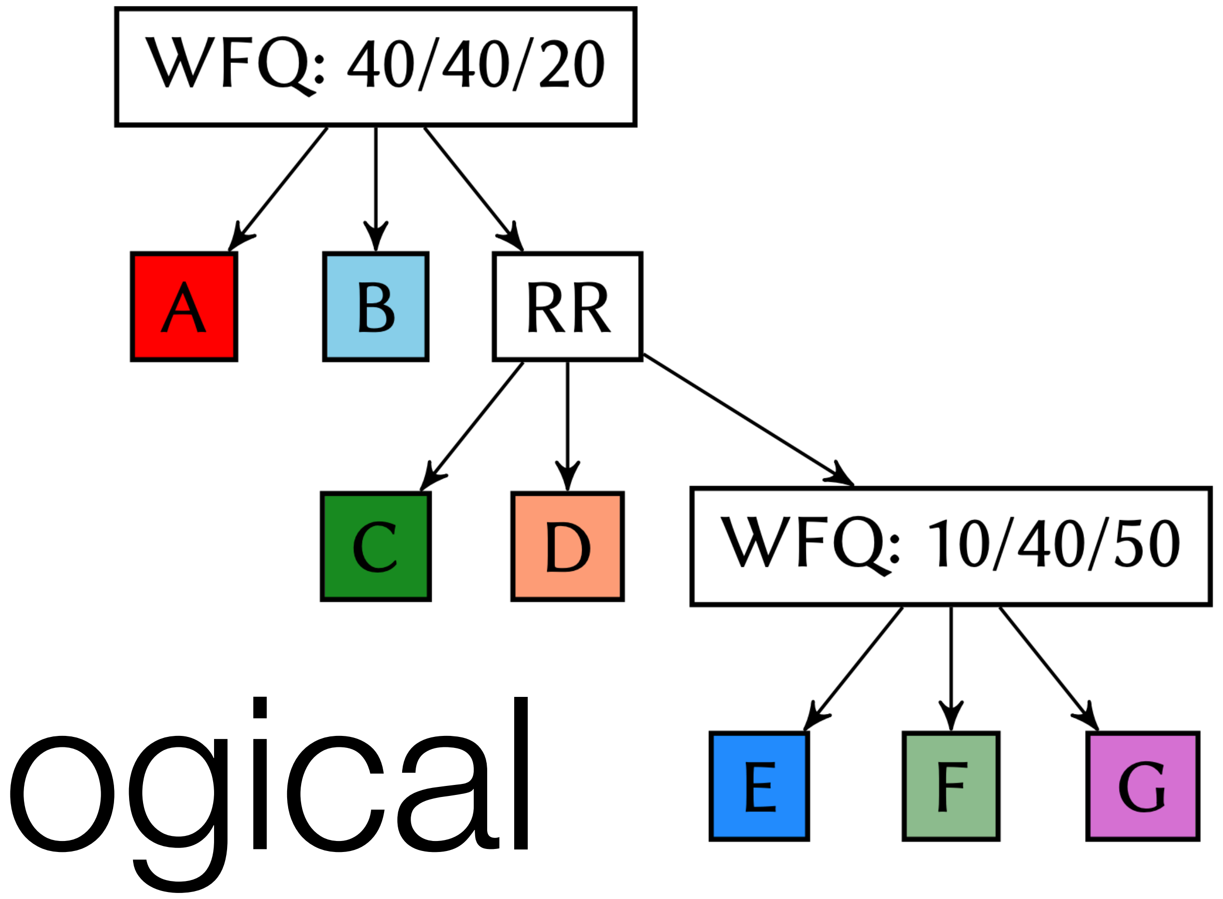


logical



actual

Simulation



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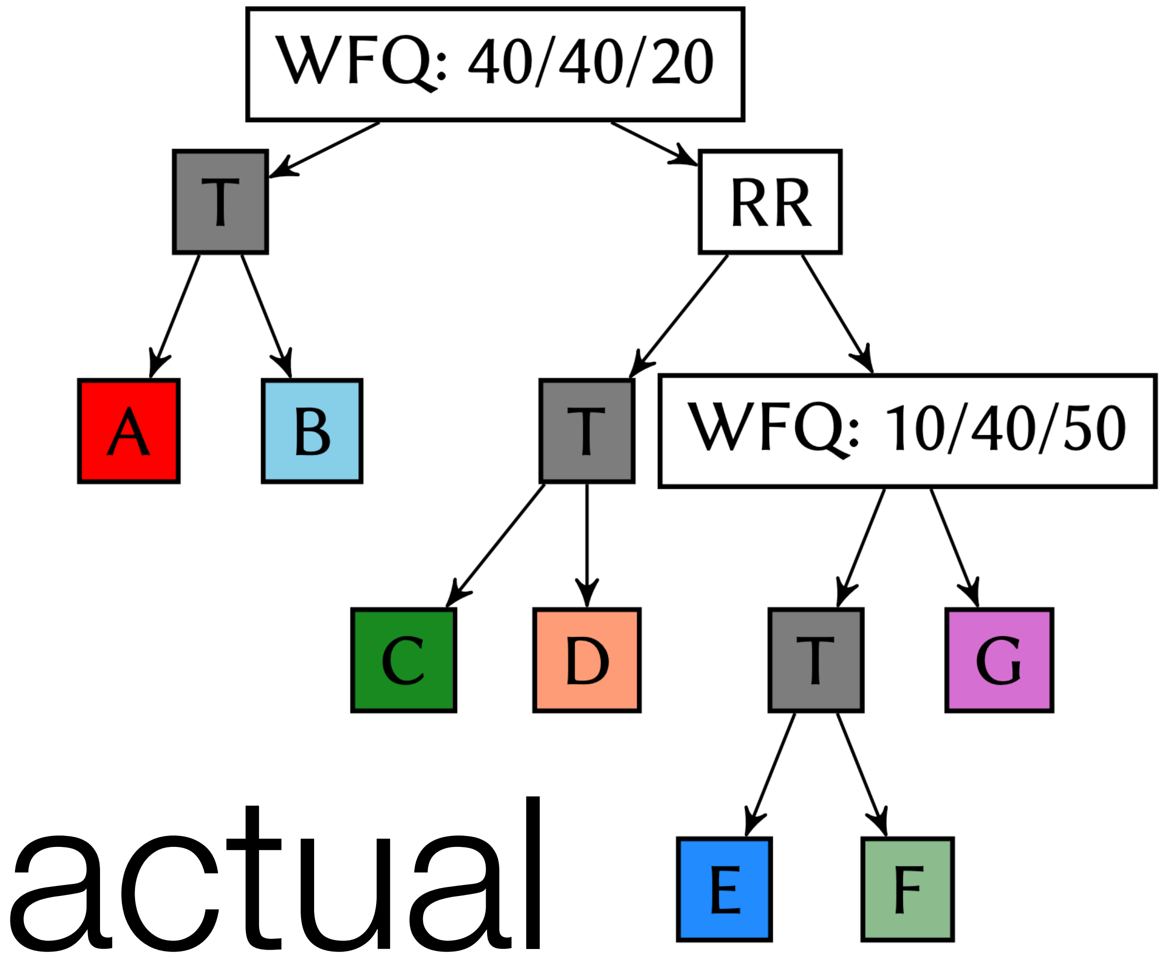

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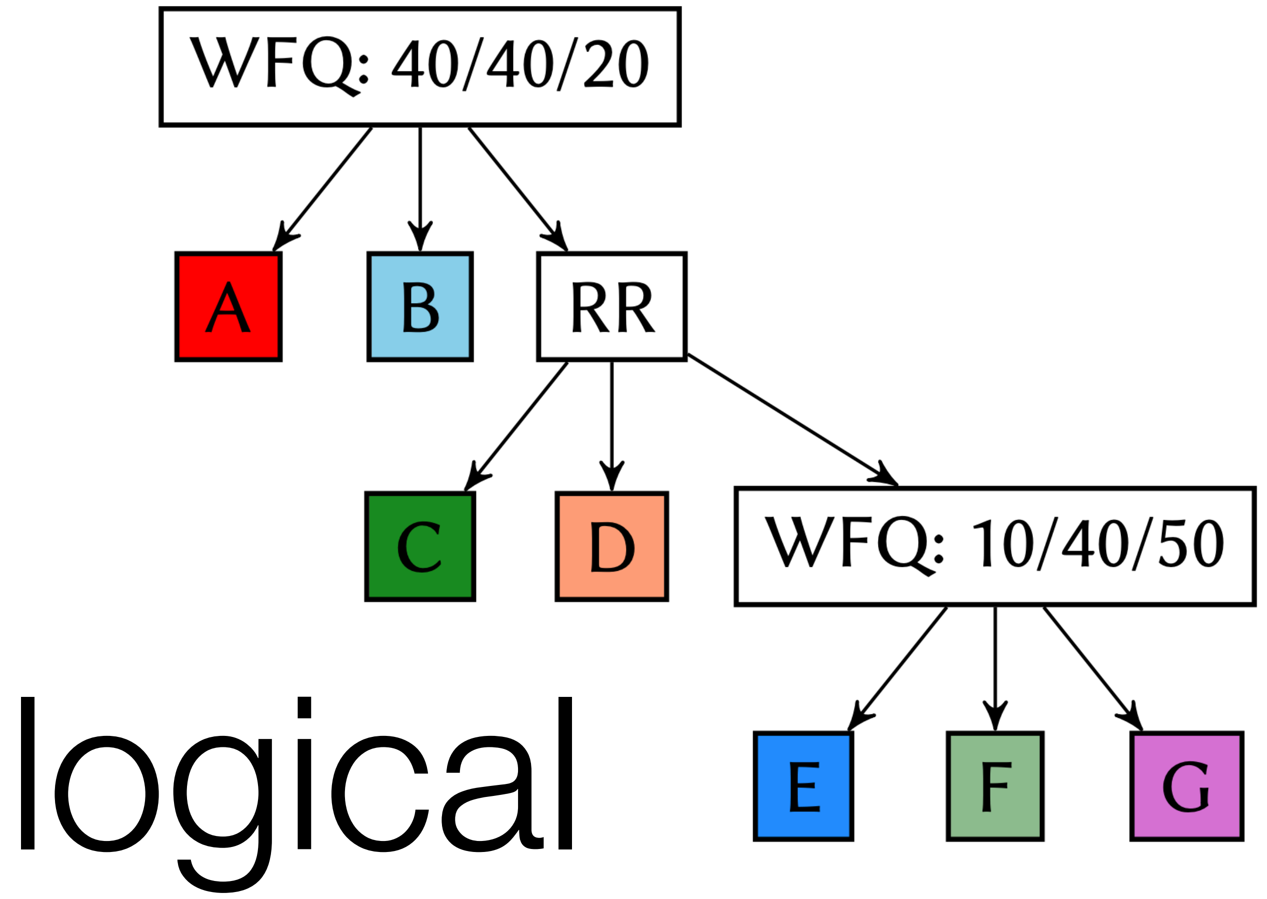

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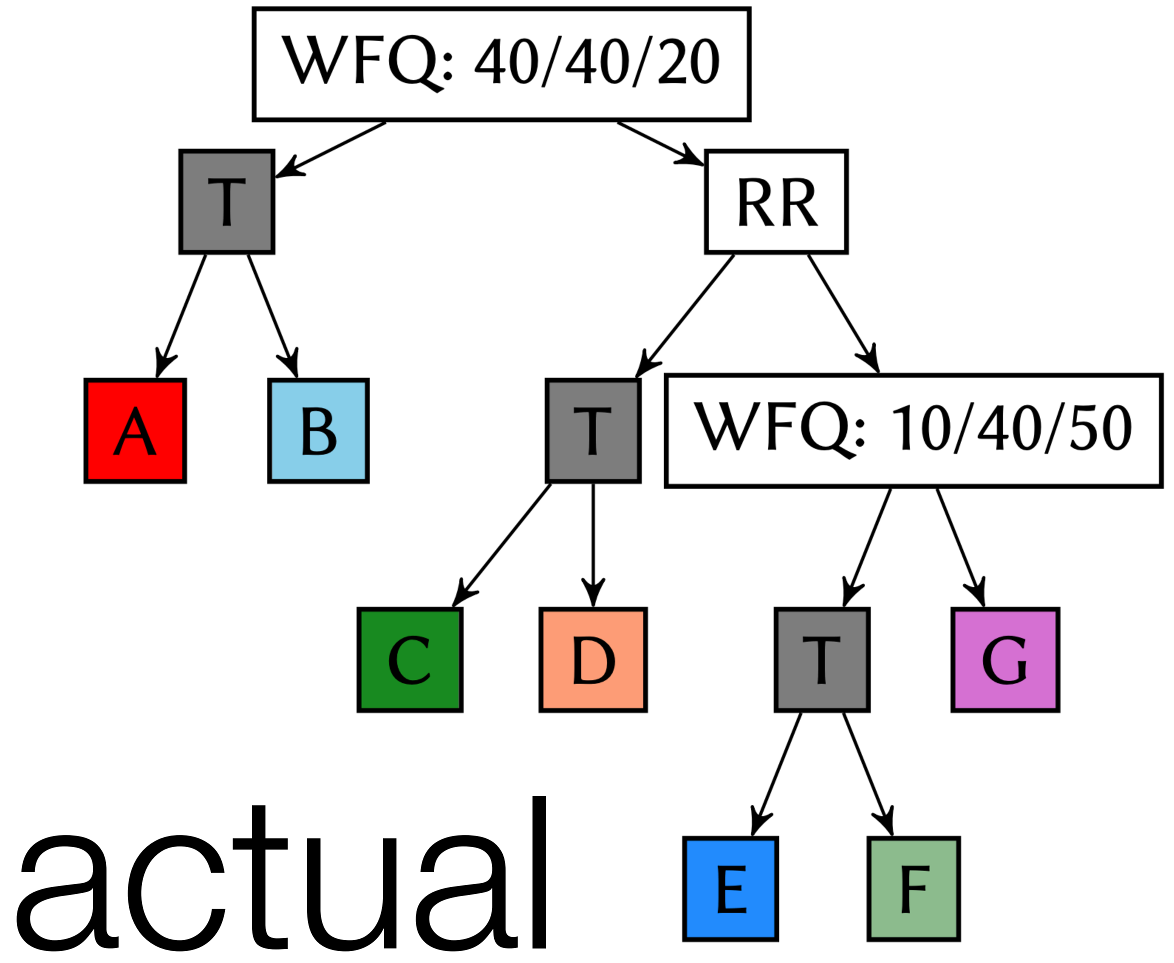
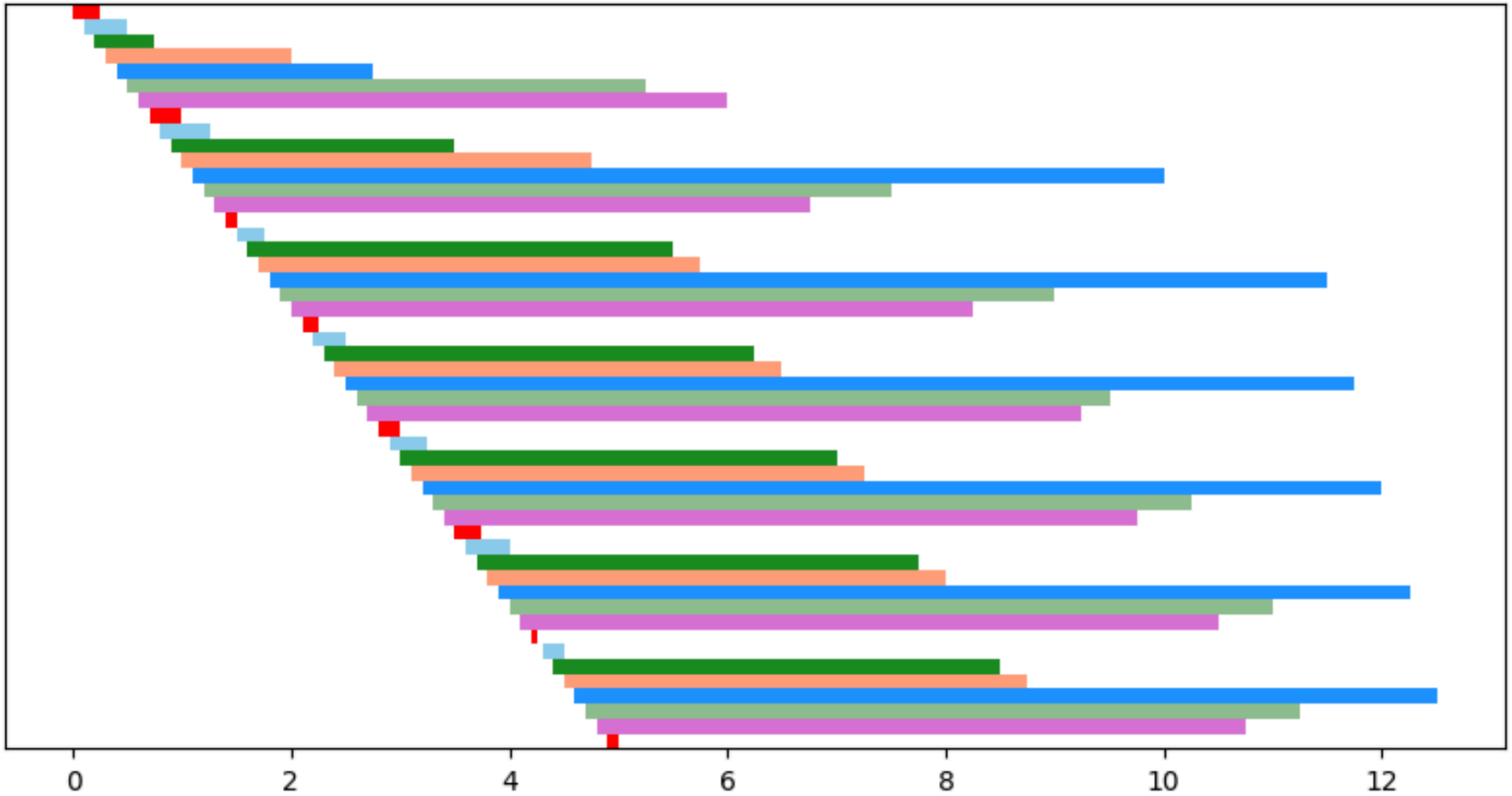


actual



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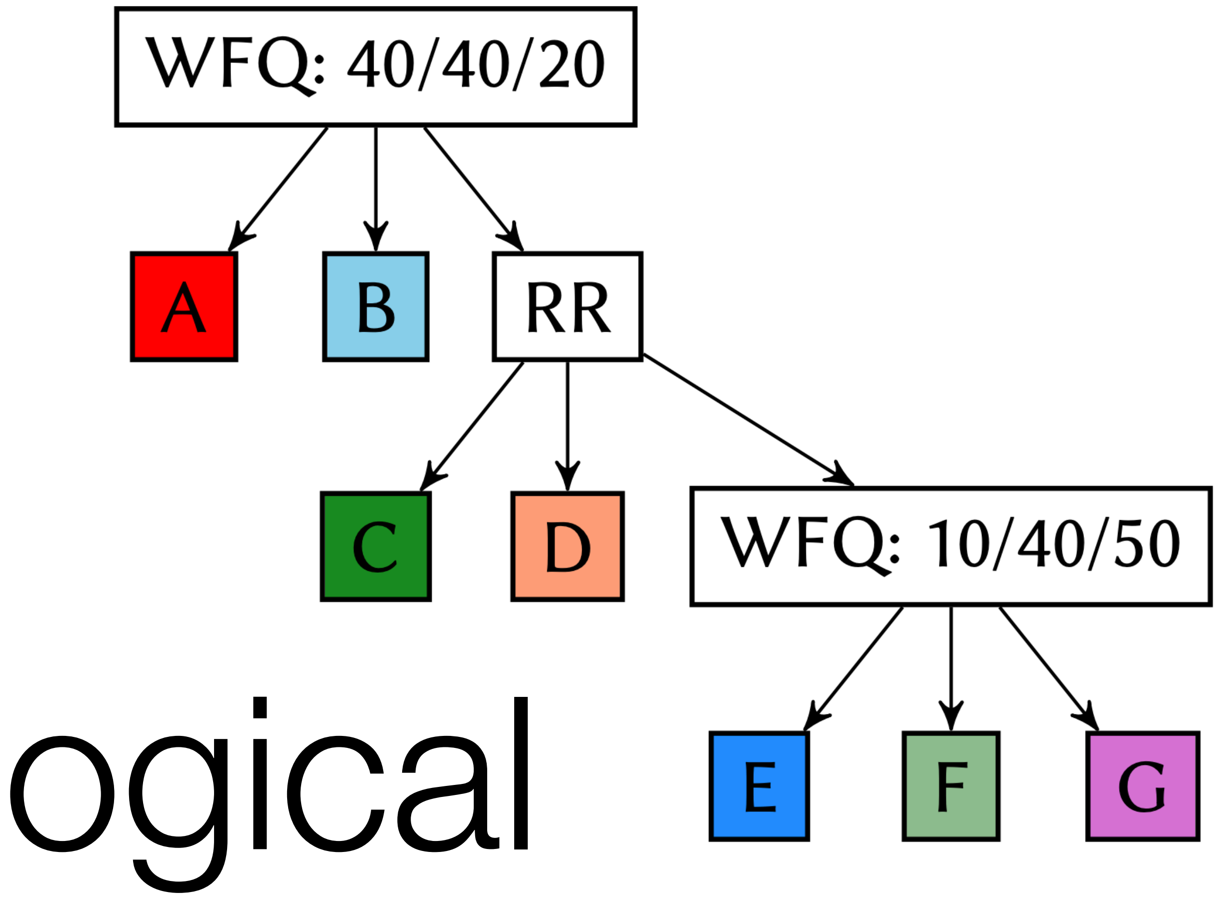

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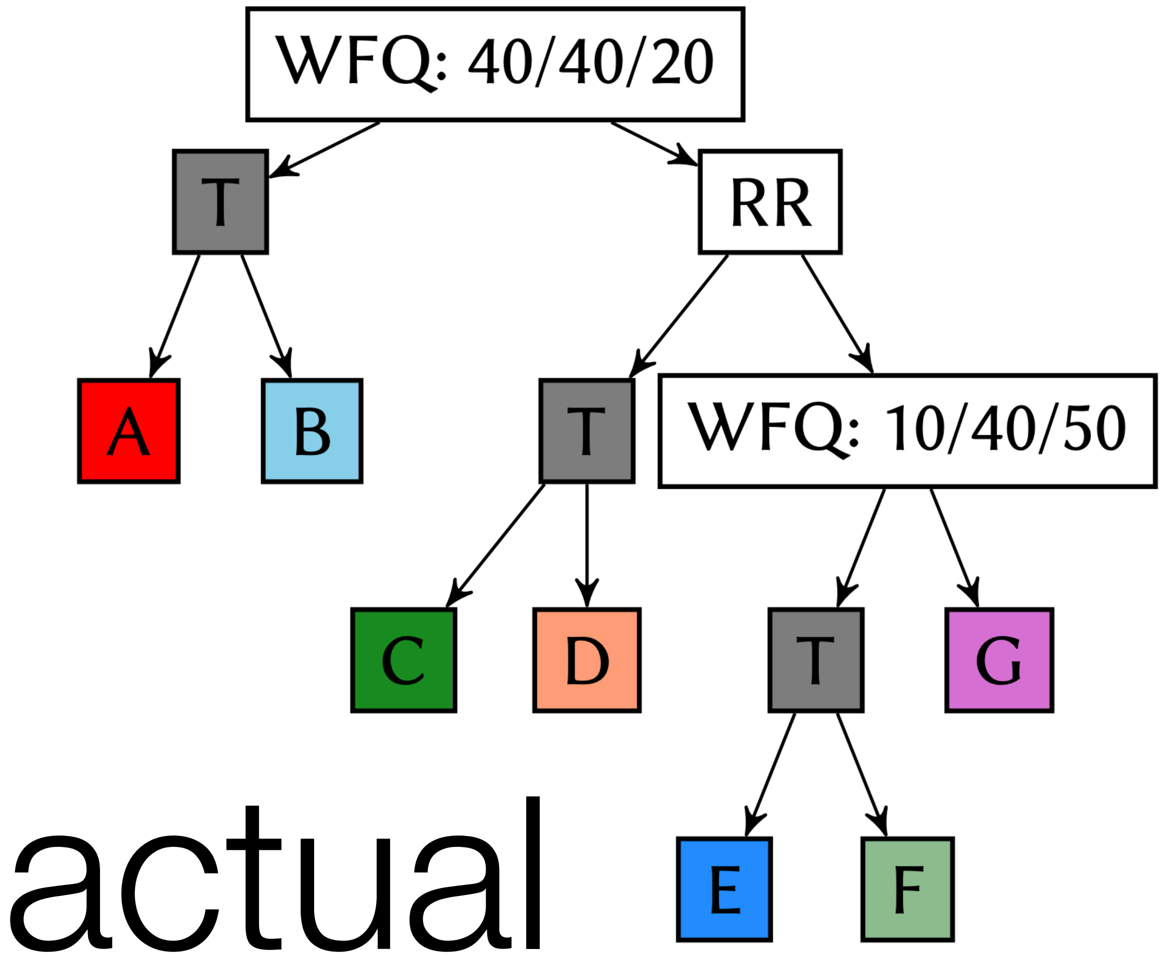

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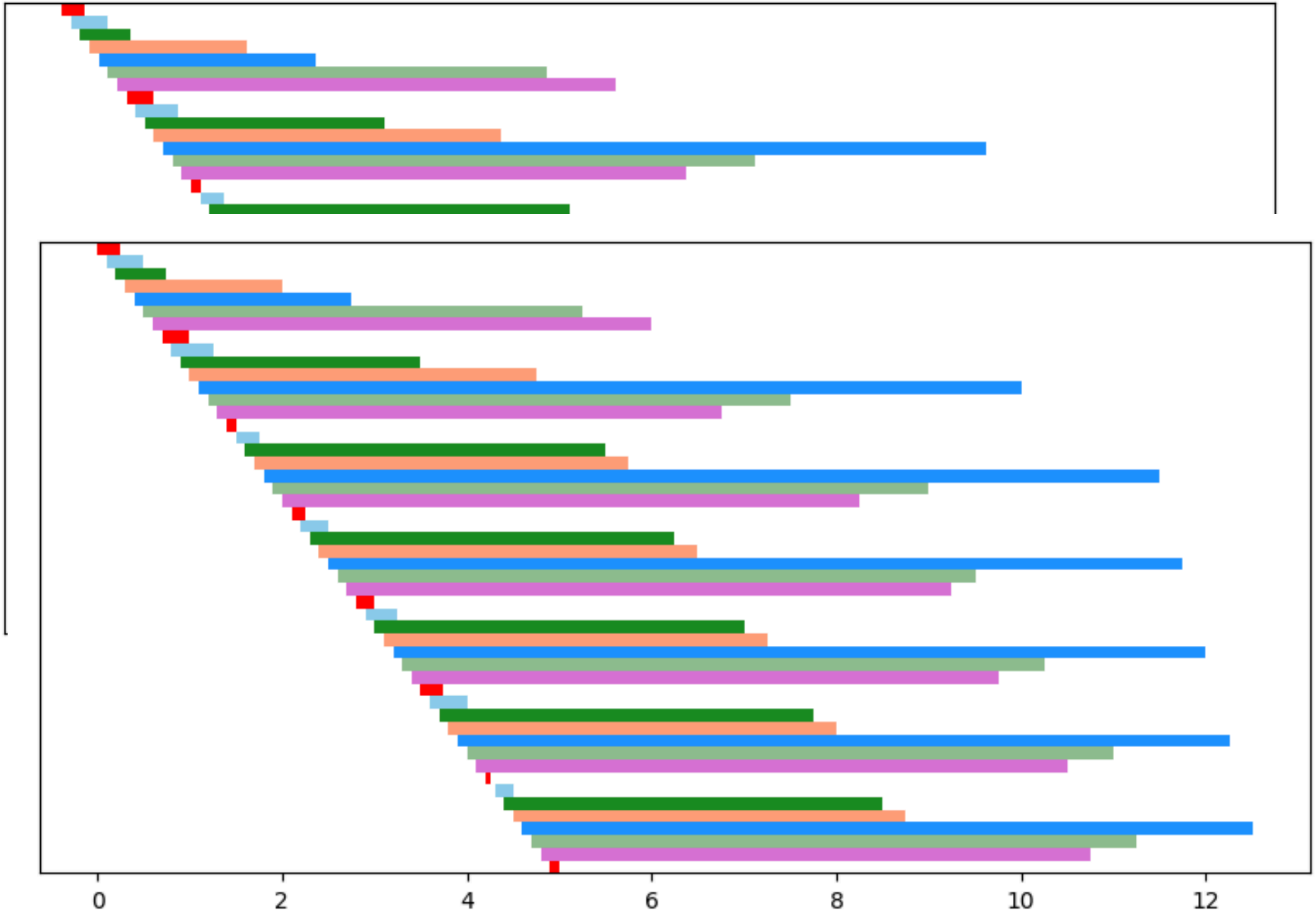

Underlying formalism

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$$\frac{\textsc{push}(p, pkt, r) = p'}{\mathrm{push}(\mathrm{Leaf}(p), pkt, r) = \mathrm{Leaf}(p')} \qquad \frac{\mathrm{push}(qs[i], pkt, pt) = q' \qquad \textsc{push}(p, i, r) = p'}{\mathrm{push}(\mathrm{Internal}(qs, p), pkt, (i, r) :: pt) = \mathrm{Internal}(qs[i/q'], p')}$$

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Underlying formalism

$$\frac{r \in \mathsf{Rk}}{r \in \mathsf{Path}(*)} \qquad \frac{ts \in \mathsf{Topo}^{n} \qquad 1 \le i \le n \qquad r \in \mathsf{Rk} \qquad pt \in \mathsf{Path}(ts[i])}{(i, r) :: pt \in \mathsf{Path}(\mathsf{Node}(ts))}$$

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Underlying formalism

$$\frac{}{* \in \mathsf{Topo}} \qquad \frac{n \in \mathbb{N} \qquad ts \in \mathsf{Topo}^{n}}{\mathsf{Node}(ts) \in \mathsf{Topo}}$$

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A general way to deploy PIFO trees

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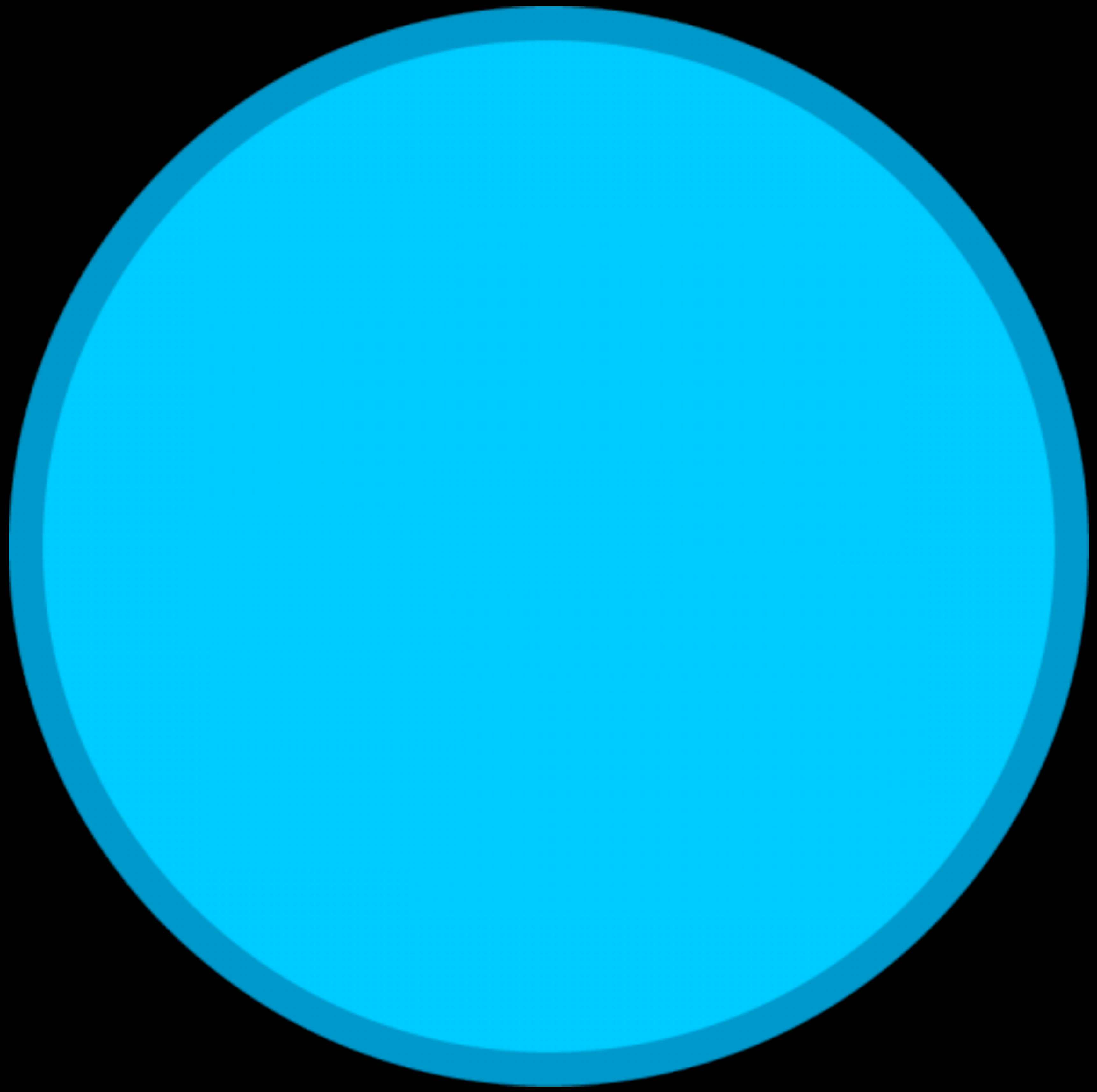

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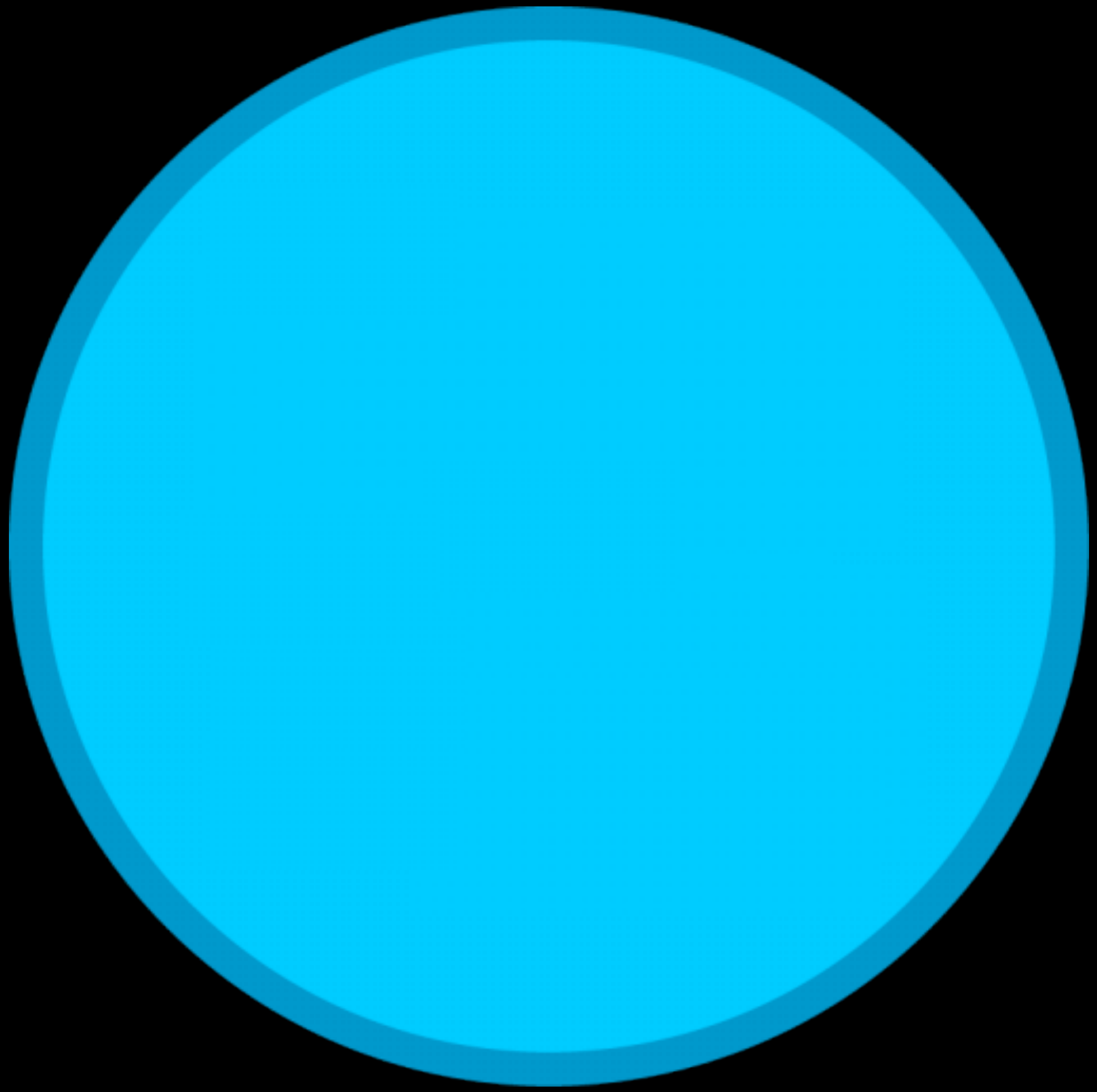

Let the hardware
support some tree.

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Let the human program against some tree.

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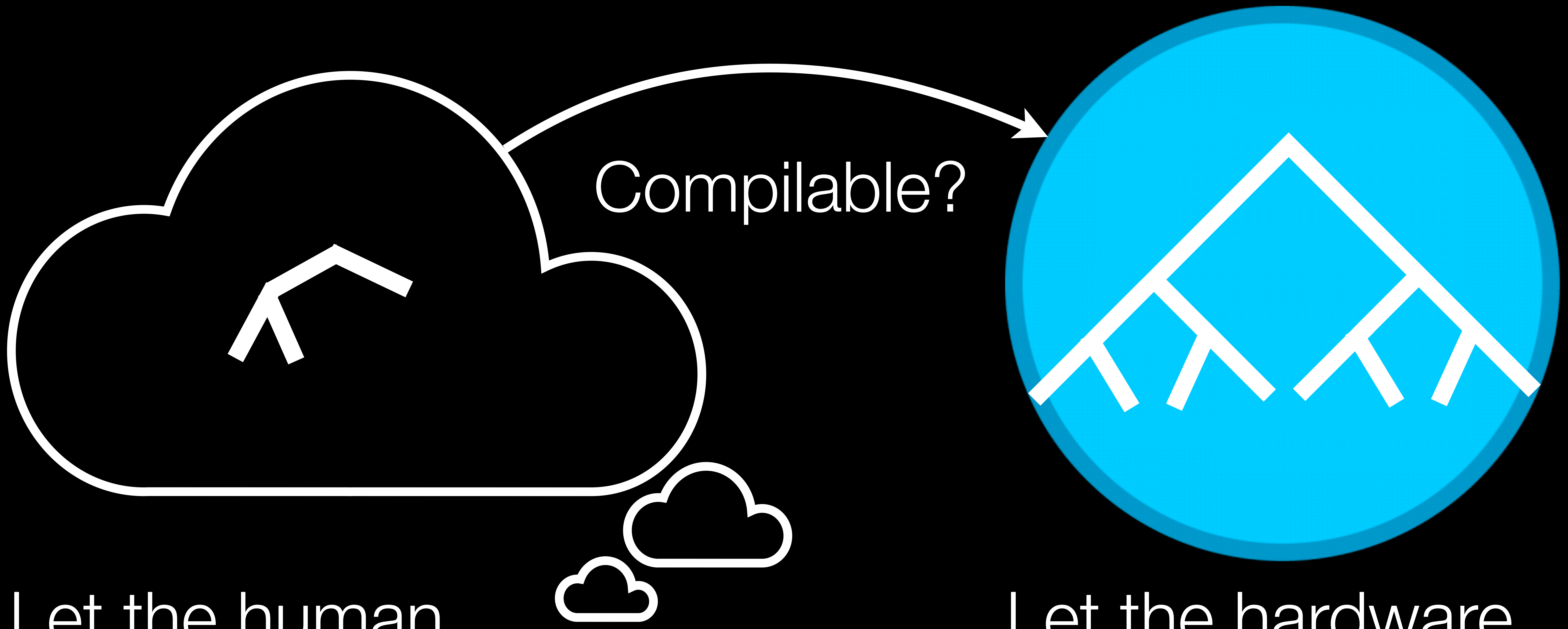

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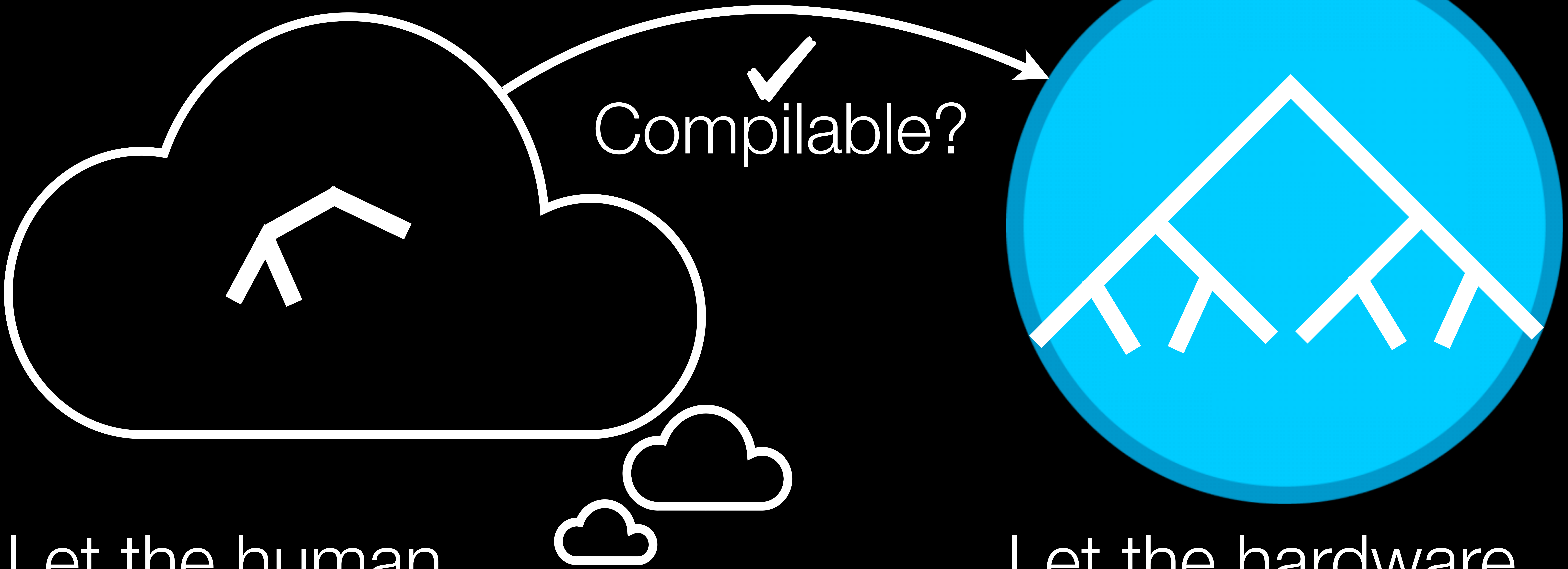

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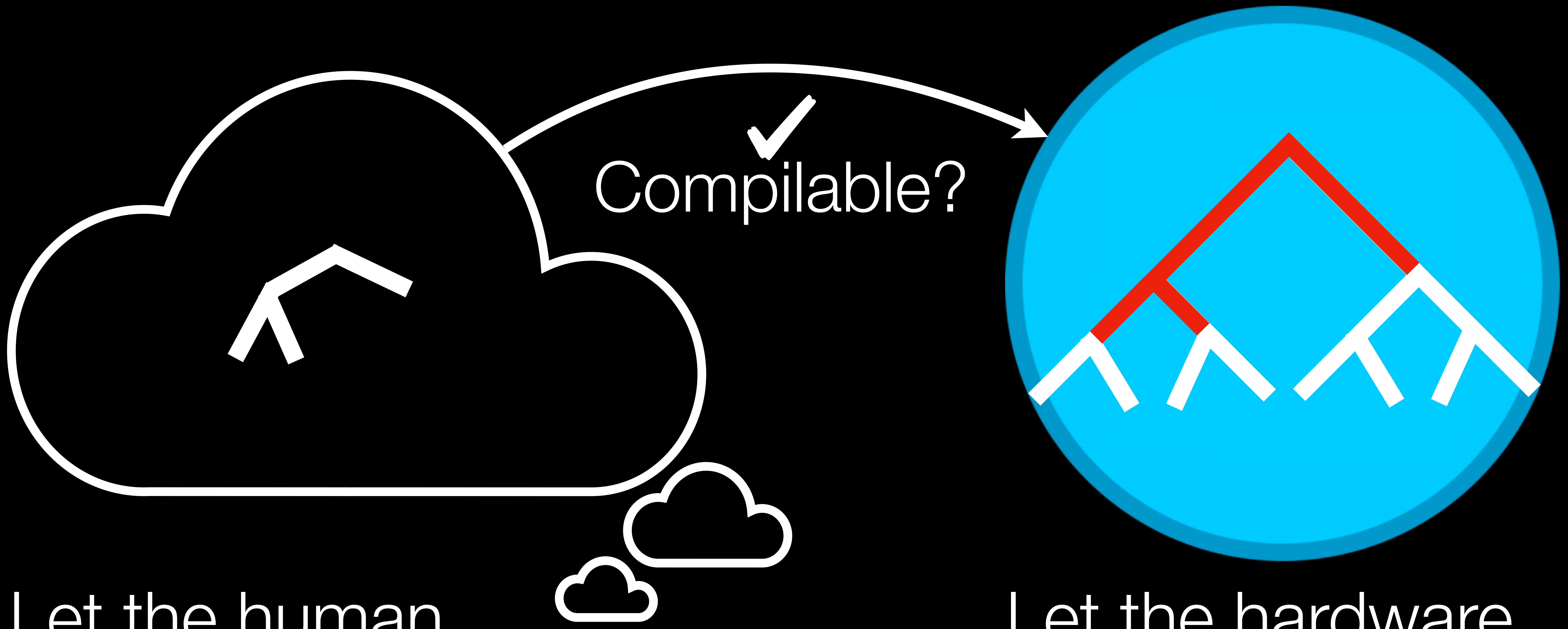

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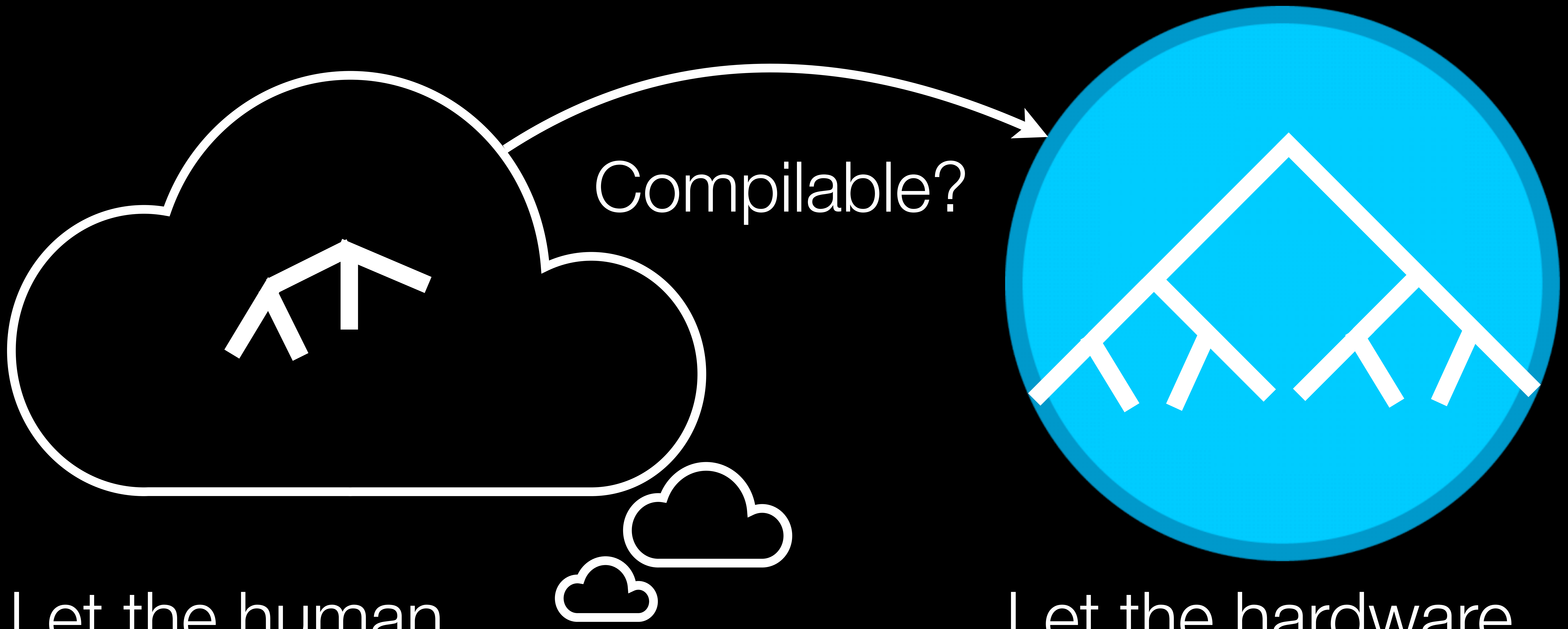

Compilable?

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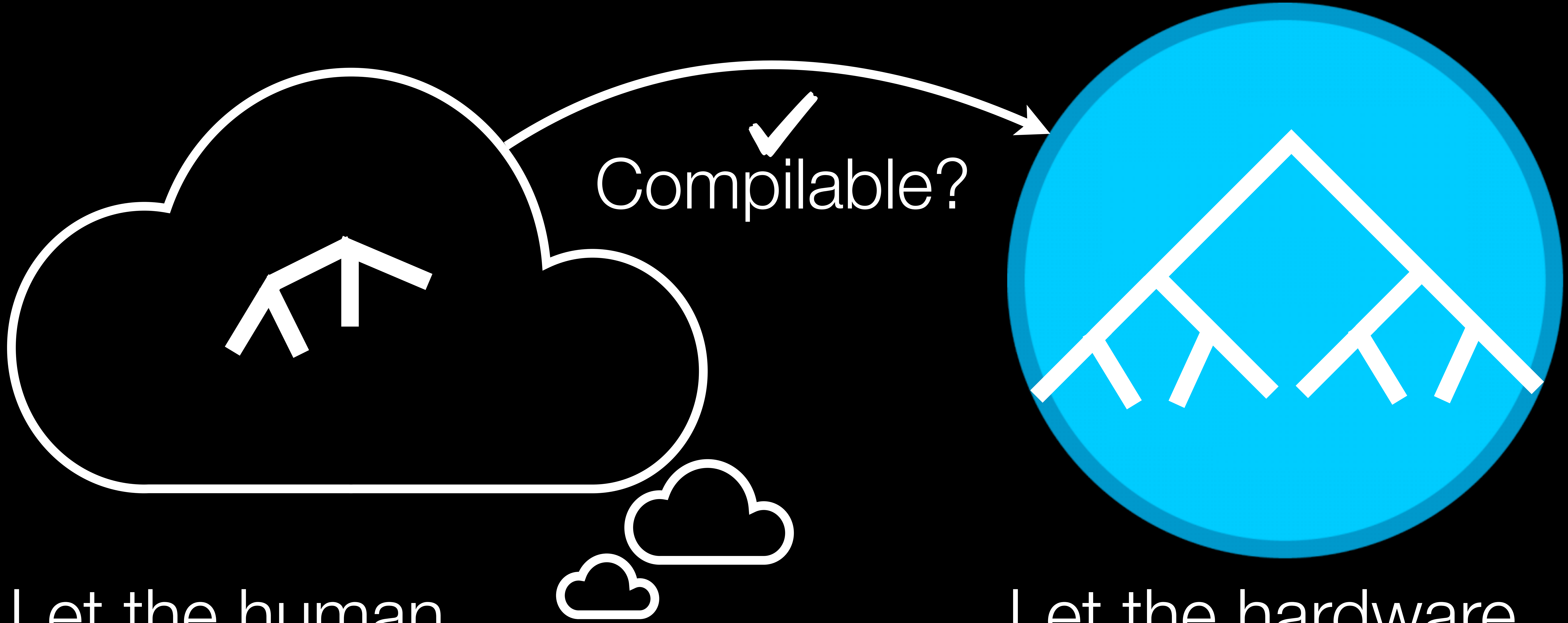

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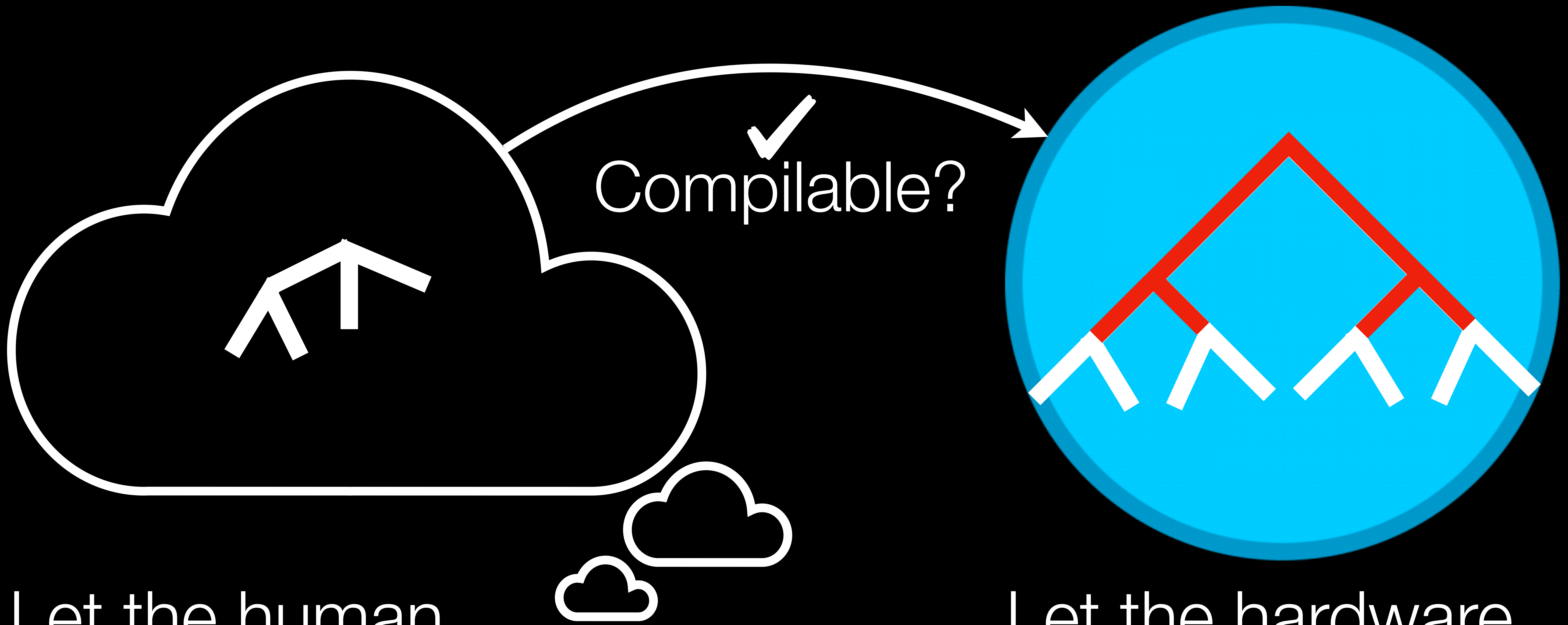

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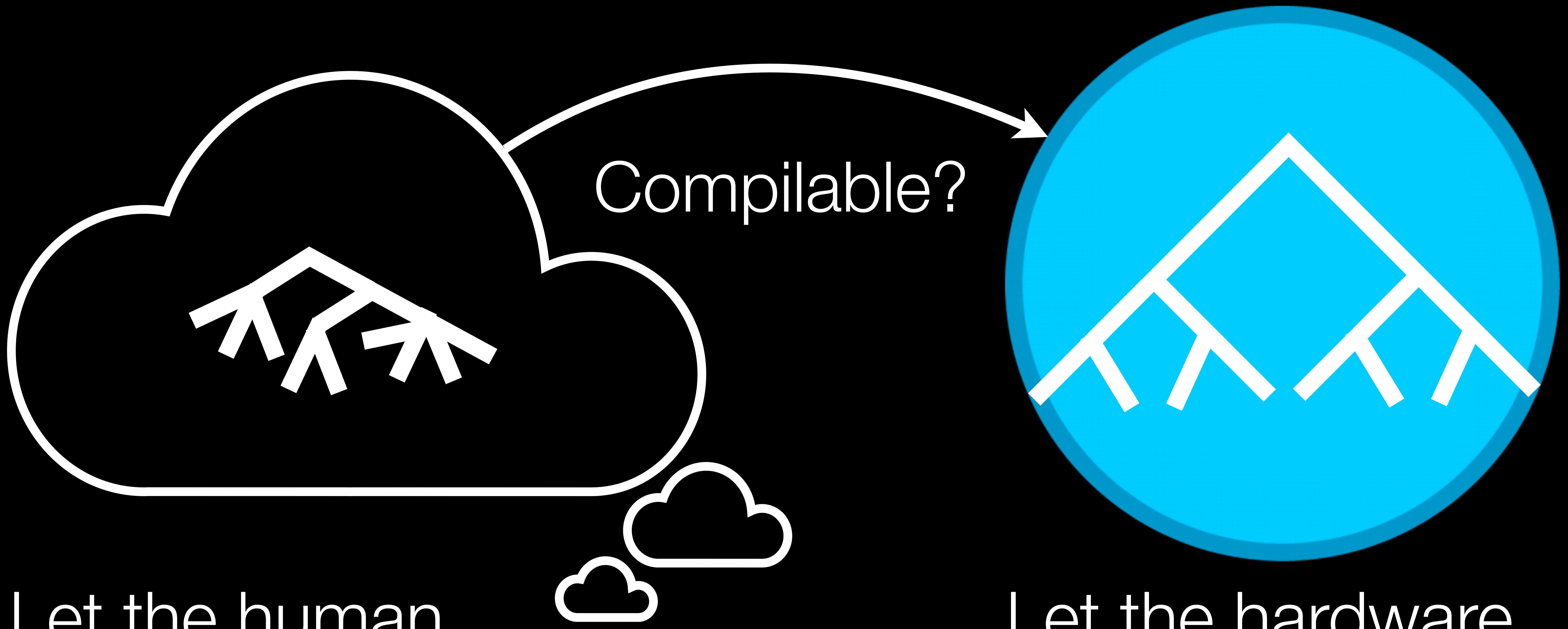

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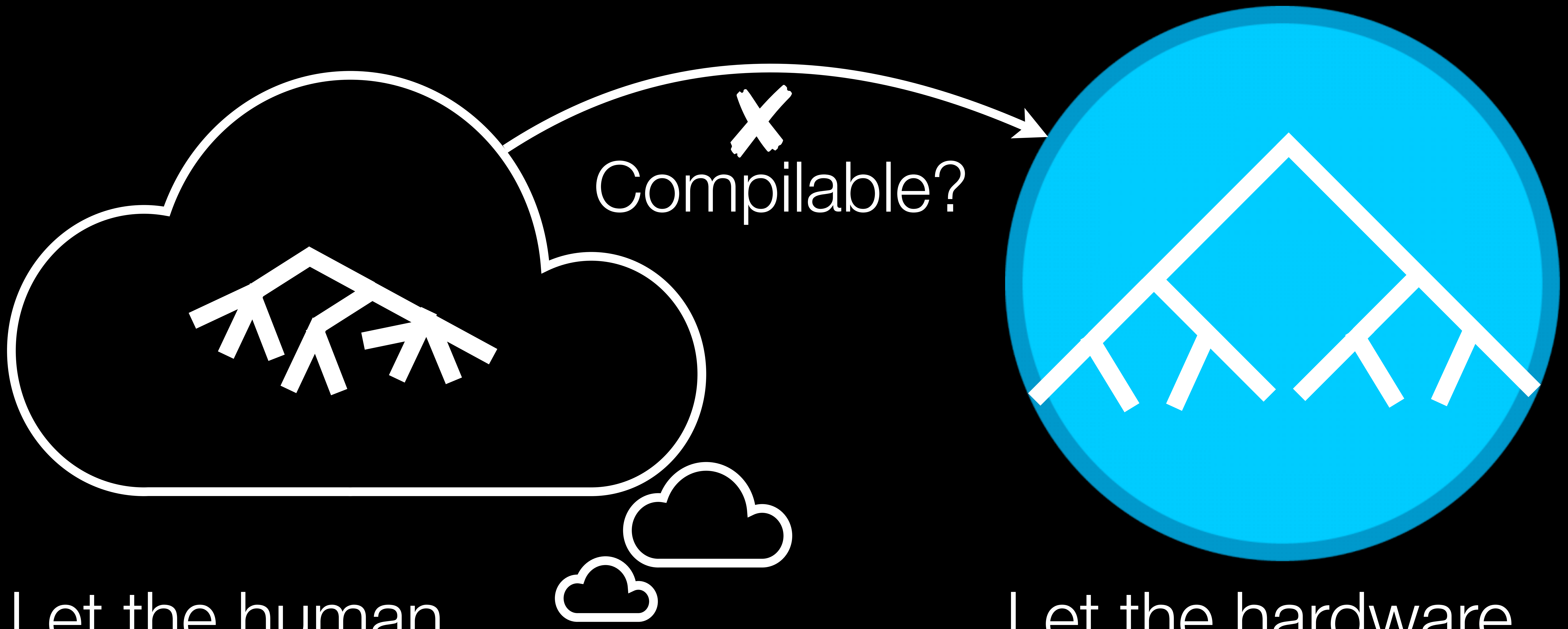

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Formal Abstractions for Packet Scheduling

Mohan, Liu, Foster, Kappé, Kozen

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